

(N)LSP Decays and Gravitino Dark Matter Relic Abundance in Big Divisor (nearly) SLagy $D3/D7$ μ -Split SUSY

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Abstract

Using the (nearly) Ricci-flat Swiss-Cheese metric of [1], in the context of a mobile space-time filling $D3$ -brane restricted to a nearly special Lagrangian sub-manifold (in the large volume limit, the pull-back of the Kähler form close to zero and the real part of the pull-back of $e^{-i\theta}$, $\theta = \frac{\pi}{2}$ times the nowhere-vanishing holomorphic three-form providing the volume form on the three-cycle) of the “big” divisor with (fluxed stacks of) space-time filling $D7$ -branes also wrapping the “big” divisor (corresponding to a local minimum), we provide an explicit identification of the electron and the u -quark, as well as their $SU(2)_L$ -singlet cousins, with fermionic super-partners of four Wilson line moduli; their superpartners turn out to be very heavy, the Higgsino-mass parameter turns out to be large, one obtains one light (with a mass of 125 GeV) and one heavy Higgs and the gluino is long lived (from a collider point of view) providing a possible realization of “ μ -Split Supersymmetry”. By explicitly calculating the lifetimes of decays of the co-NLSPs - the first generation squark/slepton and a neutralino - to the LSP - the gravitino - as well as gravitino decays, we verify that BBN constraints relevant to the former as well as the requirement of the latter to be (more than) the age of the universe, are satisfied. For the purpose of calculation of the gravitino relic density in terms of the neutralino/slepton relic density, we evaluate the latter by evaluating the neutralino/slepton (co-)annihilation cross sections and hence show that the former satisfies the requirement for a Dark Matter candidate.

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1 Introduction

It is challenging to provide a suitable choice of vacuum (local minimum) of string theory to infer various cosmological and phenomenological issues, and other fundamental physics. One of the most exciting unresolved issues in particle physics and cosmology is the nature of dark matter (DM) in the Universe. Over the years, astronomical and cosmological observations have put significant constraints on its expected properties. The recent Wilkinson Microwave Anisotropy Probe (WMAP) observations provide the relic abundance of a cold dark matter (CDM) [2] to be $\Omega_{CDM}h^2 = 0.1109 \pm 0.0056$. Theoretical status of DM generally emerges in the context of theories beyond Standard Model. It is well known that supersymmetric models with conserved R-parity contain one stable neutralino which is a candidate for cold dark matter. However, in models coupled to gravity and even in other scenarios of supersymmetry breaking mediation including gauge mediation, the gravitino (the supersymmetric partner of Graviton) stands out as probably the most natural and attractive candidate for the LSP and DM although it generically suffers from a well-known “cosmological gravitino problem” [3] the resolution of which depends on whether one considers gravitino to be LSP or unstable particle. The long-lived gravitino’s are generated in the early Universe. In the standard big-bang cosmology, they were in thermal equilibrium and then because of their weak gravitational interactions, frozen out while they were relativistic. In this case their abundance might overclose the Universe. Even if during inflationary epoch of the universe, primordial abundance of gravitino is completely diluted, the problem can not be solved because gravitinos are regenerated in the thermal bath after the reheating if the reheating temperature is high enough though overclosure of universe by dark matter constrain the exact value of reheating temperature [4, 5, 6]. However, in addition to it, the production of gravitino also depends on non-thermal production mechanism, the abundance of which is independent of the reheating temperature. i.e $\Omega_{3/2}^{total} = \Omega_{3/2}^{th} + \Omega_{3/2}^{NLSP}$. Generically in case of heavy gravitino, reheating temperature is low enough to produce appropriate abundance of Gravitino. Therefore sufficient number of gravitinos can be produced after NLSP decays to gravitino after its decoupling from the thermal plasma [7]. In other words, annihilation density does not depend on freeze out of gravitino (which causes a gravitino problem), instead depends on freeze out of NLSP. Hence relic density of gravitino is given by relic density of NLSP [7, 8] according to the relation $\Omega_{\tilde{G}} = \frac{m_{3/2}}{m_{\tilde{G}}} \Omega_{3/2}$. Therefore it is interesting to study the thermal cross-section and hence relic density of the (co-)NLSP(s) to get the the right order of relic density of gravitino.

In spirit of above discussion, the purpose of present work is to give the signatures of gravitino as a potential dark matter in the context of the large volume limit of type IIB (“big divisor”) $D3/D7$ Swiss cheese phenomenology. The consistent compactification scheme and phenomenological features of the model were initiated in [9, 10] and “ μ split-SUSY scenario”-like realization of the same was shown to be possible to be realized in [11] where in addition to getting one light Higgs (one of the primary motivation of μ split SUSY Scenario), we evaluated the life time of gluino which came out to be long and hence satisfied one of the important phenomenological features of μ split-SUSY scenario. In general, stability of LSP is governed by conserved R-parity. However, in this paper, we have considered cosmology of the gravitino LSP scenario based on R-parity violation. We argue that in addition to non-zero R-parity violating couplings, high squark masses helps to reduce the decay width and hence the lifetime of the gravitino decays become very long, typically of or larger than the age of the Universe. The explicit calculation of matrix amplitudes and hence life times of N(LSP) candidates requires the complete identification of the SM particles. Therefore we build up a set up which is able to provide an explicit identification of the first generation $SU(2)_L$ leptonic and quark doublets as well as their $SU(2)_L$ -singlets, with fermionic super-partners of four Wilson line moduli.

Further, soft SUSY parameters obtained in the context of gravity mediation provides sleptons and gaugino/lightest neutralino (being almost degenerate in mass) as co-NLSPs while gravitino naturally appears as Lightest Supersymmetric particle (LSP) which traditionally can be considered as viable dark matter candidate. The more explicit realization of the same requires life time of the LSP to be around and preferably more than the age of the universe.

The organization of the paper is as follows: In section **2**, we start off with details of an improved version of the large volume scenario set up discussed in [9]. Here, we construct four harmonic distribution one-forms supported on a sub-locus of big divisor localized along the mobile space-time filling $D3$ -brane, and by utilizing the geometric Kähler potential of [1] we show that the aforementioned sub-locus in the large volume limit is nearly a special Lagrangian sub-manifold of the “big” divisor. By calculating the intersection matrix valued in the Wilson line moduli sub-space $a_{I=1,\dots,h_-^{(0,1)}(\Sigma_B)}$, appearing in Kähler coordinate T_B , we write (the Kähler sector of the) Kähler potential that includes four Wilson line moduli a_1, a_2, a_3, a_4 and two position moduli of a mobile space-time filling $D3$ -brane restricted to the abovementioned (nearly) SLAG (corresponding to a local minimum); for the purposes of evaluation of “physical”/normalized Yukawa couplings, soft supersymmetry breaking parameters, etc., this Kähler potential is diagonalized to produce the $\mathcal{A}_I, \mathcal{Z}_i$ -basis. Further, the estimate of the Dirac mass terms appearing in the $\mathcal{N} = 1$ supergravity action of [12] calculated from superpotential and Kähler potential suggest that the fermionic superpartners of \mathcal{A}_1 and \mathcal{A}_3 correspond respectively to the first generation leptons: e_L and e_R , and the fermionic superpartners of \mathcal{A}_2 and \mathcal{A}_4 correspond respectively to the first generation quarks: u_L and u_R . We also provide explicit bi-fundamental representations for the four Wilson line moduli and two $D3$ -brane position moduli (after having turned on appropriate two-form fluxes on the $D7$ -brane world volume decomposing adjoint-valued matter fields into bi-fundamental matter fields) and speculate about the possible modification in the relevant $\mathcal{N} = 1$ chiral coordinate in the presence of the same. We also show that the (effective) physical Yukawas change only by $\mathcal{O}(1)$ under an RG-flow from the string to the EW scale. Building up on this identification, we have evaluated the contribution of various three-point vertices in the context of gauged $N = 1$ supergravity action, in section **3**. In **3.1**, we study decay width and life time of two-body and hadronic three-body decays of gauginos (NLSP), life times of which, except for gluino-to-Goldstino decay, ensure that energy release from both gauge boson and hadronic decays do not spoil the predictions arising from Big Bang Nucleosynthesis (BBN) and cosmic microwave background, etc.; gluino is hence ruled out as an NLSP. In **3.2-3.3**, we consider R-parity violating decays of the co-NLSPs - (Wino/Bino-dominated)neutralino and sleptons/squarks to ordinary particles (even R-parity) which are harmless because they rapidly decay to ordinary particles and do not disturb the beautiful predictions of BBN. In section **4**, in order to meet the requirement of an appropriate DM candidate, we calculate life time of three-body R-parity violating decays of gravitino LSP which comes out to be more than the age of the Universe. However, more exact determination requires that relic density of dark matter must be compatible with present observations and should not overclose the Universe. Therefore, in order to be able to perform a reliable comparison between theoretical predictions and improving measurements of the relic abundance from underground DM searches, in section **5**, we calculate the relic density of co-NLSP’s and hence relic abundance of gravitino. Strictly speaking, we consider very conservative approach where one assumes that decays of NLSP’s account for almost all of the dark matter of the Universe. Utilizing extensively the results of [13], [14] for providing the exact analytical expressions of various annihilation channels of neutralino and slepton, we obtain estimates of thermal cross-section and hence relic density of N(LSP) candidates. Section **6** has the conclusions. There are two Appendices; appendix A has some details of the geometric

Kähler potential calculation and appendix B has details of calculations of various soft supersymmetry breaking parameters and the fact that its possible to obtain around 10^2 GeV masses for W/Z vector gauge bosons and 125 GeV for the light Higgs.

2 Setup

From Sen's orientifold-limit-of-F-theory point of view corresponding to type IIB compactified on a Calabi-Yau three fold Z -orientifold with $O3/O7$ planes, one requires an elliptically fibered Calabi-Yau four-fold X_4 (with projection π) over a 3-fold $B_3(\equiv CY_3\text{-orientifold})$ where B_3 is taken be an n -twisted \mathbf{CP}^1 -fibration over \mathbf{CP}^2 such that pull-back of the divisors in CY_3 automatically satisfy Witten's unit-arithmetic genus condition. For $n = 6$ [15], the Calabi-Yau three-fold Z then turns out to be a unique Swiss-Cheese Calabi Yau in $\mathbf{WCP}^4_{[1,1,1,6,9]}[x_1 : x_2 : x_3 : x_4 : x_5]$ given by a smooth degree-18 hypersurface in $\mathbf{WCP}^4[1, 1, 1, 6, 9]$; the exceptional divisor corresponding to resolution of a \mathbf{Z}_3 -singularity $x_1 = x_2 = x_3 = 0$, via the monomial-divisor map, is encoded as the $\phi x_1^6 x_2^6 x_3^6$ (ϕ being an involutively odd complex structure modulus) polynomial deformation in the defining hypersurface. Also, throughout we will be working in the non-singular coordinate patch $x_2 = 1$ in the large volume limit of the Swiss-Cheese Calabi-Yau.

In the presence of a space-time filling $D3$ -brane and a space-time filling $D7$ -brane wrapping Σ_B , the Kähler potential relevant to all the calculations in this paper (without being careful about $\mathcal{O}(1)$ constant factors):

$$K \sim -2\ln \left[(\sigma_B + \bar{\sigma}_B - \gamma K_{\text{geom}})^{\frac{3}{2}} - (\sigma_S + \bar{\sigma}_S - \gamma K_{\text{geom}})^{\frac{3}{2}} \right] + \frac{\chi}{2} \sum_{m,n \in \mathbf{Z}^2/(0,0)} \frac{(\bar{\tau} - \tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3} - 4 \sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0 \sum_{m,n \in \mathbf{Z}^2/(0,0)} \frac{(\bar{\tau} - \tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3} \cos(mk \cdot \mathcal{B} + nk \cdot c) \Bigg], \quad (1)$$

will be rewritten in terms of the $\mathcal{N} = 1$ chiral coordinates: $T_{\alpha=B,S}, \mathcal{G}^{a=1,2}, \zeta^A, z^{i=1,2}$ (See [16, 9] for definitions of all) - in particular:

$$\sigma_\alpha \sim T_\alpha - \left(i\kappa_{abc} c^b \mathcal{B}^c + \kappa_\alpha + \frac{i}{(\tau - \bar{\tau})} \kappa_{abc} \mathcal{G}^b (\mathcal{G}^c - \bar{\mathcal{G}}^c) + i\delta_\alpha^B \kappa_4^2 \mu_7 l^2 C_\alpha^{I\bar{J}} a_I \bar{a}_{\bar{J}} + \frac{3i}{4} \delta_\alpha^B \tau Q_{\bar{f}} + i\mu_3 l^2 (\omega_\alpha)_{i\bar{j}} z^i (\bar{z}^{\bar{j}} - \frac{i}{2} \bar{z}^{\bar{a}} (\bar{\mathcal{P}}_{\bar{a}})^{\bar{j}} z^l) \right), \quad (2)$$

where

- κ_4 is related to four-dimensional Newton's constant, μ_3 and μ_7 are $D3$ and $D7$ -brane tensions;
- $\kappa_{\alpha ab}$'s are triple intersection integers of the CY orientifold;
- c^a and b^a are coefficients of RR and NS-NS two forms expanded in odd basis of $H_{\bar{\partial},-}^{(1,1)}(CY)$;
- $C_\alpha^{I\bar{J}} = \int_{\Sigma_B} i^* \omega_\alpha \wedge A^I \wedge A^{\bar{J}}$, $\omega_\alpha \in H_{\bar{\partial},+}^{(1,1)}(CY_3)$ and A^I forming a basis for $H_{\bar{\partial},-}^{(0,1)}(\Sigma^B)$ - immersion map is defined as: $i : \Sigma^B \hookrightarrow CY_3$, a_I is defined via a Kaluza-Klein reduction of the $U(1)$ gauge field (one-form) $A(x, y) = A_\mu(x) dx^\mu P_-(y) + a_I(x) A^I(y) + \bar{a}_{\bar{J}}(x) \bar{A}^{\bar{J}}(y)$, where $P_-(y) = 1$ if $y \in \Sigma^B$ and -1 if $y \in \sigma(\Sigma^B)$;

- $z^{\tilde{a}}, \tilde{a} = 1, \dots, h_-^{2,1}(CY_3)$, are $D = 4$ complex structure deformations of the CY orientifold, $(\mathcal{P}_{\tilde{a}})_j^i \equiv \frac{1}{\|\Omega\|^2} \bar{\Omega}^{ikl} (\chi_{\tilde{a}})_{klj}$, i.e., $\mathcal{P} : TCY_3^{(1,0)} \rightarrow TCY_3^{(0,1)}$ via the transformation: $z^i \xrightarrow{\text{c.s. deform}} z^i + \frac{i}{2} z^{\tilde{a}} (\mathcal{P}_{\tilde{a}})_j^i \bar{z}^{\tilde{j}}$, z^i are scalar fields corresponding to geometric fluctuations of $D3$ -brane inside the Calabi-Yau and defined via: $z(x) = z^i(x) \partial_i + \bar{z}^{\tilde{i}}(\bar{x}) \bar{\partial}_{\tilde{i}}$, and
- $Q_{\tilde{f}} \equiv l^2 \int_{\Sigma^B} \tilde{f} \wedge \tilde{f}$, where $\tilde{f} \in \tilde{H}_-^2(\Sigma^B) \equiv \text{coker} \left(H_-^2(CY_3) \xrightarrow{i^*} H_-^2(\Sigma^B) \right)$.

The closed string moduli-dependent Kähler potential, includes perturbative (using [17]) and non-perturbative (using [18]) α' -corrections. Written out in (discrete subgroup of) $SL(2, \mathbf{Z})$ (expected to survive orientifolding)-covariant form, the perturbative corrections are proportional to $\chi(CY_3)$ and non-perturbative α' corrections are weighted by $\{n_\beta^0\}$, the genus-zero Gopakumar-Vafa invariants that count the number of genus-zero rational curves $\beta \in H_2^-(CY_3, \mathbf{Z})$. In fact, the closed string moduli-dependent contributions are dominated by the genus-zero Gopakumar-Vafa invariants which using Castelnuovo's theory of moduli spaces can be shown to be extremely large for compact projective varieties [?] such as the one used.

Based on the study initiated in [10, 11], one us (AM), inspired by the Donaldson's algorithm - see [19] - had obtained in [1], an estimate of a nearly Ricci-flat Swiss Cheese metric for points finitely separated from Σ_B based on the following ansatz:

$$\begin{aligned}
K = \ln & \left[h^{z_4^2 \bar{z}_4^2} z_4^2 \bar{z}_4^2 + h^{z_4^2 \bar{z}_4^2} \sqrt[3]{V} z_4 \bar{z}_4 + h^{z_4^2 \bar{z}_4^2} V^{23/36} (z_1 + \bar{z}_1 + z_2 + \bar{z}_2) + h^{z_4^2 \bar{z}_4^2} V^{11/18} (z_1 \bar{z}_1 + z_2 \bar{z}_2 + z_1 \bar{z}_2 + z_2 \bar{z}_1) \right. \\
& + h^{z_4^2 \bar{z}_4^2} V^{11/18} (z_1^2 + z_2 z_1 + \bar{z}_1^2 + z_2^2 + \bar{z}_2^2 + \bar{z}_1 \bar{z}_2) + h^{z_4^2 \bar{z}_4^2} V^{7/12} (\bar{z}_2 z_1^2 + \bar{z}_1^2 z_1 + \bar{z}_2^2 z_1 + \bar{z}_1 z_2^2 + z_2 \bar{z}_2^2 + \bar{z}_1^2 z_2) \\
& + h^{z_4^2 \bar{z}_4^2} V^{5/9} (z_1^2 \bar{z}_1^2 + z_2^2 \bar{z}_1^2 + z_1^2 \bar{z}_2^2 + z_2^2 \bar{z}_2^2) + h^{z_4^2 \bar{z}_4^2} \sqrt{V} (z_4 + \bar{z}_4) + h^{z_4^2 \bar{z}_4^2} V^{17/36} (\bar{z}_1 z_4 + \bar{z}_2 z_4 + z_1 \bar{z}_4 + z_2 \bar{z}_4) \\
& + h^{z_4^2 \bar{z}_4^2} V^{4/9} (\bar{z}_4 z_1^2 + \bar{z}_1^2 z_4 + \bar{z}_2^2 z_4 + z_2^2 \bar{z}_4) + h^{z_4^2 \bar{z}_4^2} V^{5/12} (\bar{z}_1 \bar{z}_4 z_1^2 + \bar{z}_2 \bar{z}_4 z_1^2 + \bar{z}_1^2 z_4 z_1 + z_2 \bar{z}_2^2 z_4 + \bar{z}_1 z_2^2 \bar{z}_4 + z_2^2 \bar{z}_2 \bar{z}_4) \\
& + h^{z_4^2 \bar{z}_4^2} V^{5/18} (z_1 \bar{z}_1 z_4 \bar{z}_4 + \bar{z}_1 z_2 z_4 \bar{z}_4 + z_1 \bar{z}_2 z_4 \bar{z}_4 + z_2 \bar{z}_2 z_4 \bar{z}_4) + h^{z_4^2 \bar{z}_4^2} V^{11/36} ((z_1 + \bar{z}_1) z_4 \bar{z}_4 + (z_2 + \bar{z}_2) z_4 \bar{z}_4) \\
& + h^{z_4^2 \bar{z}_4^2} \sqrt[3]{V} (z_4^2 + \bar{z}_4^2) + h^{z_4^2 \bar{z}_4^2} V^{11/36} (z_4^2 \bar{z}_1 + z_1 \bar{z}_4^2 + z_2 \bar{z}_4^2 + \bar{z}_2 z_4^2) + h^{z_4^2 \bar{z}_4^2} V^{5/18} (\bar{z}_1^2 z_4^2 + \bar{z}_2^2 z_4^2 + z_1^2 \bar{z}_4^2 + z_2^2 \bar{z}_4^2) \\
& + h^{z_4^2 \bar{z}_4^2} \sqrt[6]{V} (\bar{z}_4 z_4^2 + \bar{z}_4^2 z_4) + h^{z_4^2 \bar{z}_4^2} V^{5/36} ((\bar{z}_1 + \bar{z}_2) \bar{z}_4 z_4^2 + (z_1 + z_2) \bar{z}_4^2 z_4) + h^{z_4^2 \bar{z}_4^2} V^{4/9} (z_1 \bar{z}_2 z_4 + \bar{z}_1 z_2 \bar{z}_4 \\
& \left. + |z_1|^2 (z_4 + \bar{z}_4) + z_1 \bar{z}_2 (z_4 + \bar{z}_4) + |z_2|^2 (z_4 + \bar{z}_4)) + \sqrt[3]{V} \right] \quad (3)
\end{aligned}$$

From (A3), one sees that $h^{11} \sim h^{z_4^2 \bar{z}_4^2} \mathcal{V}^{\frac{2}{3}}$. Using (25), one can show that:

$$R_{z_i \bar{z}_j} \sim \frac{\sum_{n=0}^8 a_n \left(h^{z_4^2 \bar{z}_4^2} \right)^n \mathcal{V}^{\frac{n}{3}}}{\left(1 + \mathcal{O}(1) h^{z_4^2 \bar{z}_4^2} \mathcal{V}^{\frac{1}{3}} \right)^2 \mathcal{V}^{\frac{1}{18}} \left(\sum_{n=0}^3 b_n \left(h^{z_4^2 \bar{z}_4^2} \right)^n \mathcal{V}^{\frac{n}{3}} \right)^2}. \quad (4)$$

Solving numerically: $\sum_{n=0}^8 a_n \left(h^{z_4^2 \bar{z}_4^2} \right)^n \mathcal{V}^{\frac{n}{3}} = 0$, as was assumed, one (of the eight values of) $h^{z_4^2 \bar{z}_4^2}$, up to a trivial Kähler transformation, turns out to be $\mathcal{V}^{-\frac{1}{3}}$, $\mathcal{V} \sim 10^6$. Curiously, from GLSM-based analysis, we had seen in [9] that on $\Sigma_B(z_4 = 0)$, the argument of the logarithm received the most dominant contribution from the FI-parameter $r_2 \sim \mathcal{V}^{\frac{1}{3}}$ which using this value of $h^{z_4^2 \bar{z}_4^2}$, is in fact, precisely what one obtains even now. Using this value of $h^{z_4^2 \bar{z}_4^2}$, one obtains: $R_{z_i \bar{z}_4}, R_{z_4 \bar{z}_4} \sim 10^{-1}$. Further, as has been assumed that the metric components $g_{z_1, 2 \bar{z}_4}$ are negligible as compared to $g_{z_i \bar{z}_j}$ - this was used in

[10] in showing the completeness of the basis spanning $H_-^{1,1}$ for a large volume holomorphic isometric involution restricted to (5), $z_1 \rightarrow -z_1, z_{2,3} \rightarrow z_{2,3}$, which is now born out explicitly, wherein the latter turn out to about 10% of the former. One does not need the aforementioned restriction on the geometric Swiss-Cheese metric for large volume involutions restricted to (5), of the following type. In the $u_2 \neq 0$ -coordinate patch, the defining degree-18 hypersurface in $\mathbf{WCP}_{1,1,1,6,9}^4[u_1 : u_2 : u_3 : u_4 : u_5]$ can be written as: $1 + z_1^{18} + z_2^{18} + z_3^3 - \psi z_1 z_2 z_3 z_4 - \phi z_1^6 z_2^6 = 0, z_1 = \frac{u_1}{u_2}, z_2 = \frac{u_3}{u_2}, z_3 = \frac{u_4}{u_1^6}, z_4 = \frac{u_5}{u_1^9}$.

The same can be rewritten as: $(iz_4)^2 - \psi z_1 z_2 z_3 z_4 = z_3^3 + \phi z_1^6 z_2^6 - z_1^{18} - z_2^{18}$. This is can therefore be thought of as the following Weierstrass variety:

$$\begin{aligned} T^2(z_3, z_4(z_3)) &\rightarrow \mathbf{WCP}_{1,1,1,6,9}^4(z_1, z_2, z_3) \\ &\downarrow \pi \\ &\mathbf{CP}^2(z_1, z_2) \end{aligned} \quad . \text{ Now,}$$

assuming that the complex structure modulus ψ has been stabilized to an infinitesimal value so that one can disregard the polynomial deformation proportional to ψ , and defining $\chi_{1,2} \equiv z_{1,2}^6$, this elliptic fibration structure can thought also of as:

$$\begin{aligned} (\text{Complex curve}) \mathcal{C}(\chi_1) &\longrightarrow \mathbf{CP}^2(\chi_1, \chi_2) \\ &\downarrow \pi' \\ &\mathbf{CP}^1(\chi_2) \end{aligned} \quad ,$$

corresponding to $(iz_4)^2 \approx z_3^3 + \phi \chi_1 \chi_2 - \chi_1^3 - \chi_2^3$. Alternatively, one can also think of the Weierstrass variety as:

$$\begin{aligned} T^2(-\chi_1, z_4(-\chi_1)) &\rightarrow \mathbf{WCP}_{1,1,1,6,9}^4(\chi_1, \chi_2, z_3) \\ &\downarrow \pi \\ (\text{Complex curve}) \mathcal{C}(-z_3) &\longrightarrow \mathbf{CP}^2(-z_3, \chi_2) \\ &\downarrow \pi' \\ &\mathbf{CP}^1(\chi_2) \end{aligned} \quad , \text{ corresponding to } (iz_4)^2 \approx$$

$(-\chi_1)^3 + \phi \chi_1 \chi_2 + z_3^3 - \chi_2^3$. Therefore, near (5), one can define a large volume holomorphic involution: $\chi_1 \leftrightarrow -z_3, \phi \rightarrow -\phi$ (the fact that complex structure modulus ϕ is involutively odd, is also used in (2)).

After $z_i \rightarrow z_i + \mathcal{V}^{\frac{1}{36}}$ and $a_I \rightarrow a_I + \langle a_I \rangle$, the EW symmetry is spontaneously broken to $U(1)_{\text{em}}$. We therefore require to construct four harmonic distribution one-forms A_I localized on a sub-locus of Σ_B :

$$C_3 : |z_1| \sim \mathcal{V}^{\frac{1}{36}}, |z_2| \sim \mathcal{V}^{\frac{1}{36}}, |z_3| \sim \mathcal{V}^{\frac{1}{6}}. \quad (5)$$

This is a toroidal three-cycle and the Calabi-Yau can be thought as a T^3 (swept out by $(arg z_1, arg z_2, arg z_3)$)-fibration over a large base $(|z_1|, |z_2|, |z_3|)$; precisely apt for application of mirror symmetry as three T-dualities a la S(trominger) Y(au) Z(aslow) [20]. Interestingly, (5) is almost a s(pecial) Lag(rangian) sub-manifold. The requirements for the same are that $f^*J = 0, f^*\Omega = e^{i\theta} \text{vol}(C_3)$, where $f : C_3 \rightarrow CY_3$ [21]. Let us see if these requirements hold up. Using the geometric Kähler potential of (25), the geometric metric near C_3 is estimated to be:

$$g_{i\bar{j}} \sim \frac{\mathcal{O}(10^{-1})}{2} \begin{pmatrix} \frac{1}{18\sqrt[3]{V}} & \frac{1}{18\sqrt[3]{V}} & \frac{1}{V^{7/36}} \\ \frac{1}{18\sqrt[3]{V}} & \frac{1}{18\sqrt[3]{V}} & \frac{1}{V^{7/36}} \\ \frac{1}{V^{7/36}} & \frac{1}{V^{7/36}} & \frac{1}{3\sqrt[3]{V}} \end{pmatrix}, \quad (6)$$

which justifies the assumption made in [10] about the block-diagonal form of $g_{i\bar{j}}$. The same was necessary in explicitly demonstrating $h_-^{1,1} \neq 0$ in [10] for the large volume involution: $z_1 \rightarrow -z_1, z_{2,3} \rightarrow z_{2,3}$. The most non-trivial example of involutions which are meaningful only at large volumes is mirror symmetry implemented as three T-dualities in [20] (Also see the first paragraph of section 4.3 of [22]

for implementation of SYZ proposal and importance of large base for doing so); mirror symmetry could be thought of as an involution since the mirror of a mirror is the original manifold.

1. Using (6), one sees that:

$$\begin{aligned} f^*(ds_6^2) &\sim 0.05 \left[\sum_{i,j=1}^2 \mathcal{V}^{-\frac{1}{18}} (dz_i d\bar{z}_j) + \mathcal{V}^{-\frac{1}{3}} |dz_3|^2 \right] \Big|_{C_3} \\ &\sim 0.05 \left[\sum_{i,j=1}^2 d(\arg z_i) d(\arg \bar{z}_j) + d(\arg z_3) d(\arg \bar{z}_3) \right]. \end{aligned} \quad (7)$$

This implies $f^*J \sim 0.05$, which is small.

2. In the $u_2 \neq 0$ - coordinate patch with $z_1 = \frac{u_1}{u_2}, z_2 = \frac{u_3}{u_2}, z_3 = \frac{u_4}{u_2^6}, z_4 = \frac{u_5}{u_2^6}$, by the Griffiths residue formula, one obtains:

$$\Omega = \frac{dz_1 \wedge dz_2 \wedge dz_4}{\frac{\partial P}{\partial z_3}} = \frac{dz_1 \wedge dz_2 \wedge dz_4}{3z_3^2 - \psi z_1 z_2 z_4}. \quad (8)$$

The coordinate z_4 can be solved for using:

$$z_4 = \frac{\psi z_1 z_2 z_3 \pm \sqrt{\psi^2 (z_1 z_2 z_3)^2 - 4(1 + z_1^{18} + z_2^{18} + z_3^3 - \phi(z_1 z_2)^6)}}{2} \sim \psi \mathcal{V}^{\frac{2}{9}} \pm i \mathcal{V}^{\frac{1}{4}}, \quad (9)$$

which implies that the T^3 will never degenerate as the roots will never coincide:

$$\begin{aligned} dz_4 &\sim dz_3 \left(\frac{\psi z_1 z_2}{2} \pm \frac{(2\psi^2 z_1^2 z_2^2 z_3 - 12z_3^2)}{4\sqrt{\psi^2 (z_1 z_2 z_3)^2 - 4(1 + z_1^{18} + z_2^{18} + z_3^3 - \phi(z_1 z_2)^6)}} \right) + (\dots) dz_1 + (\dots) dz_2 \\ &\sim \left(\frac{\psi z_1 z_2}{2} \pm 1.5i \frac{z_3^2}{\sqrt{z_1^{18} + z_2^{18} + z_3^3}} \right) dz_3 + (\dots) dz_1 + (\dots) dz_2, \text{ near (5), } |\psi| \ll 1, |\phi| \ll 1. \end{aligned} \quad (10)$$

Therefore, restricted to (5) and substituting (9) and (10) in (8), one obtains:

$$\begin{aligned} \Omega &\sim \frac{\left(\psi \mathcal{V}^{\frac{1}{18}} \pm 1.5i \mathcal{V}^{\frac{1}{12}} \right)}{3\mathcal{V}^{\frac{1}{3}} - \psi \mathcal{V}^{\frac{1}{18}} \left[\psi \mathcal{V}^{\frac{2}{9}} \pm i \mathcal{V}^{\frac{1}{4}} \right]} dz_1 \wedge dz_2 \wedge dz_3 \\ &\sim \left(\frac{\psi \mathcal{V}^{-\frac{5}{18}}}{3} \pm 0.5i \mathcal{V}^{-\frac{1}{4}} \right) dz_1 \wedge dz_2 \wedge dz_3. \end{aligned} \quad (11)$$

Hence, if one could estimate the pull-back of the nowhere vanishing holomorphic three-form as:

$$\begin{aligned} f^*\Omega &\sim \left(\frac{\psi}{3} \mathcal{V}^{-\frac{5}{18} + \frac{1}{18} + \frac{1}{6}} + 0.5i \mathcal{V}^{-\frac{1}{4} + \frac{1}{18} + \frac{1}{6}} \right) d(\arg z_1) \wedge d(\arg z_2) \wedge d(\arg z_3) \Big|_{\mathcal{V} \sim 10^6} \\ &\sim (0.2\psi \pm 0.3i) d(\arg z_1) \wedge d(\arg z_2) \wedge d(\arg z_3), \end{aligned} \quad (12)$$

and:

$$\begin{aligned}
\text{vol}(C_3) &\sim \sqrt{f^*g}f^*(dz_1 \wedge dz_2 \wedge dz_3) \\
&\sim \sqrt{(0.05)^3 \mathcal{V}^{\frac{1}{18} + \frac{1}{6}}} d(\arg z_1) \wedge d(\arg z_2) \wedge d(\arg z_3) \Big|_{\mathcal{V} \sim 10^6} \\
&\sim 0.2 d(\arg z_1) \wedge d(\arg z_2) \wedge d(\arg z_3),
\end{aligned} \tag{13}$$

then relative to a phase $e^{-i\theta}$, $\theta = \frac{\pi}{2}$, $\Im m(f^*e^{-i\theta}\Omega) \sim 0.2\psi d(\arg z_1) \wedge d(\arg z_2) \wedge d(\arg z_3)$, which for $|\psi| \ll 1$, is close to zero; (equivalently) $\text{Re}(f^*e^{-i\theta}\Omega) \sim \text{vol}(C_3)$. This implies that C_3 is nearly/almost a special Lagrangian sub-manifold [23].

Following our previous constructions in [9, 10], the harmonic distribution one-forms can be constructed by integrating:

$$dA_I = (P_{\Sigma_B}(z_{1,2,3}))^I dz_1 \wedge dz_2, \tag{14}$$

with $I = 1, 2(\text{done}), 3, 4$ where

$$A_I \sim \delta(|z_3| - \mathcal{V}^{\frac{1}{6}}) \delta(|z_1| - \mathcal{V}^{\frac{1}{36}}) \delta(|z_2| - \mathcal{V}^{\frac{1}{36}}) [\omega_I(z_1, z_2) dz_1 + \tilde{\omega}_I(z_1, z_2) dz_2]. \tag{15}$$

From (14), one sees that A_I is harmonic only on Σ_B and not at any other generic locus in the Calabi-Yau manifold; (15) shows that A_I are distribution one-forms on Σ_B localized along the $D3$ -brane which is localized on the three-cycle C_3 of (5).

Writing $A_I(z_1, z_2) = \omega_I(z_1, z_2) dz_1 + \tilde{\omega}_I(z_1, z_2) dz_2$ where $\omega(-z_1, z_2) = \omega(z_1, z_2)$, $\tilde{\omega}(-z_1, z_2) = -\tilde{\omega}(z_1, z_2)$ and $\partial_1 \tilde{\omega} = -\partial_2 \omega$, one obtains:

$$\begin{aligned}
A_1(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{18} z_2^{19} dz_1 + z_1^{19} z_2^{18} dz_2, \\
A_2(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -\left(\frac{z_2^{19}}{19} + z_1^{18} z_2\right) dz_1 + \left(\frac{z_1^{19}}{19} + z_2^{18} z_1\right) dz_2, \\
A_3(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -\left(\frac{z_2^{55}}{55} + 3\frac{z_2^{37}}{37}(\sqrt{\mathcal{V}} + z_1^{18}) + z_2(\sqrt{\mathcal{V}} + z_1^{18})^3 + \frac{z_2^{19}}{19}(3\mathcal{V} + 6\sqrt{\mathcal{V}}z_1^{18} + 3z_1^{36})\right) dz_1 + (1 \leftrightarrow 2) \\
&\sim -z_1^{18} z_2^{37} dz_1 + (1 \leftrightarrow 2), \\
A_4(z_1, z_2, z_3 \sim \mathcal{V}^{\frac{1}{6}}) &\sim -z_1^{36} z_2^{37} dz_1 + z_2^{36} z_1^{37} dz_2.
\end{aligned} \tag{16}$$

Hence,

$$C_{1\bar{1}} \sim \frac{1}{\mathcal{V}} \int z_1^{18} z_2^{19} \bar{z}_1^{18} \bar{z}_2^{19} \delta(|z_1| - \mathcal{V}^{\frac{1}{36}}) \delta(|z_2| - \mathcal{V}^{\frac{1}{36}}) dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2, \tag{17}$$

which using $dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2 \sim |z_1|d|z_1| \wedge d(\arg z_1) \wedge |z_2|d|z_2| \wedge d(\arg z_2)$ yields:

$$\begin{aligned}
C_{1\bar{1}} &\sim \frac{1}{\mathcal{V}} \int |z_1|^{36} |z_1| \delta(|z_1| - \mathcal{V}^{\frac{1}{36}}) d|z_1| \int |z_2|^{38} |z_2| \delta(|z_2| - \mathcal{V}^{\frac{1}{36}}) d|z_2| \\
&\sim \mathcal{V}^{\frac{37}{36} + \frac{39}{36} - 1} = \mathcal{V}^{\frac{10}{9}}; \\
C_{2\bar{2}} &\sim \frac{1}{\mathcal{V}} \int z_1^{18} z_2^{18} \bar{z}_1^{18} \bar{z}_2^{18} \delta(|z_1| - \mathcal{V}^{\frac{1}{36}}) \delta(|z_2| - \mathcal{V}^{\frac{1}{36}}) dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2 \\
&\sim \frac{1}{\mathcal{V}} \int |z_1|^{36} |z_1| \delta(|z_1| - \mathcal{V}^{\frac{1}{36}}) d|z_1| \int |z_2|^2 |z_2| \delta(|z_2| - \mathcal{V}^{\frac{1}{36}}) d|z_2| \\
&\sim \mathcal{V}^{\frac{37}{36} + \frac{3}{36} - 1} = \mathcal{V}^{\frac{1}{9}};
\end{aligned}$$

Lets now look at the term quadratic in the $D3$ -brane position moduli in $T_{\alpha=B,S}$ which is given by:

$$(\omega_\alpha)_{i\bar{j}} z^i \left(\bar{z}^{\bar{j}} - \frac{i}{2} (\mathcal{P}_{\bar{a}})^{\bar{j}}_{\bar{l}} \bar{z}^{\bar{a}} z^{\bar{l}} \right), \quad (19)$$

where

$$(\mathcal{P}_{\bar{a}})^{\bar{j}}_{\bar{l}} = \frac{\Omega^{\bar{j}\bar{k}\bar{m}} (\chi_{\bar{a}})_{\bar{k}\bar{m}\bar{l}}}{||\Omega||^2} \sim \frac{g^{j_1\bar{j}} g^{k_1\bar{k}} g^{m_1\bar{m}} \Omega_{j_1 k_1 m_1} (\chi_{\bar{a}})_{\bar{k}\bar{m}\bar{l}}}{g^{i_2\bar{i}_2} g^{j_3\bar{j}_3} g^{k_2\bar{k}_2} \Omega_{i_2 j_2 k_2} \bar{\Omega}_{\bar{i}_2 \bar{j}_2 \bar{k}_2}}. \quad (20)$$

As ω_α forms a basis of $H_+^{1,1}(CY_3)$ and $\chi_{\bar{a}}$ forms a basis of $H_-^{2,1}(CY_3)$, this does not therefore depend on the choice of the divisor α . By Griffith's residue formula, $\Omega = \Omega_{124} dz_1 \wedge dz_2 \wedge dz_4$ where

$$\Omega_{124} = \frac{1}{\frac{\partial P(z_1, z_2, z_3, z_4)}{\partial z_3}} = \frac{1}{3z_3^2 - \psi z_1 z_2 z_4}. \quad (21)$$

From $P(z_1, z_2, z_3, z_4) = 0$, one obtains:

$$z_3 \sim \frac{\psi z_1 z_2 z_4}{\left(- (z_1^{18} + z_2^{18} + z_4^2 - \phi z_1^6 z_2^6) + \sqrt{(\psi z_1 z_2 z_4)^3 + (z_1^{18} + z_2^{18} + z_4^2 - \phi z_1^6 z_2^6)^2} \right)^{\frac{1}{3}}} + \left(- (z_1^{18} + z_2^{18} + z_4^2 - \phi z_1^6 z_2^6) + \sqrt{(\psi z_1 z_2 z_4)^3 + (z_1^{18} + z_2^{18} + z_4^2 - \phi z_1^6 z_2^6)^2} \right)^{\frac{1}{3}}, \quad (22)$$

which for $z_{1,2} \sim \mathcal{V}^{\frac{1}{36}}, z_4 \sim \mathcal{V}^{\frac{1}{6}}$ yields $z_3 \sim \mathcal{V}^{\frac{1}{6}}$. Hence,

$$\Omega_{124} \sim \mathcal{V}^{-\frac{1}{3}}. \quad (23)$$

The polynomial deformation coefficient ψ is a complex structure deformation modulus and given that it must be of the type $z^{\bar{a}}$, i.e., odd under the holomorphic isometric involution $\sigma : z_1 \rightarrow -z_1, z_{2,4} \rightarrow z_{2,4}, \sigma : \psi \rightarrow -\psi$. So, given that $\sigma^* \Omega = -\Omega$, hence $\sigma : \Omega_{124} \rightarrow \Omega_{124}$.

Now,

$$\frac{1}{2} (\mathcal{P}_{\bar{a}})^{\bar{j}}_{\bar{l}} \sim \frac{(\chi_{\bar{a}})_{\bar{4}\bar{2}\bar{1}}}{\bar{\Omega}_{\bar{1}\bar{2}\bar{4}}} \sim \frac{\mathcal{V}^{\frac{1}{3}}}{2} \Big|_{\mathcal{V} \lesssim 10^6} \sim \mathcal{O}(10). \quad (24)$$

Assuming the complex structure moduli $z^{\bar{a}}$ are stabilized at values: $\mathcal{O}(10)z^{\bar{a}} \sim \mathcal{O}(1)$ for $\mathcal{V} \lesssim 10^6$, one sees that contribution quadratic in $z^i z^{\bar{j}}$ goes like $(\omega_\alpha)_{i\bar{j}} (z^i \bar{z}^{\bar{j}})$. $\omega_{B,S}$ are Poincare-duals of $\Sigma_{B,S}$ respectively. Hence, $\omega_{B,S} = \delta(P_{\Sigma_{B,S}}) dP_{\Sigma_{B,S}} \wedge \delta(\bar{P}_{\Sigma_{B,S}}) d\bar{P}_{\Sigma_{B,S}}$.

- Noting that $i_B^* dz_3 \sim \frac{\phi z_1^5 z_2^5 (z_2 dz_1 + z_1 dz_2) - (z_1^{17} dz_1 + z_2^{17} dz_2)}{(\phi z_1^6 z_2^6 - z_1^{18} - z_2^{18} - 1)^{\frac{2}{3}}} (i_B : \Sigma_B(1 + z_1^{18} + z_2^{18} + z_3^2 = \phi z_1^6 z_2^6) \hookrightarrow CY_3)$, which near $z_{1,2} \sim \mathcal{V}^{\frac{1}{36}}$ implies $dz_3 \sim \mathcal{V}^{\frac{5}{36}} (dz_1 + dz_2)$. Hence, near $z_{1,2} \sim \frac{\mathcal{V}^{\frac{1}{36}}}{\sqrt{2}}, \omega_B \sim \frac{\mathcal{V}^{\frac{17}{217}}}{2} (dz_1 + dz_2) \wedge (d\bar{z}_1 + d\bar{z}_2) \Big|_{\mathcal{V} \sim 10^6} \sim (dz_1 + dz_2) \wedge (d\bar{z}_1 + d\bar{z}_2)$.
- Noting that $i_S^* dz_4 \sim \frac{\phi z_1^5 z_2^5 (z_2 dz_1 + z_1 dz_2) - (z_1^{17} dz_1 + z_2^{17} dz_2)}{\sqrt{(\phi z_1^6 z_2^6 - z_1^{18} - z_2^{18} - 1)}} (i_S : \Sigma_S(1 + z_1^{18} + z_2^{18} + z_4^2 = \phi z_1^6 z_2^6) \hookrightarrow CY_3)$, which near $z_{1,2} \sim \mathcal{V}^{\frac{1}{36}}$ implies $dz_4 \sim \mathcal{V}^{\frac{2}{9}} (dz_1 + dz_2)$. Again, hence $z_{1,2} \sim \frac{\mathcal{V}^{\frac{1}{36}}}{\sqrt{2}}, \omega_S \sim \frac{\mathcal{V}^{\frac{17}{217}}}{2} (dz_1 + dz_2) \wedge (d\bar{z}_1 + d\bar{z}_2) \Big|_{\mathcal{V} \sim 10^6} \sim (dz_1 + dz_2) \wedge (d\bar{z}_1 + d\bar{z}_2)$.

We hence see that $(\omega_B)_{i\bar{j}} \sim (\omega_S)_{i\bar{j}} \sim \mathcal{O}(1)$ near $z_{1,2} \sim \frac{\mathcal{V}^{\frac{1}{36}}}{\sqrt{2}}$.

Therefore, near $c^a, b^a \sim \frac{\pi}{nk^a} < 1$ (in units of $M_p = 1$), $n \sim \mathcal{O}(1)$, $k^a \sim \mathcal{O}(10)$,

$$\begin{aligned} \frac{K}{M_p^2} \sim & -2\ln \left(\left[\frac{T_B + \bar{T}_B}{M_p} - \left(\mu_3 l^2 \frac{\{|z_1|^2 + |z_2|^2 + z_1 \bar{z}_2 + z_2 \bar{z}_1\}}{M_p^2} + \mathcal{V}^{\frac{10}{9}} \frac{|a_1|^2}{M_p^2} + \mathcal{V}^{\frac{11}{18}} \frac{(a_1 \bar{a}_2 + h.c.)}{M_p^2} + \mathcal{V}^{\frac{1}{9}} \frac{|a_2|^2}{M_p^2} + \mathcal{V}^{\frac{29}{18}} \frac{(a_1 \bar{a}_3 + h.c.)}{M_p^2} \right. \right. \\ & + \mathcal{V}^{\frac{10}{9}} \frac{(a_2 \bar{a}_3 + h.c.)}{M_p^2} + \mathcal{V}^{\frac{19}{9}} \frac{|a_3|^2}{M_p^2} + \mathcal{V}^{\frac{19}{9}} (a_1 \bar{a}_4 + a_4 \bar{a}_1) + \mathcal{V}^{\frac{29}{18}} (a_2 \bar{a}_4 + a_4 \bar{a}_2) + \mathcal{V}^{\frac{47}{18}} (a_3 \bar{a}_4 + a_4 \bar{a}_3) + \mathcal{V}^{\frac{28}{9}} |a_4|^2 \left. \right) \right]^{3/2} \\ & - \left(\frac{T_S + \bar{T}_S}{M_p} - \mu_3 l^2 \frac{\{|z_1|^2 + |z_2|^2 + z_1 \bar{z}_2 + z_2 \bar{z}_1\}}{M_p^2} \right)^{3/2} + \sum n_\beta^0(\dots) \Big), \end{aligned} \quad (25)$$

One can argue that near

$(|z_{1,2}| \sim \mathcal{V}^{\frac{1}{36}} M_p, |z_3| \sim \mathcal{V}^{\frac{1}{6}} M_p, |a_1| \sim \mathcal{V}^{-\frac{2}{9}} M_p, |a_2| \sim \mathcal{V}^{-\frac{1}{3}} M_p, |a_3| \sim \mathcal{V}^{-\frac{13}{18}} M_p, |a_4| \sim \mathcal{V}^{-\frac{11}{9}} M_p, \zeta^A = 0; \mathcal{G}^a \sim \frac{\pi}{\mathcal{O}(1)k^a(\sim \mathcal{O}(10))} M_p)$, one obtains a local meta-stable dS-like minimum corresponding to the positive minimum of the potential $e^K G^{T_S \bar{T}_S} |D_{T_S} W|^2$ stabilizing $\text{vol}(\Sigma_B) \sim \text{Re}(\sigma_B) \sim \mathcal{V}^{\frac{2}{3}}$, $\text{vol}(\Sigma_S) \sim \text{Re}\sigma_S \sim \mathcal{V}^{\frac{1}{18}}$ such that $\text{Re}(T)_S \sim \mathcal{V}^{\frac{1}{18}}$ and in the dilute flux approximation:

$$\begin{aligned} \frac{1}{g_{j=SU(3) \text{ or } SU(2)}^2} &= \text{Re}(T_B) + \ln \left(P(\Sigma_S)|_{D3|_{\Sigma_B}} \right) + \ln \left(\bar{P}(\Sigma_S)|_{D3|_{\Sigma_B}} \right) \\ &+ \mathcal{O} \left(\text{U}(1) - \text{Flux}_j^2 \right) \sim \mathcal{V}^{\frac{1}{18}}. \end{aligned} \quad (26)$$

Indeed, near the aforementioned stabilized values of the open string moduli,

$$\begin{aligned} |C_{1\bar{1}}|a_1|^2| &\sim \mathcal{V}^{\frac{2}{3}}, \quad |C_{1\bar{2}}(a_1 \bar{a}_2 + h.c.)| \sim \mathcal{V}^{\frac{1}{18}}, \quad |C_{1\bar{3}}(a_1 \bar{a}_3 + h.c.)| \sim \mathcal{V}^{\frac{2}{3}}, \\ |C_{1\bar{4}}(a_1 \bar{a}_4 + h.c.)| &\sim \mathcal{V}^{\frac{2}{3}}, \quad |C_{2\bar{2}}|a_2|^2| \sim \mathcal{V}^{-\frac{5}{9}}, \quad |C_{2\bar{3}}(a_2 \bar{a}_3 + h.c.)| \sim \mathcal{V}^{\frac{1}{18}}, \\ |C_{2\bar{4}}(a_2 \bar{a}_4 + h.c.)| &\sim \mathcal{V}^{\frac{1}{18}}, \quad |C_{3\bar{3}}|a_3|^2| \sim \mathcal{V}^{\frac{2}{3}}, \quad |C_{3\bar{4}}(a_3 \bar{a}_4 + h.c.)| \sim \mathcal{V}^{\frac{2}{3}}, \\ |C_{4\bar{4}}|a_4|^2| &\sim \mathcal{V}^{\frac{2}{3}}, \end{aligned} \quad (27)$$

we see that there is the possibility that:

$$\text{Vol}(\Sigma_B) + C_{I\bar{J}} a_I \bar{a}_{\bar{J}} + h.c. \sim \mathcal{V}^{\frac{1}{18}}, \quad (28)$$

and hence (26) could be implemented.

To obtain the “physical”/normalized Yukawa couplings, soft SUSY breaking parameters, vertices later in the paper, etc., one needs to diagonalize the matrix generated from the mixed double derivative of the Kähler potential, the same given as under:

$$\begin{pmatrix} a_{11} \frac{1}{\mathcal{V}^{2/3}} & a_{12} \frac{1}{\mathcal{V}^{2/3}} & a_{13} \frac{1}{\mathcal{V}^{5/12}} & a_{14} \frac{1}{\mathcal{V}^{11/12}} & a_{15} \frac{1}{\sqrt[12]{\mathcal{V}}} & a_{16} \mathcal{V}^{7/12} \\ a_{12} \frac{1}{\mathcal{V}^{2/3}} & a_{22} \frac{1}{\mathcal{V}^{2/3}} & a_{23} \frac{1}{\mathcal{V}^{5/12}} & a_{24} \frac{1}{\mathcal{V}^{11/12}} & a_{25} \frac{1}{\sqrt[12]{\mathcal{V}}} & a_{26} \mathcal{V}^{7/12} \\ a_{13} \frac{1}{\mathcal{V}^{5/12}} & a_{23} \frac{1}{\mathcal{V}^{5/12}} & a_{33} \mathcal{V}^{4/9} & a_{34} \frac{1}{\sqrt[18]{\mathcal{V}}} & a_{35} \mathcal{V}^{17/18} & a_{36} \mathcal{V}^{13/9} \\ a_{14} \frac{1}{\mathcal{V}^{11/12}} & a_{24} \frac{1}{\mathcal{V}^{11/12}} & a_{34} \frac{1}{\sqrt[18]{\mathcal{V}}} & a_{44} \frac{1}{\mathcal{V}^{5/9}} & a_{45} \mathcal{V}^{4/9} & a_{46} \mathcal{V}^{17/18} \\ a_{15} \frac{1}{\sqrt[12]{\mathcal{V}}} & a_{25} \frac{1}{\sqrt[12]{\mathcal{V}}} & a_{35} \mathcal{V}^{17/18} & a_{45} \mathcal{V}^{4/9} & a_{55} \mathcal{V}^{13/9} & a_{56} \mathcal{V}^{35/18} \\ a_{16} \mathcal{V}^{7/12} & a_{26} \mathcal{V}^{7/12} & a_{36} \mathcal{V}^{13/9} & a_{46} \mathcal{V}^{17/18} & a_{56} \mathcal{V}^{35/18} & a_{66} \mathcal{V}^{22/9} \end{pmatrix} \quad (29)$$

With $\mathcal{V} \sim 10^6$ and some fine tuning in $a_{11,12,22,14,13,23,24,36,44,46,56,66}$, the numerical eigenvalues are estimated to be:

$$10^{12}, 10^7, 10^4, 10^{-2}, 10^{-3}, 10^{-5}, \quad (30)$$

with the corresponding eigenvectors given by:

$$\begin{aligned} \mathcal{A}_4 &\sim a_4 + \mathcal{V}^{-\frac{3}{5}}a_3 + \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{9}{5}}a_2 + \mathcal{V}^{-2}(z_1 + z_2); \\ \mathcal{A}_3 &\sim -a_3 + \mathcal{V}^{-\frac{3}{5}}a_4 - \mathcal{V}^{-\frac{3}{5}}a_1 - \mathcal{V}^{-\frac{7}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}(z_1 + z_2); \\ \mathcal{A}_1 &\sim a_1 - \mathcal{V}^{-\frac{3}{5}}a_3 + \mathcal{V}^{-1}a_2 - \mathcal{V}^{-\frac{6}{5}}a_4 + \mathcal{V}^{-\frac{6}{5}}(z_1 + z_2); \\ \mathcal{A}_2 &\sim -a_2 - \mathcal{V}^{-1}a_1 + \mathcal{V}^{-\frac{7}{5}}a_3 - \mathcal{V}^{-\frac{3}{5}}(z_1 + z_2); \\ \mathcal{Z}_2 &\sim -\frac{(z_1 + z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{3}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}a_3 + \mathcal{V}^{-2}a_4; \\ \mathcal{Z}_1 &\sim \frac{(z_1 - z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}a_1 + \mathcal{V}^{-\frac{3}{5}}a_2 + \mathcal{V}^{-\frac{8}{5}}a_3 + \mathcal{V}^{-2}a_4. \end{aligned} \quad (31)$$

The system of equations (31) can be solved to yield:

$$\begin{aligned} z_1 &\sim \frac{(\mathcal{Z}_2 - \mathcal{Z}_1)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}\mathcal{A}_1 - \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_2 + \mathcal{V}^{-\frac{8}{5}}\mathcal{A}_3 + \mathcal{V}^{-2}\mathcal{A}_4; \\ z_2 &\sim -\frac{(\mathcal{Z}_2 + \mathcal{Z}_1)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}\mathcal{A}_1 - \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_2 + \mathcal{V}^{-\frac{8}{5}}\mathcal{A}_3 + \mathcal{V}^{-2}\mathcal{A}_4; \\ a_1 &\sim \mathcal{V}^{-\frac{6}{5}}\mathcal{Z}_1 + \mathcal{V}^{-\frac{7}{5}}\mathcal{Z}_2 - \mathcal{A}_1 - \frac{\mathcal{A}_2}{\mathcal{V}} + \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_3 + \mathcal{V}^{-\frac{6}{5}}\mathcal{A}_4; \\ a_2 &\sim \mathcal{V}^{-\frac{3}{5}}\mathcal{Z}_1 + \frac{\mathcal{Z}_2}{\mathcal{V}} - \frac{\mathcal{A}_1}{\mathcal{V}} + \mathcal{A}_2 - \mathcal{V}^{-\frac{7}{5}}\mathcal{A}_3 + \mathcal{V}^{-\frac{9}{5}}\mathcal{A}_4; \\ a_3 &\sim \mathcal{V}^{-\frac{7}{5}}\mathcal{Z}_1 + \mathcal{V}^{-\frac{9}{5}}\mathcal{Z}_2 - \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_1 - \mathcal{V}^{-\frac{7}{5}}\mathcal{A}_2 - \mathcal{A}_3 + \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_4; \\ a_4 &\sim \frac{\mathcal{Z}_1}{\mathcal{V}^2} + \mathcal{V}^{-\frac{11}{5}}\mathcal{Z}_2 + \frac{\mathcal{A}_1}{\mathcal{V}} - \mathcal{V}^{-\frac{9}{5}}\mathcal{A}_2 + \mathcal{V}^{-\frac{3}{5}}\mathcal{A}_3 + \mathcal{A}_4; \end{aligned} \quad (32)$$

The non-perturbative superpotential we will be using is given by:

$$W \sim \left(1 + z_1^{18} + z_2^{18} + (3\phi_0 z_1^6 z_2^6 - z_1^{18} - z_2^{18})^{\frac{2}{3}} - 3\phi_0 z_1^6 z_2^6\right)^{n^s} e^{-n^s \text{vol}(\Sigma_S) - \mu_3(\alpha_S z_1^2 + \beta_S z_2^2 + \gamma_S z_1 z_2)}. \quad (33)$$

For $n^s = 2$, $\text{vol}(\Sigma_S)$ and z_i were stabilized at around $\mathcal{V}^{\frac{1}{18}}$ and $\mathcal{V}^{\frac{1}{36}}$. Now, considering fluctuations in $z_i : z_i \rightarrow \mathcal{V}^{\frac{1}{36}} + \delta z_i$, with the fluctuations expressible in terms of $\delta \mathcal{Z}_{1,2}, \delta \mathcal{A}_{1,2,3,4}$ using (32), and with the understanding that $\mathcal{V}^{\frac{1}{18}}(1 + \alpha_S + \beta_S + \gamma_S) \sim \ln \mathcal{V}$, one obtains the following non-zero physical effective Yukawa couplings: $\hat{Y}_{C_i C_j C_k}^{\text{eff}} \equiv \frac{e^{\frac{K}{2}} Y_{C_i C_j C_k}^{\text{eff}}}{\sqrt{K_{C_i \bar{C}_i} K_{C_j \bar{C}_j} K_{C_k \bar{C}_k}}}$, C_i being an open string modulus which for us is $\delta \mathcal{Z}_{1,2}, \delta \mathcal{A}_{1,2,3,4}$, where, e.g., $Y_{\mathcal{Z}_i \mathcal{A}_{1/2} \mathcal{A}_{3/4}}^{\text{eff}}$ is the $\mathcal{O}(\mathcal{Z}_i)$ -coefficient in the following mass term in the

$\mathcal{N} = 1$ SUGRA action of [12]: $e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_{1/2}} D_{\bar{\mathcal{A}}_{3/4}} \bar{W} \bar{\chi}^{\mathcal{A}_{1/3}} \chi^{\mathcal{A}_{2/4}}$. Using (32), and:

$$\begin{aligned} e^{\frac{K}{2}} \mathcal{D}_{z_1} D_{z_1} W &= \mathcal{V}^{-\frac{43}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{z_1} D_{a_1} W = \mathcal{V}^{-\frac{89}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{z_1} D_{a_2} W = \mathcal{V}^{-\frac{125}{72}}, \\ e^{\frac{K}{2}} \mathcal{D}_{z_1} D_{a_3} W &= \mathcal{V}^{-\frac{25}{36}}, \quad e^{\frac{K}{2}} \mathcal{D}_{z_1} D_{a_4} W = \mathcal{V}^{-\frac{17}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_1} D_{a_1} W = \mathcal{V}^{-\frac{71}{72}}, \\ e^{\frac{K}{2}} \mathcal{D}_{a_1} D_{a_2} W &= \mathcal{V}^{-\frac{107}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_1} D_{a_3} W = \mathcal{V}^{-\frac{35}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_1} D_{a_4} W = \mathcal{V}^{-\frac{1}{72}}, \\ e^{\frac{K}{2}} \mathcal{D}_{a_2} D_{a_2} W &= \mathcal{V}^{-\frac{1}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_2} D_{a_3} W = \mathcal{V}^{-\frac{3}{4}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_2} D_{a_4} W = \mathcal{V}^{-\frac{5}{9}}, \\ e^{\frac{K}{2}} \mathcal{D}_{a_3} D_{a_3} W &= \mathcal{V}^{-\frac{1}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_3} D_{a_4} W = \mathcal{V}^{-\frac{37}{72}}, \quad e^{\frac{K}{2}} \mathcal{D}_{a_4} D_{a_4} W = \mathcal{V}^{-\frac{73}{72}}, \end{aligned} \quad (34)$$

one can verify that $e^{\frac{K}{2}} \mathcal{D}_{\mathcal{A}_I} D_{\mathcal{A}_J} \bar{W} \sim e^{\frac{K}{2}} \mathcal{D}_{a_I} D_{a_J} \bar{W}$. Now, $\left(e^{\frac{K}{2}} \mathcal{D}_{\bar{\lambda}} D_{\bar{\Sigma}} \bar{W} \right) \bar{\chi}_L^\lambda \chi_R^\Sigma = \left(e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{I}}} D_{\bar{\mathcal{J}}} \bar{W} \right) \bar{\chi}_L^{\mathcal{I}} \chi_R^{\mathcal{J}}$, where λ/Σ index $\mathcal{A}_I, \mathcal{Z}_J$ and \mathcal{I}/\mathcal{J} index a_I, z_i . What is interesting is that using (31), (32), e.g.,

$$\begin{aligned} \left(e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_{1,2}} D_{\bar{\mathcal{A}}_{3,4}} \bar{W} \right) \bar{\chi}_L^{\mathcal{A}_{1,2}} \chi_R^{\mathcal{A}_{3,4}} &\sim \left(e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_{1,2}} D_{\bar{a}_{3,4}} \bar{W} \right) \bar{\chi}_L^{\mathcal{A}_{1,2}} \chi_R^{\mathcal{A}_{3,4}} = \left(\frac{\partial \bar{\mathcal{A}}_{1,2}}{\partial \mathcal{M}_{\mathcal{I}}} \right) \left(\frac{\partial \bar{\mathcal{A}}_{3,4}}{\partial \mathcal{M}_{\mathcal{J}}} \right) \left(e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_{1,2}} D_{\bar{a}_{3,4}} \bar{W} \right) \bar{\chi}_L^{\mathcal{M}_{\mathcal{I}}} \chi_R^{\mathcal{M}_{\mathcal{J}}} \\ &\sim \left(e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_{1,2}} D_{\bar{a}_{3,4}} \bar{W} \right) \bar{\chi}_L^{a_{1,2}} \chi_R^{a_{3,4}}. \end{aligned} \quad (35)$$

This implies that the mass terms, in the large volume limit, are invariant under diagonalization of the open string moduli.

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$$\frac{\mathcal{O}(\mathcal{Z}_i) \text{ term in } e^{\frac{K}{2}} \mathcal{D}_{\mathcal{A}_1} D_{\mathcal{A}_3} W}{\sqrt{K_{\mathcal{Z}_i \bar{\mathcal{Z}}_i} K_{\mathcal{A}_1 \bar{\mathcal{A}}_1} K_{\mathcal{A}_3 \bar{\mathcal{A}}_3}}} \equiv \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} \sim 10^{-3} \times \mathcal{V}^{-\frac{4}{9}}, \quad (36)$$

which implies that for $\mathcal{V} \sim 10^5$, $\langle \mathcal{Z}_i \rangle \sim 246 \text{ GeV}$, $\langle \mathcal{Z}_i \rangle \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_3} \sim \text{MeV}$ - about the mass of the electron!

•

$$\frac{\mathcal{O}(\mathcal{Z}_i) \text{ term in } e^{\frac{K}{2}} \mathcal{D}_{\mathcal{A}_2} D_{\mathcal{A}_4} W}{\sqrt{K_{\mathcal{Z}_i \bar{\mathcal{Z}}_i} K_{\mathcal{A}_2 \bar{\mathcal{A}}_2} K_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \equiv \hat{Y}_{\mathcal{Z}_i \mathcal{A}_2 \mathcal{A}_4}^{\text{eff}} \sim 10^{-\frac{5}{2}} \times \mathcal{V}^{-\frac{4}{9}}, \quad (37)$$

which implies that for $\mathcal{V} \sim 10^5$, $\langle \mathcal{Z}_i \rangle \sim 246 \text{ GeV}$, $\langle \mathcal{Z}_i \rangle \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_2} \sim 10 \text{ MeV}$ - close to the mass of the up quark!

There is an implicit assumption that the vev of the $D3$ -brane position moduli, identified with the neutral components of two Higgs doublets, can RG-flow down to 246 GeV - that the same is possible was shown in [10]. The RG-flow of the effective physical Yukawas are expected to change by $\mathcal{O}(1)$ under an RG flow from the string scale down to the EW scale. This can be motivated by looking at RG-flows of the physical Yukawas $\hat{Y}_{\Lambda \Sigma \Delta}$ in MSSM-like models:

$$\frac{d\hat{Y}_{\Lambda \Sigma \Delta}}{dt} = \gamma_{\Lambda}^{\kappa} \hat{Y}_{\kappa \Sigma \Delta} + \gamma_{\Sigma}^{\kappa} \hat{Y}_{\Lambda \kappa \Delta} + \gamma_{\Delta}^{\kappa} \hat{Y}_{\Lambda \Sigma \kappa}, \quad (38)$$

where the anomalous dimension matrix $\gamma_{\Lambda}^{\kappa}$, at one loop, is defined as:

$$\gamma_{\Lambda}^{\kappa} = \frac{1}{32\pi^2} \left(\hat{Y}_{\Lambda \Psi \Upsilon} \hat{Y}_{\kappa \Psi \Upsilon}^* - 2 \sum_{(a)} g_{(a)}^2 C_{(a)}(\Phi_{\Lambda}) \delta_{\mathcal{I}}^{\kappa} \right), \quad (39)$$

where (a) indexes the three gauge groups and Λ , etc. index the diagonalized basis fields $\mathcal{A}_I, \mathcal{Z}_i$. Using (33), one can show that at M_s (string scale):

$$\begin{aligned}
\hat{Y}_{\mathcal{Z}_i \mathcal{Z}_j \mathcal{Z}_k} &\sim \mathcal{V}^{-\frac{1}{8}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{A}_1} \sim \mathcal{V}^{-\frac{79}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{A}_2} \sim \mathcal{V}^{-\frac{31}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{A}_4} \sim \mathcal{V}^{-\frac{143}{40}}, \\
\hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{A}_3} &\sim \mathcal{V}^{-\frac{107}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_1} \sim \mathcal{V}^{-\frac{163}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_2} \sim \mathcal{V}^{-\frac{23}{8}}, \\
\hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_3} &\sim \mathcal{V}^{-\frac{191}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_1 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{227}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_2 \mathcal{A}_2} \sim \mathcal{V}^{-\frac{67}{40}}, \\
\hat{Y}_{\mathcal{Z}_i \mathcal{A}_2 \mathcal{A}_3} &\sim \mathcal{V}^{-\frac{143}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_2 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{179}{40}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_3 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{219}{40}}, \\
\hat{Y}_{\mathcal{Z}_i \mathcal{A}_3 \mathcal{A}_4} &\sim \mathcal{V}^{-\frac{51}{8}}, \quad \hat{Y}_{\mathcal{Z}_i \mathcal{A}_4 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{55}{8}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_1 \mathcal{A}_1} \sim \mathcal{V}^{-\frac{247}{40}}, \\
\hat{Y}_{\mathcal{A}_1 \mathcal{A}_1 \mathcal{A}_2} &\sim \mathcal{V}^{-\frac{199}{40}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_1 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{55}{8}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_1 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{311}{40}}, \\
\hat{Y}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_2} &\sim \mathcal{V}^{-\frac{151}{40}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{227}{40}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_2 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{263}{40}}, \\
\hat{Y}_{\mathcal{A}_2 \mathcal{A}_4 \mathcal{A}_5} &\sim \mathcal{V}^{-\frac{339}{40}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_4 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{75}{8}}, \quad \hat{Y}_{\mathcal{A}_2 \mathcal{A}_2 \mathcal{A}_2} \sim \mathcal{V}^{-\frac{103}{40}}, \\
\hat{Y}_{\mathcal{A}_2 \mathcal{A}_2 \mathcal{A}_3} &\sim \mathcal{V}^{-\frac{179}{40}}, \quad \hat{Y}_{\mathcal{A}_2 \mathcal{A}_2 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{43}{8}}, \quad \hat{Y}_{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{299}{60}}, \\
\hat{Y}_{\mathcal{A}_2 \mathcal{A}_3 \mathcal{A}_4} &\sim \mathcal{V}^{-\frac{263}{40}}, \quad \hat{Y}_{\mathcal{A}_4 \mathcal{A}_3 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{367}{40}}, \quad \hat{Y}_{\mathcal{A}_1 \mathcal{A}_3 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{283}{40}}, \\
\hat{Y}_{\mathcal{A}_2 \mathcal{A}_4 \mathcal{A}_4} &\sim \mathcal{V}^{-\frac{327}{40}}, \quad \hat{Y}_{\mathcal{A}_3 \mathcal{A}_4 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{403}{40}}, \quad \hat{Y}_{\mathcal{A}_3 \mathcal{A}_3 \mathcal{A}_3} \sim \mathcal{V}^{-\frac{331}{40}}, \quad \hat{Y}_{\mathcal{A}_4 \mathcal{A}_4 \mathcal{A}_4} \sim \mathcal{V}^{-\frac{439}{40}}.
\end{aligned} \tag{40}$$

From (40), one sees that $\hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{Z}_i}(M_s) \sim \mathcal{V}^{-\frac{1}{8}}$ is the most dominant physical Yukawa at the string scale. Hence,

$$\gamma_\Lambda^\Gamma \hat{Y}_{\Gamma \Sigma \Delta} \sim \frac{1}{32\pi^2} \hat{Y}_{\Lambda \mathcal{Z}_i \mathcal{Z}_i} \hat{Y}_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{Z}_i}^* \hat{Y}_{\mathcal{Z}_i \Sigma \Delta} - \frac{2 \sum_{(a)} g_{(a)}^2 C_{(a)}(\Phi_\Gamma) \delta_\Lambda^\Gamma \hat{Y}_{\Gamma \Sigma \Delta}}{32\pi^2}. \tag{41}$$

Now, the first term in (41) is volume suppressed as compared to the second term at M_s ; let us assume that the same will be true up to the EW scale. Using the one-loop solution for $\frac{g_{(a)}^2(t)}{16\pi^2} = \frac{\frac{\beta_{(a)}}{b_{(a)}}}{1+\beta_{(a)}t}$ in (41) and therefore (38), one obtains:

$$\frac{d \ln \hat{Y}_{\Lambda \Sigma \Delta}}{dt} \sim -2 \left(\sum_{(a)} \frac{C_{(a)}(\Lambda) \frac{\beta_{(a)}}{b_{(a)}}}{1 + \beta_{(a)}t} + \sum_{(a)} \frac{C_{(a)}(\Sigma) \frac{\beta_{(a)}}{b_{(a)}}}{1 + \beta_{(a)}t} + \sum_{(a)} \frac{C_{(a)}(\Delta) \frac{\beta_{(a)}}{b_{(a)}}}{1 + \beta_{(a)}t} \right), \tag{42}$$

whose solution yields:

$$\hat{Y}_{\Lambda \Sigma \Delta}(t) \sim \hat{Y}_{\Lambda \Sigma \Delta}(M_s) \prod_{(a)=1}^3 (1 + \beta_{(a)}t)^{\frac{-2(C_{(a)}(\Lambda) + C_{(a)}(\Sigma) + C_{(a)}(\Delta))}{b_{(a)}}}. \tag{43}$$

The solution (43) justifies the assumption that all $\hat{Y}_{\Lambda \Sigma \Delta}$ s change only by $\mathcal{O}(1)$ as one RG-flows down from the string to the EW scale.

A similar argument for the RG-evolution of $\hat{Y}_{\mathcal{Z}_i \mathcal{A}_I \mathcal{A}_J}^{\text{eff}}$ s would proceed as follows. We will for definiteness and due to relevance to the preceding discussion on lepton and quark masses, consider $\frac{d\hat{Y}_{\mathcal{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}}}{dt}$ and $\frac{d\hat{Y}_{\mathcal{Z}_1 \mathcal{A}_2 \mathcal{A}_4}^{\text{eff}}}{dt}$. Now,

$$\frac{d\hat{Y}_{\mathcal{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}}}{dt} = \gamma_{\mathcal{Z}_1}^\Lambda \hat{Y}_{\Lambda \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} + \gamma_{\mathcal{A}_1}^\Lambda \hat{Y}_{\mathcal{Z}_1 \Lambda \mathcal{A}_3}^{\text{eff}} + \gamma_{\mathcal{A}_3}^\Lambda \hat{Y}_{\mathcal{Z}_1 \mathcal{A}_1 \Lambda}^{\text{eff}}, \tag{44}$$

where:

$$\gamma_{\bar{Z}_1}^\Lambda \ni \hat{Y}_{Z_1 \Gamma \Sigma} \hat{Y}_{\Lambda \Gamma \Sigma}^* \sim \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{\Lambda Z_1 Z_1}^*, \text{ etc.}, \quad (45)$$

implying:

$$\begin{aligned} \frac{d\hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}}}{dt} \ni & \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{Z_1 Z_1 Z_1}^* \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} + \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{\mathcal{A}_1 Z_1 Z_1}^* \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} + \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{\mathcal{A}_2 Z_1 Z_1}^* \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} \\ & \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{\mathcal{A}_3 Z_1 Z_1}^* \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} + \hat{Y}_{Z_1 Z_1 Z_1} \hat{Y}_{\mathcal{A}_4 Z_1 Z_1}^* \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}}, \end{aligned} \quad (46)$$

and a similar equation for $\frac{d\hat{Y}_{\bar{Z}_1 \mathcal{A}_2 \mathcal{A}_4}^{\text{eff}}}{dt}$. Using (40) and:

$$\begin{aligned} e^{\frac{K}{2}} \mathcal{D}_{a_{1/2}} \mathcal{D}_{a_{3/4}} W \ni & \mathcal{V}^{-\frac{4}{9}} \left(z_i - \mathcal{V}^{\frac{1}{36}} \right) + \mathcal{V}^{\frac{2}{3}} \left(a_1 - \mathcal{V}^{-\frac{2}{9}} \right) + \mathcal{V}^{-\frac{5}{6}} \left(a_2 - \mathcal{V}^{-\frac{1}{3}} \right) \\ & + \mathcal{V}^{\frac{1}{6}} \left(a_3 - \mathcal{V}^{-\frac{13}{18}} \right) + \mathcal{V}^{\frac{7}{6}} \left(a_4 - \mathcal{V}^{-\frac{11}{9}} \right), \end{aligned} \quad (47)$$

and (46), one sees that at M_s :

$$\begin{aligned} \gamma_{\bar{Z}_1}^\Lambda \hat{Y}_{\Lambda \mathcal{A}_1 \mathcal{A}_3}^{\text{eff}} + \gamma_{\mathcal{A}_1}^\Lambda \hat{Y}_{\bar{Z}_1 \Lambda \mathcal{A}_3}^{\text{eff}} + \gamma_{\mathcal{A}_3}^\Lambda \hat{Y}_{\bar{Z}_1 \mathcal{A}_1 \Lambda}^{\text{eff}} \ni & \frac{\mathcal{V}^{-\frac{1}{8} - \frac{1}{8} - \frac{4}{9}}}{\sqrt{K_{Z_1 \bar{Z}_1} K_{\mathcal{A}_1 \bar{\mathcal{A}}_1} K_{\mathcal{A}_3 \bar{\mathcal{A}}_3}}} \Big|_{\mathcal{V} \sim 10^5} \sim \mathcal{V}^{-\frac{233}{180}}, \\ \gamma_{\bar{Z}_1}^\Lambda \hat{Y}_{\Lambda \mathcal{A}_2 \mathcal{A}_4}^{\text{eff}} + \gamma_{\mathcal{A}_2}^\Lambda \hat{Y}_{\bar{Z}_1 \Lambda \mathcal{A}_4}^{\text{eff}} + \gamma_{\mathcal{A}_4}^\Lambda \hat{Y}_{\bar{Z}_1 \mathcal{A}_2 \Lambda}^{\text{eff}} \ni & \frac{\mathcal{V}^{-\frac{1}{8} - \frac{1}{8} - \frac{4}{9}}}{\sqrt{K_{Z_1 \bar{Z}_1} K_{\mathcal{A}_2 \bar{\mathcal{A}}_2} K_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \Big|_{\mathcal{V} \sim 10^5} \sim \mathcal{V}^{-\frac{43}{36}}. \end{aligned} \quad (48)$$

Hence, at the string scale, the anomalous dimension matrix contribution is sub-dominant as compared to the gauge coupling-dependent contribution, and by a similar assumption (justified by the solution below)/reasoning:

$$\hat{Y}_{\Lambda \Sigma \Delta}^{\text{eff}}(t) \sim \hat{Y}_{\Lambda \Sigma \Delta}^{\text{eff}}(M_s) \prod_{(a)=1}^3 (1 + \beta_{(a)} t)^{\frac{-2(C_{(a)}(\Lambda) + C_{(a)}(\Sigma) + C_{(a)}(\Delta))}{b_{(a)}}}. \quad (49)$$

This suggests that possibly, the fermionic superpartners of \mathcal{A}_1 and \mathcal{A}_3 correspond respectively to e_L and e_R and the fermionic superpartners of \mathcal{A}_2 and \mathcal{A}_4 correspond respectively to the first generation u_L and u_R .

We will consider four stacks of $D7$ -branes: a stack of 3, a stack of 2 and two stacks of 1. The matter fields: L-quarks and their superpartners will be valued in the bifundamentals $(3, \bar{2})$ under $SU(3)_c \times SU(2)_L$; the L-leptons and their superpartners will be valued in the bifundamentals $(2, \bar{1})$ of $SU(2)_L \times U(1)_Y$. Before the fluxes (50) are turned on, the Wilson line moduli are valued in the adjoint of $U(7)$. With the following choice of fluxes:

$$F = \begin{pmatrix} f_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & f_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & f_4 \end{pmatrix}, \quad (50)$$

the $U(7)$ is broken down to $U(3) \times U(2) \times U(1) \times U(1)$. So, the bifundamental Wilson line super-moduli \mathcal{A}_I will be represented as:

$$\mathcal{A}_I = \sum a_I^{ab} e_{ab} + \sum \theta \tilde{a}_I^{ab} e_{ab}, \quad [(a, b) = 1, \dots, 7], \quad (51)$$

where $(e_{ab})_{ij} = \delta_{ai} \delta_{bj}$. Hence,

$$\mathcal{A}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\nu}_e + \theta \nu_e & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{e} + \theta e & 0 \\ 0 & 0 & 0 & \tilde{\nu}_e + \bar{\theta} \bar{\nu}_e & \tilde{e} + \bar{\theta} \bar{e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \quad (52)$$

$$\mathcal{A}_2 = \begin{pmatrix} 0 & 0 & 0 & \tilde{u} + \theta u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{u} + \bar{\theta} u^\dagger & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (53)$$

and

$$\mathcal{A}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{e}_R + \theta e_R \\ 0 & 0 & 0 & 0 & 0 & \tilde{e}_R + \bar{\theta} e_R^\dagger & 0 \end{pmatrix}. \quad (54)$$

$$\mathcal{A}_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \tilde{u}_R + \theta u_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{u}_R + \bar{\theta} u_R^\dagger & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (55)$$

Unlike the \mathcal{A}_I which correspond to matter fields corresponding to open strings stretched between two stacks of $D7$ branes (with different two-form fluxes turned on their world volumes), the Higgses would arise as a geometric moduli corresponding to the fluctuations in the position of the mobile space-time filling $D3$ -brane.

Now, we will see if one can construct appropriate a_I (bi-fundamental) and z_i (mimicking bi-

fundamental fields) such that (31), (52) - (55), as well as:

$$\mathcal{Z}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_u + \theta \tilde{H}_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{H}_u + \bar{\theta} \tilde{H}_u^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (56)$$

and

$$\mathcal{Z}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_d + \theta \tilde{H}_d & 0 \\ 0 & 0 & 0 & 0 & \bar{H}_d + \bar{\theta} \tilde{H}_d^\dagger & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (57)$$

are satisfied. Guided by (51), we will make the following ansatze for a_I and z_i :

$$a_I = \begin{pmatrix} 0 & 0 & 0 & a_I^{14} & a_I^{15} & a_I^{16} & a_I^{17} \\ 0 & 0 & 0 & a_I^{24} & a_I^{25} & a_I^{26} & a_I^{27} \\ 0 & 0 & 0 & a_I^{34} & a_I^{35} & a_I^{36} & a_I^{37} \\ \bar{a}_I^{14} & \bar{a}_I^{24} & \bar{a}_I^{34} & 0 & 0 & a_I^{46} & a_I^{47} \\ \bar{a}_I^{15} & \bar{a}_I^{25} & \bar{a}_I^{35} & 0 & 0 & a_I^{56} & a_I^{57} \\ \bar{a}_I^{16} & \bar{a}_I^{26} & \bar{a}_I^{36} & \bar{a}_I^{46} & \bar{a}_I^{56} & 0 & a_I^{67} \\ \bar{a}_I^{17} & \bar{a}_I^{27} & \bar{a}_I^{37} & \bar{a}_I^{47} & \bar{a}_I^{57} & \bar{a}_I^{67} & 0 \end{pmatrix},$$

$$z_i = \begin{pmatrix} 0 & 0 & 0 & z_i^{14} & z_i^{15} & z_i^{16} & z_i^{17} \\ 0 & 0 & 0 & z_i^{24} & z_i^{25} & z_i^{26} & z_i^{27} \\ 0 & 0 & 0 & z_i^{34} & z_i^{35} & z_i^{36} & z_i^{37} \\ \bar{z}_i^{14} & \bar{z}_i^{24} & \bar{z}_i^{34} & 0 & 0 & z_i^{46} & z_i^{47} \\ \bar{z}_i^{15} & \bar{z}_i^{25} & \bar{z}_i^{35} & 0 & 0 & z_i^{56} & z_i^{57} \\ \bar{z}_i^{16} & \bar{z}_i^{26} & \bar{z}_i^{36} & \bar{z}_i^{46} & \bar{z}_i^{56} & 0 & z_i^{67} \\ \bar{z}_i^{17} & \bar{z}_i^{27} & \bar{z}_i^{37} & \bar{z}_i^{47} & \bar{z}_i^{57} & \bar{z}_i^{67} & 0 \end{pmatrix}. \quad (58)$$

Using (58) in (31), one obtains $17 \times 6 = 102$ equations in 102 variables. The same can be solved (using

Mathematica) to yield the following solution:

$$\begin{aligned}
a_1 &= \begin{pmatrix} 0 & 0 & 0 & \xi_1^{14} \mathcal{V}^{-\frac{7}{5}} \tilde{u}_L & 0 & 0 & \xi_1^{17} \mathcal{V}^{-\frac{11}{5}} \tilde{u}_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^{14} \mathcal{V}^{-\frac{7}{5}} \tilde{u}_L & 0 & 0 & 0 & 0 & \xi_1^{46} \left(\frac{\tilde{e}_L}{2} + \mathcal{V}^{-\frac{8}{5}} H_u \right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi_1^{56} \mathcal{V}^{-\frac{8}{5}} H_d & 0 \\ 0 & 0 & 0 & \bar{\xi}_1^{46} \left(\frac{\tilde{e}_L}{2} + \mathcal{V}^{-\frac{8}{5}} \bar{H}_u \right) & \bar{\xi}_1^{56} \mathcal{V}^{-\frac{8}{5}} \bar{H}_d & 0 & -\xi_1^{67} \mathcal{V}^{-\frac{4}{5}} e_R \\ \bar{\xi}_1^{17} \mathcal{V}^{-\frac{11}{5}} \tilde{u}_R & 0 & 0 & 0 & 0 & -\bar{\xi}_1^{67} \mathcal{V}^{-\frac{4}{5}} \bar{e}_R & 0 \end{pmatrix}, \\
a_2 &= \begin{pmatrix} 0 & 0 & 0 & 0.3 \xi_2^{14} \tilde{u}_L & 0 & 0 & \xi_2^{17} \mathcal{V}^{-\frac{13}{5}} \tilde{u}_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 \xi_2^{14} \tilde{u}_L & 0 & 0 & 0 & 0 & \xi_2^{46} \left(-\mathcal{V}^{-\frac{7}{5}} \tilde{e}_L + \mathcal{V}^{-\frac{4}{5}} H_u \right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \xi_2^{56} \mathcal{V}^{-\frac{6}{5}} H_d & 0 \\ 0 & 0 & 0 & \bar{\xi}_2^{46} \left(-\mathcal{V}^{-\frac{7}{5}} \bar{e}_L + \mathcal{V}^{-\frac{4}{5}} \bar{H}_u \right) & 5 \bar{\xi}_2^{56} \mathcal{V}^{-\frac{6}{5}} \bar{H}_d & 0 & -\xi_2^{67} \mathcal{V}^{-\frac{9}{5}} e_R \\ \bar{\xi}_2^{17} \mathcal{V}^{-\frac{13}{5}} \tilde{u}_R & 0 & 0 & 0 & 0 & -\bar{\xi}_2^{67} \mathcal{V}^{-\frac{9}{5}} \bar{e}_R & 0 \end{pmatrix}, \\
a_3 &= \begin{pmatrix} 0 & 0 & 0 & \xi_3^{14} \frac{\tilde{u}_L}{\mathcal{V}} & 0 & 0 & -6 \xi_3^{17} \frac{\tilde{u}_R}{\mathcal{V}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\xi}_3^{14} \frac{\tilde{u}_L}{\mathcal{V}} & 0 & 0 & 0 & 0 & \xi_3^{46} \left(-\mathcal{V}^{-\frac{4}{5}} \tilde{e}_L + \mathcal{V}^{-\frac{8}{5}} H_u \right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi_3^{56} \mathcal{V}^{-\frac{11}{5}} H_d & 0 \\ 0 & 0 & 0 & \bar{\xi}_3^{46} \left(-\mathcal{V}^{-\frac{4}{5}} \bar{e}_L + \mathcal{V}^{-\frac{8}{5}} \bar{H}_u \right) & 5 \bar{\xi}_3^{56} \mathcal{V}^{-\frac{11}{5}} \bar{H}_d & 0 & 0.3 \xi_3^{67} e_R \\ \bar{\xi}_3^{17} \mathcal{V}^{-\frac{13}{5}} \tilde{u}_R & 0 & 0 & 0 & 0 & 0.3 \bar{\xi}_3^{67} \bar{e}_R & 0 \end{pmatrix}, \\
a_4 &= \begin{pmatrix} 0 & 0 & 0 & 0.08 \xi_4^{14} \tilde{u}_L & 0 & 0 & 0.2 \xi_4^{17} \tilde{u}_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.08 \bar{\xi}_4^{14} \tilde{u}_L & 0 & 0 & 0 & 0 & \xi_4^{46} \mathcal{V}^{-\frac{7}{5}} \tilde{e}_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi_4^{56} \mathcal{V}^{-\frac{14}{5}} H_d & 0 \\ 0 & 0 & 0 & \bar{\xi}_4^{46} \mathcal{V}^{-\frac{7}{5}} \bar{e}_L & \bar{\xi}_4^{56} \mathcal{V}^{-\frac{14}{5}} \bar{H}_d & 0 & -7 \xi_4^{67} e_R \\ 0.2 \bar{\xi}_4^{17} \tilde{u}_R & 0 & 0 & 0 & 0 & -7 \bar{\xi}_4^{67} \bar{e}_R & 0 \end{pmatrix}, \tag{59}
\end{aligned}$$

and

$$\begin{aligned}
z_1 = & \begin{pmatrix} 0 & 0 & 0 & \alpha_1^{14} \frac{\tilde{u}_L}{\mathcal{V}} & 0 & 0 & 5\alpha_1^{17} \mathcal{V}^{-\frac{14}{5}} \tilde{u}_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\alpha}_1^{14} \frac{\tilde{u}_L}{\mathcal{V}} & 0 & 0 & 0 & 0 & \alpha_1^{46} \left(\mathcal{V}^{-\frac{9}{5}} \tilde{e}_L - \frac{H_u}{\sqrt{2}} \right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_1^{56} \frac{H_d}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \bar{\alpha}_1^{46} \left(\mathcal{V}^{-\frac{9}{5}} \tilde{e}_L - \frac{H_u}{\sqrt{2}} \right) & \bar{\alpha}_1^{56} \frac{\tilde{H}_d}{\sqrt{2}} & 0 \\ 5\bar{\alpha}_1^{17} \mathcal{V}^{-\frac{14}{5}} \tilde{u}_R & 0 & 0 & 0 & 0 & \bar{\alpha}_1^{67} \mathcal{V}^{-\frac{11}{5}} \tilde{e}_R & \alpha_1^{67} \mathcal{V}^{-\frac{11}{5}} e_R \end{pmatrix}, \\
z_2 = & \begin{pmatrix} 0 & 0 & 0 & \alpha_2^{14} \mathcal{V}^{-\frac{4}{5}} \tilde{u}_L & 0 & 0 & 5\alpha_2^{17} \mathcal{V}^{-\frac{13}{5}} \tilde{u}_R \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\alpha}_2^{14} \mathcal{V}^{-\frac{4}{5}} \tilde{u}_L & 0 & 0 & 0 & 0 & \alpha_2^{46} \left(6\mathcal{V}^{-\frac{8}{5}} \tilde{e}_L - \frac{H_u}{\sqrt{2}} \right) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_2^{56} \frac{H_d}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \bar{\alpha}_2^{46} \left(\mathcal{V}^{-\frac{8}{5}} \tilde{e}_L - \frac{H_u}{\sqrt{2}} \right) & -\bar{\alpha}_2^{56} \frac{\tilde{H}_d}{\sqrt{2}} & 0 \\ 5\bar{\alpha}_2^{17} \mathcal{V}^{-\frac{13}{5}} \tilde{u}_R & 0 & 0 & 0 & 0 & \bar{\alpha}_2^{67} \frac{\tilde{e}_R}{\mathcal{V}^{-2}} & \alpha_2^{67} \frac{\tilde{e}_R}{\mathcal{V}^{-2}} \end{pmatrix}
\end{aligned} \tag{60}$$

In (59) and (60), $\xi_{I,i}^{ab}$, $1 \leq a \leq 6, 1 \leq b \leq 7$ are $\mathcal{O}(1)$ numbers. Now, from (18), the intersection matrix can be written as:

$$C^{I\bar{J}} = \begin{pmatrix} c_{11} \mathcal{V}^{10/9} & c_{12} \mathcal{V}^{11/18} & c_{13} \mathcal{V}^{29/18} & c_{14} \mathcal{V}^{19/9} \\ c_{12} \mathcal{V}^{11/18} & c_{22} \sqrt[9]{V} & c_{23} \mathcal{V}^{10/9} & c_{24} \mathcal{V}^{29/18} \\ c_{13} \mathcal{V}^{29/18} & c_{23} \mathcal{V}^{10/9} & c_{33} \mathcal{V}^{10/9} & c_{34} \mathcal{V}^{47/18} \\ c_{14} \mathcal{V}^{19/9} & c_{24} \mathcal{V}^{29/18} & c_{34} \mathcal{V}^{47/18} & c_{44} \mathcal{V}^{28/9} \end{pmatrix}, \tag{61}$$

where c_{ab} , $1 \leq a, b \leq 4$ are $\mathcal{O}(1)$ numbers. Assuming that the complex structure moduli $z^{\bar{a}=1, \dots, h_-^{2,1}(CY_3)}$ are stabilized at very small values, which is in fact already assumed in writing (2) which has been written upon inclusion of terms up to linear in the complex structure moduli, let us define a modified intersection matrix in the $a_I - z_i$ moduli space:

$$\begin{aligned}
\mathcal{C}^{\mathcal{I}\mathcal{J}} &= C^{I\bar{J}}, \quad \mathcal{I} = I, \mathcal{J} = \bar{J}; \\
\mathcal{C}^{\mathcal{I}\mathcal{J}} &= \mu_3 (2\pi\alpha')^2 (\omega_\alpha)^{i\bar{j}}, \quad \mathcal{I} = i, \mathcal{J} = \bar{j}; \\
\mathcal{C}^{\mathcal{I}\mathcal{J}} &= 0, \quad \mathcal{I} = I, \mathcal{J} = \bar{j}, \text{ etc..}
\end{aligned} \tag{62}$$

Now, from (59) and (60), one can show that for $|z_i| \sim 0.8\mathcal{V}^{\frac{1}{36}}$, $\mathcal{V} \sim 10^5$:

$$\mathcal{C}^{\mathcal{I}\bar{\mathcal{J}}} \text{Tr}(\mathcal{M}_{\mathcal{I}} \mathcal{M}_{\bar{\mathcal{J}}}^\dagger) \sim C^{a_1 \bar{a}_1} |\tilde{e}_L|^2 + C^{a_2 \bar{a}_2} |\tilde{u}_L|^2 + C^{a_3 \bar{a}_3} |\tilde{e}_R|^2 + C^{a_4 \bar{a}_4} |\tilde{u}_R|^2 + \mu_3 (2\pi\alpha')^2 |H_u|^2, \tag{63}$$

where $\mathcal{M}_{\mathcal{I}} \equiv a_I, z_i$. Now, using (31), one can show that in the large volume limit: $\mathcal{C}^{\mathcal{A}_I \bar{\mathcal{A}}_{\bar{J}}} \sim C^{a_I \bar{a}_{\bar{J}}}, \mathcal{C}^{\mathcal{Z}_i \bar{\mathcal{Z}}_{\bar{j}}} \sim \mathcal{C}^{z_i \bar{z}_{\bar{j}}}$. Clubbing together the Wilson line moduli and the $D3$ -brane position moduli into a single vector: $\mathcal{M}_\Lambda \equiv \mathcal{A}_I, \mathcal{Z}_i$, then one sees from (52) - (57), and (63):

$$\mathcal{C}^{\Lambda \bar{\Sigma}} \text{Tr}(\mathcal{M}_\Lambda \mathcal{M}_{\bar{\Sigma}}^\dagger) \sim \mathcal{C}^{\mathcal{I}\bar{\mathcal{J}}} \text{Tr}(\mathcal{M}_{\mathcal{I}} \mathcal{M}_{\bar{\mathcal{J}}}^\dagger). \tag{64}$$

In the large volume and rigid limit of $\Sigma_B(\zeta^A = 0$ which corresponds to a local minimum), perhaps $\mathcal{C}^{\Lambda\bar{\Sigma}} Tr(\mathcal{M}_\Lambda \mathcal{M}_\Sigma^\dagger)$ is invariant under moduli transformations in the $(\mathcal{A}_I, \mathcal{Z}_i)/(a_I, z_i)$ -subspace of the open-string moduli space, which would imply that $C^{I\bar{J}} a_I \bar{a}_{\bar{J}} + \mu_3 (\alpha')^2 (\omega_B)_{i\bar{j}} z^i \bar{z}^{\bar{j}}$ for multiple $D7$ -branes, in a basis that diagonalizes $g_{\mathcal{M}_I \mathcal{M}_{\bar{J}}}$ at stabilized values of the open string moduli, is replaced by (63).

For the purpose of evaluation of (N)LSP decays in the subsequent sections, we will be using the following terms (written out in four-component notation or their two-component analogs and utilizing/generalizing results of [16]) in the $\mathcal{N} = 1$ gauged supergravity action of Wess and Bagger [12] with the understanding that $m_{\text{moduli/modulini}} \ll m_{\text{KK}} \left(\sim \frac{M_s}{\nu^{\frac{1}{6}}} \Big|_{\nu \sim 10^{5/6}} \sim 10^{14} \text{GeV} \right)$, $M_s = \frac{M_p}{\sqrt{V}} \Big|_{\nu \sim 10^{5/6}} \sim 10^{15} \text{GeV}$, and that for multiple $D7$ -branes, the non-abelian gauged isometry group³, corresponding to the killing vector $6i\kappa_4^2 \mu_7 (2\pi\alpha') Q_B \partial_{T_B}$, $Q_B = (2\pi\alpha') \int_{\Sigma_B} i^* \omega_B \wedge P_- \tilde{f}$ arising due to the elimination of the two-form axions $D_B^{(2)}$ in favor of the zero-form axions ρ_B under the KK-reduction of the ten-dimensional four-form axion [16] (which results in a modification of the covariant derivative of T_B by an additive shift given by $6i\kappa_4^2 \mu_7 (2\pi\alpha') Tr(Q_B A_\mu)$) can be identified with the SM group (i.e. A_μ is the SM-like adjoint-valued gauge field [12]):

$$\begin{aligned} \mathcal{L} = & g_{YM} g_{T_B \bar{J}} Tr \left(X^{T_B} \bar{\chi}_L^{\bar{J}} \lambda_{\tilde{g}, R} \right) + i g_{\mathcal{I} \bar{J}} Tr \left(\bar{\chi}_L^{\bar{J}} \left[\not{\partial} \chi_L^{\mathcal{I}} + \Gamma_{Mj}^i \not{\partial} a^M \chi_L^{\mathcal{J}} + \frac{1}{4} (\partial_{a_M} K \not{\partial} a_M - \text{c.c.}) \chi_L^{\mathcal{I}} \right] \right) \\ & + \frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\bar{\mathcal{I}}} D_{\mathcal{J}} \bar{W}) Tr (\chi_L^{\mathcal{I}} \chi_R^{\mathcal{J}}) + g_{T_B \bar{T}_B} Tr \left[(\partial_\mu T_B - A_\mu X^{T_B}) (\partial^\mu T_B - A^\mu X^{T_B})^\dagger \right] \\ & + g_{T_B \mathcal{J}} Tr (X^{T_B} A_\mu \bar{\chi}_L^{\mathcal{J}} \gamma^\nu \gamma^\mu \psi_{\nu, R}) + \bar{\psi}_{L, \mu} \sigma^{\rho\lambda} \gamma^\mu \lambda_{\tilde{g}, L} F_{\rho\lambda} + \bar{\psi}_{L, \mu} \sigma^{\rho\lambda} \gamma^\mu \lambda_{\tilde{g}, L} W_\rho^+ W_\lambda^- \\ & + Tr \left[\bar{\lambda}_{\tilde{g}, L} A \left(6\kappa_4^2 \mu_7 (2\pi\alpha') Q_B K + \frac{12\kappa_4^2 \mu_7 (2\pi\alpha') Q_B v^B}{\mathcal{V}} \right) \lambda_{\tilde{g}, L} \right] \\ & + \frac{e^K G^{T_B \bar{T}_B}}{\kappa_4^2} 6i\kappa_4^2 (2\pi\alpha') Tr \left[Q_B A^\mu \partial_\mu \left(\kappa_4^2 \mu_7 (2\pi\alpha')^2 C^{I\bar{J}} a_I \bar{a}_{\bar{J}} \right) \right] + \text{h.c.}, \end{aligned} \quad (65)$$

3 NLSP Decays

The generation of mass scales of superpartners and realization of μ split SUSY in the context of Big Divisor $D3 - D7$ type IIB string compactifications provides us with the gravitino as the L(ightest) S(upersymmetric) P(article) and sleptons/squarks as N(ext-to) L(ightest) S(upersymmetric) P(article)s with (Bino/Wino-type)gaugino/(Bino/Wino-type)gaugino-dominant neutralino, in the dilute-flux approximation, only differing in their masses by an $\mathcal{O}(1)$ factor. This helps in shedding some light on identifying a viable dark matter candidate in this framework. Decays of the neutralino, are, in general, driven by any of the trilinear R-violating couplings which necessarily involve squark or slepton as propagators. In MSSM, stability of LSP is governed by conserved R-parity however same is not a restrictive condition in “ μ split SUSY”. Some of the R-parity-violating interactions are not necessarily

³As explained in [16], one of the two Pecci-Quinn/shift symmetries along the RR two-form axions c^a and the zero-form axion ρ_B gets gauged due to the dualization of the Green-Schwarz term $\int_{\mathbf{R}^{1,3}} dD_B^{(2)} \wedge A$ coming from the KK reduction of the Chern-Simons term on $\Sigma_B \cup \sigma(\Sigma_B) - D_B^{(2)}$ being an RR two-form axion. In the presence of fluxes (50) for multiple $D7$ -brane fluxes, the aforementioned Green-Schwarz is expected to be modified to $Tr \left(Q_B \int_{\mathbf{R}^{1,3}} dD_B^{(2)} \wedge A \right)$, which after dualization in turn modifies the covariant derivative of T_B and hence the killing isometry.

negligible and the consideration of LSP to be a viable dark matter candidate needs the contribution of the same to be evaluated. Here, the large squark masses in μ -split SUSY helps to suppress the decay width. In BSM/2HDM models, one considers R-parity violating coupling generated from R-parity violating superpotential. However, here we consider the possible R-parity violating couplings in the effective $\mathcal{N} = 1$ gauged Supergravity action and by evaluating the same, we explicitly calculate the decay widths and hence life times for neutralino/gaugino and slepton decays. The neutralino mass matrix, for the four-Wilson-line-moduli setup, similar to [11], can be diagonalized and the smallest eigenvalue $\mathcal{V}^{\frac{2}{3}} m_{3/2}$ corresponds to the neutralino:

$$\tilde{\chi}_3^0 \sim -\lambda^0 + \tilde{f} \left(\tilde{H}_1^0 + \tilde{H}_2^0 \right) + \text{charge conjugate}, \quad (66)$$

where $\tilde{H}_{1,2}^0$ are the Higgsinos. We will first evaluate the lifetimes corresponding to two- and three-body decays of Wino/Bino-type gauginos and the gluino. We will then evaluate the lifetime for three-body decays of the neutralino followed by the lifetime corresponding to two- and three-body decays of the slepton.

The Big bang nucleosynthesis predicts the universal abundances of D, 3He, 4He, and 7Li, essentially fixed by average lifetime $\tau \sim 180 \text{ sec}$. In our study, we are considering both radiative as well as hadronic decay modes of N(LSP)'s. Therefore, if life time of NLSP's are more than 10^2 sec , the high energy photons emitted via radiative decay might destroy the abundance of light elements by inducing photo-dissociation of same, hence reducing the standard baryon-to-photon ratio and resulting in baryon-poor universe. Similarly, during the hadronic decay, the released hadronic energy can produce mesons/charged pions causing interconversions between background proton and neutron (p / n conversion) that alters the neutron-to proton ratio, resulting in the change of 4He abundance and hence destroying this success of standard BBN. Therefore, if one assumes that all of the dark matter density are produced by non thermal decays of NLSP decaying into LSP, it is first important to determine whether life time of these decays do not effect standard abundance of D, 3He, 4He, and 7Li, and hence predictions of BBN. We will see that one-loop gluino decay into gravitino violates the BBN requirements, and hence the gluino can not be an NLSP.

3.1 Gaugino Decays

In this sub-section, we discuss two- and three-body decay modes of the Wino/Bino-type gauginos as well as the gluino.

3.1.1 Two- and Three-Body Gaugino(Wino/Bino) Decays

The relevant gravitino-gaugino-photon/Z-boson vertex in the $\mathcal{N} = 1$ SUGRA action of [12] is obtained from $\frac{1}{M_p} \bar{\psi}_\rho \sigma^{\mu\nu} \gamma^\rho \lambda_L^{(a)} F_{\mu\nu}^{(a)}$, (a) corresponding to the three gauge groups. The decay width for $\tilde{B} \rightarrow \psi_\mu + \gamma$ is given by (See [24]):

$$\begin{aligned} \Gamma \left(\tilde{B} \rightarrow \psi_\mu + \gamma \right) &= \frac{\cos^2 \theta_W}{48\pi M_p^2} \frac{m_{\tilde{B}}^5}{m_{3/2}^2} \left(1 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right)^3 \left(1 + 3 \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right) \\ &\sim \frac{m_{\tilde{B}}^5}{m_{3/2}^2 M_p^2} \sim \mathcal{V}^{-\frac{8}{3}} M_p. \end{aligned} \quad (67)$$

So, (67) implies a lifetime of around 10^{-30} s .

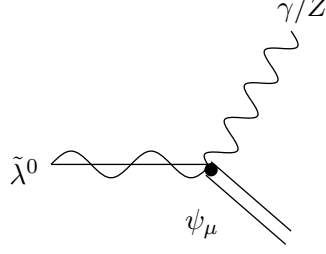


Figure 1: Two-body gaugino-decay diagram

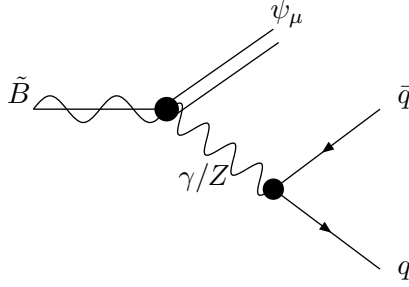


Figure 2: Three-body gaugino-decay diagram

Similarly, the decay width for $\tilde{B} \rightarrow \psi_\mu + Z$ is given by (See [24]):

$$\begin{aligned} \Gamma(\tilde{B} \rightarrow \psi_\mu + Z) &= \frac{\cos^2 \theta_W}{48\pi M_p^2} \frac{m_{\tilde{B}}^5}{m_{3/2}^2} \sqrt{1 - 2 \left(\frac{m_{\psi_\mu}^2}{m_{\tilde{B}}^2} + \frac{m_Z^2}{m_{\tilde{B}}^2} \right) + \left(\frac{m_{\psi_\mu}^2}{m_{\tilde{B}}^2} - \frac{m_Z^2}{m_{\tilde{B}}^2} \right)^2} \\ &\times \left[\left(1 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right)^2 \left(1 + 3 \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right) - \frac{m_Z^2}{m_{\tilde{B}}^2} \left\{ 3 + \frac{m_{3/2}^3}{m_{\tilde{B}}^3} \left(\frac{m_{3/2}}{m_{\tilde{B}}} - 12 \right) - \frac{x_Z^2}{x_{\tilde{B}}^2} \left(3 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} - \frac{m_Z^2}{m_{\tilde{B}}^2} \right) \right\} \right] \\ &\sim \frac{m_{\tilde{B}}^5}{m_{3/2}^2 M_p^2} \sim \mathcal{V}^{-\frac{8}{3}} M_p. \end{aligned} \quad (68)$$

So, (68) implies a lifetime of around $10^{-30} s$.

Given that the gauge boson-quark-anti-quark vertex will be accompanied by $\frac{g_{\mathcal{A}_2 \tilde{\mathcal{A}}_2} g_{YM} (X^{TB} K + i D^{TB})}{(\sqrt{K_{\mathcal{A}_2 \tilde{\mathcal{A}}_2}})^2} \sim \tilde{f} \mathcal{V}^{-\frac{1}{36} - \frac{2}{3}} \ln \mathcal{V}$, which for $\mathcal{V} \sim 10^5$ is approximately $\tilde{f} \mathcal{V}^{-\frac{2}{3}}$, using further the results of [24] one obtains the following result for the decay width $\lambda^0 \rightarrow \psi_\mu + u + \bar{u}$:

$$\Gamma(\tilde{B} \rightarrow \psi_\mu + u + \bar{u}) \sim \frac{g_{YM}^2 \tilde{f}^2 \mathcal{V}^{-\frac{4}{3}} m_{\lambda^0}^5}{32 (2\pi)^3 m_{3/2}^2 M_p^2} \sim \tilde{f}^2 \mathcal{V}^{-4} M_p, \quad (69)$$

which yields a lifetime of around $10^{-13} s$.

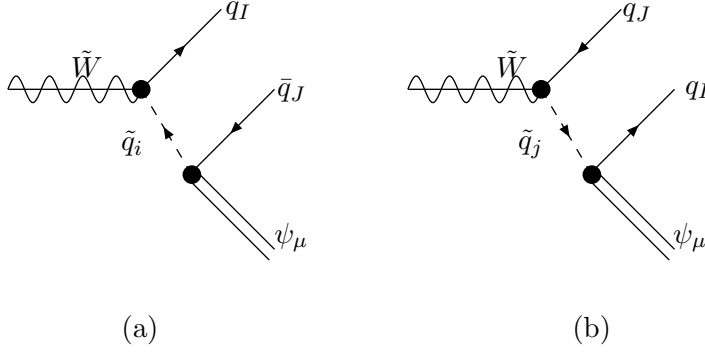


Figure 3: Three-body gaugino decays into the gravitino

We will now discuss three-body gaugino decays into gravitino mediated by squarks. To avoid channel overlap, for simplicity, we will assume that the gaugino decays mediated by vector bosons and those mediated by squarks, involve different gauginos. Utilizing the conditions $\bar{\psi}_\mu^{(+)}\gamma^\mu = \gamma^\mu\psi_\mu^{(-)} = 0$, the amplitudes for the above two diagrams can be written as:

$$\begin{aligned}
M_{(a)} &\sim 2p_{\tilde{q}}^\mu \frac{1}{M_p} \left(\overline{\psi_\mu^{(+)}(p_\psi)} v(p_{\tilde{q}}) \right) \left(\frac{i}{p_{\tilde{q}}^2 - m_{\tilde{q}}^2 + i\epsilon} \right) \left(\bar{u}(p_q) V_{\lambda-q-\tilde{q}} \lambda_{\tilde{g}}^{(+)} \right) \\
M_{(b)} &\sim 2p_{\tilde{q}}^\mu \frac{1}{M_p} \left(\bar{u}(p_q) \psi_\mu^{(-)}(p_\psi) v(p_{\tilde{q}}) \right) \left(\frac{i}{p_{\tilde{q}}^2 - m_{\tilde{q}}^2 + i\epsilon} \right) \left(\overline{\lambda_{\tilde{g}}^{(-)}} V_{\lambda-q-\tilde{q}} v(p_{\tilde{q}}) \right), \quad (70)
\end{aligned}$$

$V_{\lambda-q-\tilde{q}} \sim \tilde{f}\mathcal{V}^{-\frac{4}{5}}$ being the gaugino-quark-squark vertex. From (70), one obtains the following helicities averaged sum:

$$\begin{aligned}
\sum_{s_\psi=\pm\frac{3}{2}, \pm\frac{1}{2}, s_q, s_{\tilde{q}}, s_{\lambda_g}=\pm\frac{1}{2}} M_{(a)} M_{(a)}^\dagger &\sim |V_{\lambda-q-\tilde{q}}|^2 \frac{p_{\tilde{q}}^\mu p_{\tilde{q}}^\nu}{M_p^2} \text{Tr} \left[\mathcal{P}_{\mu\nu}^{(+)} (-\not{p}_{\tilde{q}} + m_{\tilde{q}}) \right] \text{Tr} \left[(\not{p}_{\lambda_{\tilde{g}}} + m_{\lambda_{\tilde{g}}}) (\not{p}_q + m_q) \right], \\
\sum_{s_\psi=\pm\frac{3}{2}, \pm\frac{1}{2}, s_q, s_{\tilde{q}}, s_{\lambda_g}=\pm\frac{1}{2}} M_{(b)} M_{(b)}^\dagger &\sim |V_{\lambda-q-\tilde{q}}|^2 \frac{p_{\tilde{q}}^\mu p_{\tilde{q}}^\nu}{M_p^2} \text{Tr} \left[\mathcal{P}_{\mu\nu}^{(-)} (\not{p}_q + m_q) \right] \text{Tr} \left[(-\not{p}_{\lambda_{\tilde{g}}} + m_{\lambda_{\tilde{g}}}) (-\not{p}_{\tilde{q}} + m_{\tilde{q}}) \right], \quad (71)
\end{aligned}$$

and utilizing $\bar{u}(p_q) \psi_\mu^{(-)}(p_\psi) = -\overline{\psi_\mu^{(+)}}(p_\psi) v(p_{\tilde{q}})$, $\overline{\lambda_{\tilde{g}}^{(-)}} v(p_{\tilde{q}}) = -\bar{u}(p_q) \lambda_{\tilde{g}}^{(+)}$ to rewrite $M_{(b)}$ and therefore obtain:

$$\sum_{s_\psi=\pm\frac{3}{2}, \pm\frac{1}{2}, s_q, s_{\tilde{q}}, s_{\lambda_g}=\pm\frac{1}{2}} \text{Re} M_{(a)} M_{(b)}^\dagger \sim |V_{\lambda-q-\tilde{q}}|^2 \frac{p_{\tilde{q}}^\mu p_{\tilde{q}}^\nu}{M_p^2} \text{Tr} \left[\mathcal{P}_{\mu\nu}^{(+)} (-\not{p}_{\tilde{q}} + m_{\tilde{q}}) \right] \text{Tr} \left[(\not{p}_{\lambda_{\tilde{g}}} + m_{\lambda_{\tilde{g}}}) (\not{p}_q + m_q) \right]. \quad (72)$$

In (70) - (72), the positive and negative energy gravitino projectors are given by:

$$\mathcal{P}^{(\pm)} \equiv -(\not{p}_{3/2} \pm m_{3/2}) \left[\left(\eta_{\mu\nu} - \frac{p_{3/2,\mu} p_{3/2,\nu}}{m_{3/2}^2} \right) - \frac{1}{3} \left(\eta_{\mu\sigma} - \frac{p_{3/2,\mu} p_{3/2,\sigma}}{m_{3/2}^2} \right) \left(\eta_{\nu\lambda} - \frac{p_{3/2,\nu} p_{3/2,\lambda}}{m_{3/2}^2} \right) \gamma^\sigma \gamma^\lambda \right]. \quad (73)$$

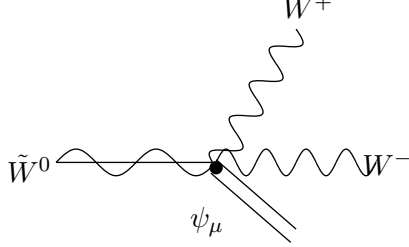


Figure 4: Contact-vertex three-body decay diagram

A typical term that one would need to calculate is:

$$\begin{aligned}
& p_{\bar{q}}^\mu p_{\bar{q}}^\nu \text{Tr} \left[\mathcal{P}_{\mu\nu}^{(\pm)} \left(\eta \not{p}_\eta + m_{\bar{q}} \right) \right], \quad \eta = \pm \text{ corresponding to } p_\eta = p_q/p_{\bar{q}} \\
& \sim 4 \left\{ \left(m_{\bar{q}}^2 - \frac{(p_{\bar{q}} \cdot p_{3/2})^2}{m_{3/2}^2} \right) (\eta p_{3/2} \cdot p_\eta \pm m_{3/2} m_\eta) \right. \\
& \quad \left. - \left(p_{\bar{q}}^\mu - \frac{p_{\bar{q}} \cdot p_{3/2} p_{3/2}^\mu}{m_{3/2}^2} \right) \left(p_{\bar{q}}^\mu - \frac{p_{\bar{q}} \cdot p_{3/2} p_{3/2}^\mu}{m_{3/2}^2} \right) \left(\frac{\eta}{3} p_\eta \cdot p_{3/2} \pm m_{3/2} m_\eta \right) \right\}. \quad (74)
\end{aligned}$$

Using results of [25], to get an estimate of the decay width, one sees that:

$$\begin{aligned}
& \Gamma \left(\tilde{W}^0 \rightarrow q + \bar{q} + \psi_\mu \right) \\
& \sim \text{Max} \left[\frac{1}{m_{\lambda_{\tilde{g}}}^3} \int_{s_{23}=m_{3/2}^2}^{m_{\lambda_{\tilde{g}}}^2} ds_{23} \int_{s_{13}=\frac{m_{3/2}^2 m_{\lambda_{\tilde{g}}}^2}{s_{23}}}^{m_{3/2}^2 + m_{\lambda_{\tilde{g}}}^2 - s_{23}} ds_{13} \frac{(74) |V_{\lambda-\bar{q}-q}|^2 \times (p_{\lambda_{\tilde{g}}} \cdot p_q + m_{\lambda_{\tilde{g}}} m_q)}{(s_{23}^2 - m_{\bar{q}}^2)^2 (s_{13}^2 - m_{\bar{q}}^2)^2 (s_{23}^2 - m_{\bar{q}}^2) (s_{13}^2 - m_{\bar{q}}^2)} \right]_{\mathcal{V} \sim 10^5, m_q, \bar{q}=0} \\
& \sim \text{Max} (10^{-32}, 10^{-30}, 10^{-32}) M_p = 10^{-28} M_p, \quad (75)
\end{aligned}$$

implying that the corresponding lifetime would be $10^{-15} s$.

In the approximation, $\frac{m_W}{m_{\tilde{W}}} \approx 0$, $\frac{m_{3/2}}{m_{\tilde{W}}} \approx 0$, the decay width for $\tilde{W}^0 \rightarrow \psi_\mu + W^+ + W^-$ is given by (See [24]):

$$\Gamma \left(\tilde{W}^0 \rightarrow \psi_\mu + W^+ + W^- \right) \approx \frac{g_Y^2 M m_{\tilde{W}}^9}{34560 (2\pi)^3 m_W^4 M_p^2} \sim \frac{\mathcal{V}^{-8} M_p^5}{M_W^4}, \quad (76)$$

which implies a lifetime of around $10^{-66} s$. Treating the Wino/Bino gauginos as the co-NLSPs, we hence conclude that the two- and three-body gaugino decays into the gravitino (LSP), respect the BBN bounds. If one were to calculate similar neutralino decays into the gravitino, then for the Higgsino contribution to the neutralino-gauge-gravitino vertex, one will have the following suppression factor: $\tilde{f} \left(\tilde{f} \mathcal{V}^{-\frac{2}{3}} \right) \times \mathcal{O}(z_i)$ term in $g_{YM} \frac{g_{T_B \tilde{z}_i}}{\sqrt{K_{\tilde{z}_i \tilde{z}_i}}} \times \frac{\langle z_i \rangle}{M_p}$, which is $\tilde{f}^2 \mathcal{V}^{-\frac{2}{3}-\frac{5}{3}} \times 10^{\frac{5}{2}}$. For $\mathcal{V} \sim 10^6$, this is approximately, 10^{-19} . The corresponding gaugino-gauge-gravitino vertex will, without worrying about the gamma matrices, be of $\mathcal{O} \left(\frac{p_Z}{M_p} \right)$. Now, even at low Z -momenta, this is around 10^{-16} for $M_Z 10^2 GeV$. Hence, the neutralino and gaugino decay (e.g., into gravitino and Z boson) widths, will approximately be the same.

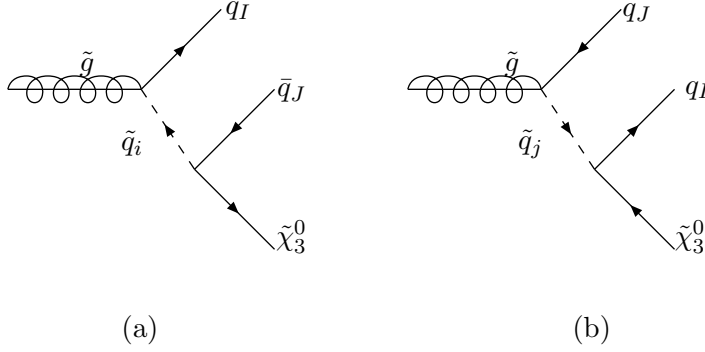


Figure 5: Three-body gluino decay diagrams

3.1.2 Gluino Decays Revisited

In this sub-section, we will readdress the gluino decays in the light of previous discussion done in [11]. The earlier discussion was made with an ambiguity in identifying the quarks with right set of wilson line moduli. In the four Wilson line moduli framework where the first generation left handed quarks can be identified with Wilson line modulus a_2 (strictly speaking \mathcal{A}_2) and the first generation right handed quarks get identified with a_4 (strictly speaking \mathcal{A}_4) as mentioned above, by recalculating the moduli space metric, we will briefly estimate Gluino decay life time results for tree-level as well as one-loop decays into neutralino and Goldstino.

$\tilde{g} \rightarrow q\bar{q}\chi_n$

We first discuss gluino three-body decays that involve the process like $\tilde{g} \rightarrow q\bar{q}\chi_n$ - \tilde{g} being a gaugino, q/\bar{q} being quark/anti-quark and χ_n being a neutralino. More specifically, e.g., the gluino decays into an anti-quark and an off-shell squark and the off-shell squark decays into a quark and neutralino. The Feynman diagrams involve gluino-quark/antivertex- squark vertex and neutralino-quark/antiquark-squark vertex. The gluino-quark/antiquark- squark in gauged supergravity action as also given in (148), is proportional to:

$$G_{\tilde{q}\mathcal{A}_2}^{q/\bar{q}} \sim \tilde{f}\mathcal{V}^{-\frac{3}{2}}\tilde{q}\chi^{q/\bar{q}}\lambda_{\tilde{g}}, \lambda_{\tilde{g}} \text{ corresponds to gluino} \quad (77)$$

and from (149), contribution of physical neutralino-quark/antiquark-squark is proportional to

$$X_{\tilde{q}}^{q/\bar{q}} \sim \tilde{f}\mathcal{V}^{-\frac{4}{5}}\tilde{q}\chi^{q/\bar{q}}\tilde{\chi}_3^0 \quad (78)$$

Using RG analysis of coefficients of the effective dimension-six gluino decay operators as given in [26], it was shown in [11] that these coefficients at the EW scale are of the same order as that at the squark mass scale.

The analytical formulae to calculate decay width for three-body tree-level gluino decay channel as given in [25] is:-

$$\Gamma(\tilde{g} \rightarrow \chi_n^o q_I \bar{q}_J) = \frac{g_s^2}{256\pi^3 M_{\tilde{g}}^3} \sum_{i,j} \int ds_{13} ds_{23} \frac{1}{2} \text{Re} \left(A_{ij}(s_{23}) + B_{ij}(s_{13}) - 2\epsilon_n \epsilon_g C_{ij}(s_{23}, s_{13}) \right) \quad (79)$$

where the integrand is the square of the spin-averaged total amplitude and $i, j = 1, 2, \dots, 6$ are the indices of the squarks mediating the decay. The limits of integration in (79) are given in [11]. The A_{ij} terms in (79) represent the contributions from the gluino decay channel involving gluino \rightarrow squark + quark and squark \rightarrow neutralino + anti-quark, whereas the B_{ij} terms come from channel gluino \rightarrow squark + anti-quark and squark \rightarrow neutralino + quark. The same are defined in [11]. Utilizing the results as given in (77) and (78), $A_{ij} \left(Tr \left[G_{\tilde{q}}^{\tilde{q}} G_{\tilde{q}}^{\tilde{q} \dagger} \right] Tr \left[X_{\tilde{q}}^q X_{\tilde{q}}^q \dagger \right] \right) \sim \tilde{f}^4 \mathcal{V}^{-\frac{23}{5}} \sim B_{ij} \left(Tr \left[G_{\tilde{q}_{a1}}^{q_{a1}} G_{\tilde{q}_{a1}}^{q_{a1} \dagger} \right] Tr \left[X_{\tilde{q}_{a1}}^{\tilde{q}_{a1}} X_{\tilde{q}_{a1}}^{\tilde{q}_{a1} \dagger} \right] \right) \sim \tilde{f}^4 \mathcal{V}^{-\frac{23}{5}}$,
 $C \left(Tr \left[G_{\tilde{q}_{a1}}^{\tilde{q}} G_{\tilde{q}}^q \dagger X_{\tilde{q}}^q X_{\tilde{q}}^{\tilde{q} \dagger} \right] \right) \sim \tilde{f}^4 \mathcal{V}^{-\frac{23}{5}}$. Substituting these values for various vertex elements and solving equation (79) by using the same limits of integration as given in [11], dominating contribution of decay width is:

$$\begin{aligned} \Gamma(\tilde{g} \rightarrow \chi_n^0 q_I \bar{q}_J) &\sim \frac{g_s^2}{256\pi^3 m_{\frac{3}{2}}^3} \left[\tilde{f}^4 \mathcal{V}^{-\frac{23}{5}} m_{\frac{3}{2}}^4 \mathcal{V}^{\frac{10}{3}} + \tilde{f}^4 \mathcal{V}^{-\frac{61}{12}} \mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^4 + \tilde{f}^4 \mathcal{V}^{-\frac{23}{5}} \mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^4 \right] \\ &\sim \frac{g_s^2}{256\pi^3 \mathcal{V}^2 m_{\frac{3}{2}}^3} \left(\tilde{f}^4 \mathcal{V}^{-\frac{23}{5}} \mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^4 \right) \sim O(10^{-4}) \tilde{f}^4 \mathcal{V}^{-\frac{10}{3}} M_p \\ &\sim O(10^{-2}) \tilde{f}^4 GeV \end{aligned} \quad (80)$$

Considering lower bound on $f^2 \sim 10^{-8}$ as calculated in [11] in dilute flux approximation, decay width of gluino i.e equation (80) becomes: $\Gamma(\tilde{g} \rightarrow \chi_n^0 q_I \bar{q}_J) \sim O(10^{-2}) \tilde{f}^4 < O(10^{-18}) GeV$. Further, Life time of gluino is given as

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} Jsec}{10^{-2} \tilde{f}^4 GeV} \sim \frac{10^{-23}}{\tilde{f}^4} > 10^{-5} sec \quad (81)$$

$$\tilde{\mathbf{g}} \rightarrow \tilde{\chi}_3^0 + \mathbf{g}$$

Relevant to Figs.4(a) and 4(b), the gluino-quark-squark vertex and the neutralino-quark-squark vertex will be given by (77)- (78). As explained in [11], the quark-quark-gluon vertex relevant to figure 4, from [12] is given by:

$$\begin{aligned} &g_{I\bar{J}} \bar{\chi}^{\bar{J}} \bar{\sigma} \cdot A \text{Im} (X^B K + iD^B) \chi^I, \\ &\sim g_{YM} g_{\mathcal{A}_2} \bar{\mathcal{A}}_2 \bar{\chi}^{\bar{\mathcal{A}}_2} \bar{\sigma} \cdot A \left\{ 6\kappa_4^2 \mu_7 2\pi\alpha' Q_B K + \frac{12\kappa_4^2 \mu_7 2\pi\alpha' Q_B v^B}{\mathcal{V}} \right\} \end{aligned} \quad (82)$$

$\chi^{\mathcal{A}_2}$ and $\bar{\chi}^{\bar{\mathcal{A}}_2}$ to quarks and antiquarks. $X = X^B \partial_B = -12i\pi\alpha' \kappa_4^2 \mu_7 Q_B \partial_{T_B}$ corresponding to the killing isometry vector and D term generated is given by:

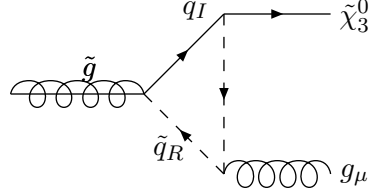
$$D^B = \frac{4\pi\alpha' \kappa_4^2 \mu_7 Q_B v^B}{\mathcal{V}} \quad (83)$$

which utilizing the fact (see equation 124):

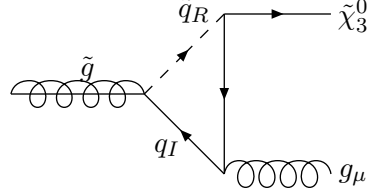
$$g_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \sim \mathcal{V}^{-\frac{5}{9}}, \quad (84)$$

as well as $v^B \sim \mathcal{V}^{\frac{1}{3}}$, $Q_B \sim \mathcal{V}^{\frac{1}{3}} (2\pi\alpha')^2 \tilde{f}$, yields for the quark-quark-gluon vertex:

$$\frac{\left(\mathcal{V}^{-\frac{7}{6}} \delta_{\mathcal{A}_2}^I \delta_{\mathcal{A}_2}^J \right) \tilde{f} \bar{\sigma} \cdot \epsilon}{\left(\sqrt{\hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2}} \right)^2 \sim O(10^{-2})} \sim \left(O(10)^2 \mathcal{V}^{-\frac{7}{6}} \delta_{\mathcal{A}_2}^I \delta_{\mathcal{A}_2}^J \right) \tilde{f} \bar{\sigma} \cdot \epsilon \sim \mathcal{V}^{-\frac{23}{30}} \tilde{f} \bar{\sigma} \cdot \epsilon; \text{ for } \mathcal{V} \sim 10^5 \quad (85)$$



4(a)



4(b)

Figure 6: Diagrams contributing to one-loop two-body gluino decay

The gauge kinetic term for squark-squark-gluon vertex, relevant to Fig.4(b) will be given by $\frac{1}{\kappa_4^2 \mathcal{V}^2} G^{\sigma_B \bar{\sigma}_B} \nabla_\mu T_B \tilde{\nabla}^\mu \bar{T}_{\bar{B}}$. This implies that the following term generates the required squark-squark-gluon vertex:

$$\frac{6i\kappa_4^2 \mu_7 2\pi\alpha' Q_B G^{\sigma_B \bar{\sigma}_B}}{\kappa_4^2 \mathcal{V}^2} A^\mu \partial_\mu (\kappa_4^2 \mu_7 (2\pi\alpha')^2 C_{2\bar{2}} \mathcal{A}_2 \bar{\mathcal{A}}_2) \xrightarrow[\kappa_4^2 \mu_7 (2\pi\alpha')^2 C_{2\bar{2}} \sim \mathcal{V}^{\frac{1}{9}}]{G^{\sigma_B \bar{\sigma}_B} \sim \mathcal{V}^{\frac{4}{3}}} \frac{\mathcal{V}^{\frac{7}{9}} \epsilon \cdot (2k - (p_{\tilde{\chi}_3^0} + p_{\tilde{g}}))}{\left(\sqrt{\hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2}}\right)^2} \sim O(10^2) \tilde{f} \mathcal{V}^{-\frac{2}{9}} \left[2\epsilon \cdot k - \epsilon \cdot (p_{\tilde{\chi}_3^0} + p_{\tilde{g}})\right] \sim \tilde{f} \mathcal{V}^{\frac{8}{45}}, \text{ for } \mathcal{V} \sim 10^5 \quad (86)$$

In [11], it has been discussed that behavior of Wilson coefficients corresponding to two-body gluino decay does not change much upon RG evolution to EW scale. Using the vertices calculated above relevant to Figs. 4(a) and 4(b), and the Feynman rules of [27], one obtains for the scattering

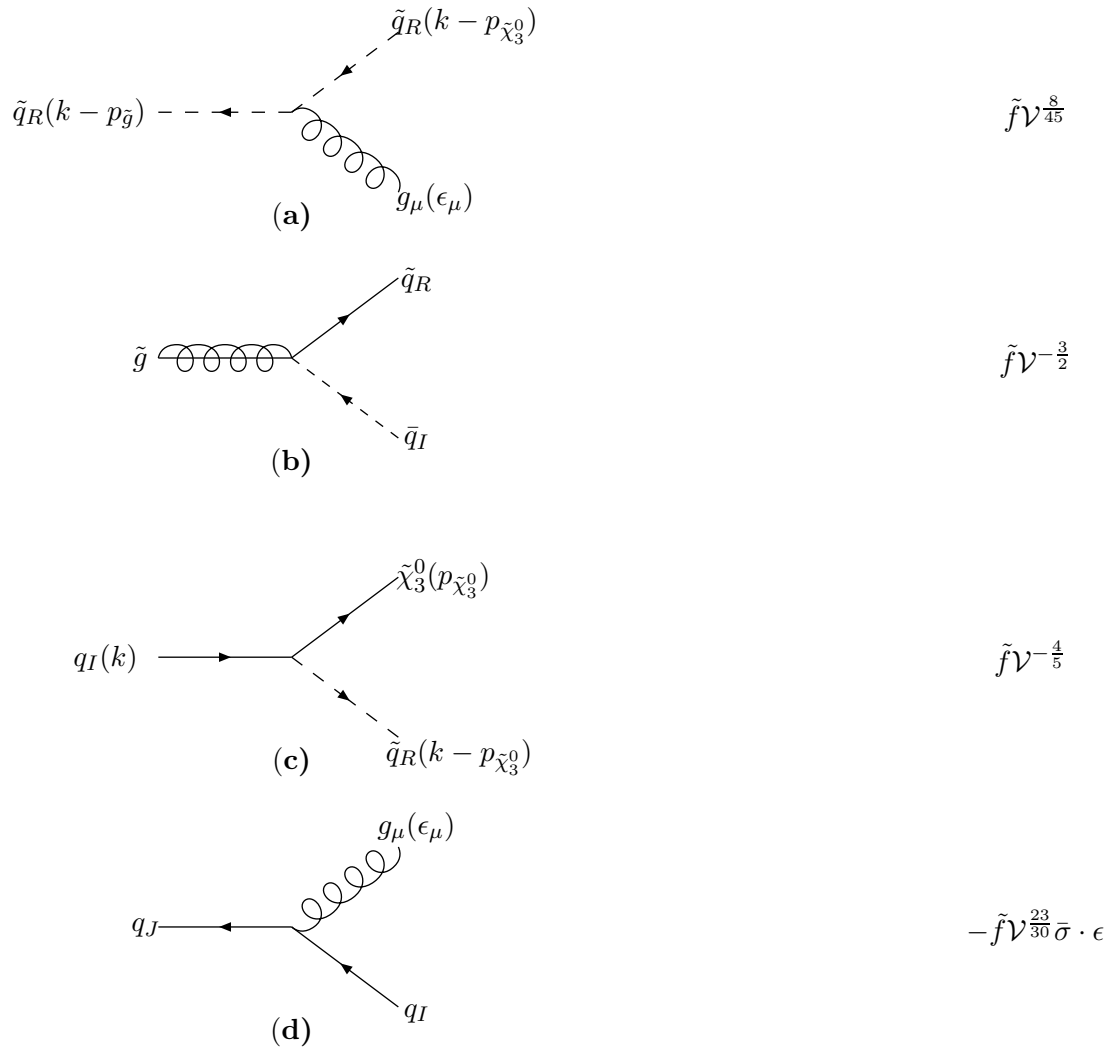


Figure 7: Different vertices relevant to one-loop gluino decay

amplitude:

$$\begin{aligned}
\mathcal{M} &\sim \tilde{f}^3 M_p \int \frac{d^4 k}{(2\pi)^4} \times \mathcal{V}^{-\frac{3}{2}} \left(\frac{i\bar{\sigma} \cdot k}{k^2 - m_q^2 + i\epsilon} \right) \left(\mathcal{V}^{-\frac{4}{5}} \right) \left(\frac{i}{[(k - p_{\bar{G}})^2 - m_{\bar{q}}^2 + i\epsilon]} \right) \\
&\times \left(\mathcal{V}^{\frac{8}{45}} \right) \left(\frac{i}{[(k - p_{\bar{g}})^2 - m_{\bar{q}}^2 + i\epsilon]} \right) \\
&+ \tilde{f}^3 M_p \int \frac{d^4 k}{(2\pi)^4} \times \mathcal{V}^{-\frac{3}{2}} \left(\frac{i}{[(k + p_{\tilde{\chi}_3^0})^2 - m_{\bar{q}}^2 + i\epsilon]} \right) \left(\mathcal{V}^{-\frac{4}{5}} \right) \left(\frac{i\bar{\sigma} \cdot k}{k^2 - m_q^2 + i\epsilon} \right) \\
&\times \left(\mathcal{V}^{-\frac{23}{30}} \bar{\sigma} \cdot \epsilon \right) \left(\frac{i\bar{\sigma} \cdot (k - p_{g_\mu})}{[(k - p_{g_\mu})^2 - m_q^2 + i\epsilon]} \right)
\end{aligned} \tag{87}$$

Using the 1-loop integrals of [28]:

$$\begin{aligned}
&\frac{1}{i} \int \frac{d^4 k}{(2\pi)^4} \frac{(k_\mu, k_\mu k_\nu)}{(k^2 - m_1^2 + i\epsilon) [(k + p_1)^2 - m_2^2 + i\epsilon] [(k + p_1 + p_2)^2 - m_3^2 + i\epsilon]} \\
&= 4\pi^2 \left[p_{1\mu} C_{11} + p_{2\mu} C_{12} + p_{1\mu} p_{1\nu} C_{21} + p_{2\mu} p_{2\nu} C_{22} + (p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu}) C_{23} + \eta_{\mu\nu} C_{24} \right]; \\
&(a) m_1 = m_q, m_2 = m_3 = m_{\bar{q}}; p_1 = -p_{\tilde{\chi}_3^0}, p_2 = -p_{g_\mu}; \\
&(b) m_1 = m_3 = m_q, m_2 = m_{\bar{q}}; p_1 = p_{\tilde{\chi}_3^0}, p_2 = -p_{\bar{g}}.
\end{aligned} \tag{88}$$

The one loop three point functions C_{ij} 's relevant for cases (a) and (b) have been calculated in [11] and are given as under:

$$\begin{aligned}
C_{24}^{(a)} &= O(10^6), C_{24}^{(b)} = O(10^6); \\
C_0^{(a)} &= \frac{1}{4\pi^2} \times 10^{-21} \sim O(1) \times 10^{-23} GeV^{-2}; \\
C_{11}^{(b)} &= O(10) \times 10^{-16} GeV^{-2}; \\
C_{12}^{(b)} &= O(10) \times 10^{-16} GeV^{-2}; \\
C_{21}^{(b)} &= C_{22}^{(b)} = C_{23}^{(b)} \sim O(1) \times 10^{-10} GeV^{-2}; \\
C_0^{(b)} &= \frac{1}{4\pi^2} \times 10^{-21} \sim O(1) \times 10^{-23} GeV^{-2}.
\end{aligned} \tag{89}$$

Now, equation(87) can be evaluated to yield:

$$\begin{aligned}
&\bar{u}(p_{\tilde{\chi}_3^0}) \left(\tilde{f}^3 \mathcal{V}^{-2.1} \left[\left\{ \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} C_{11}^{(a)} + \bar{\sigma} \cdot p_{g_\mu} C_{12}^{(a)} \right\} (2\epsilon \cdot p_{\tilde{\chi}_3^0}) + \left\{ \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} \epsilon \cdot p_{\tilde{\chi}_3^0} C_{21}^{(a)} + \bar{\sigma} \cdot p_{g_\mu} \epsilon \cdot p_{\tilde{\chi}_3^0} C_{23}^{(a)} + \bar{\sigma} \cdot \epsilon C_{24}^{(a)} \right\} \right] \right. \\
&+ \tilde{f}^3 \mathcal{V}^{-3} \left[- \left\{ \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} C_{11}^{(b)} + \bar{\sigma} \cdot p_{\bar{g}} C_{12}^{(b)} \right\} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{g_\mu} + \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} C_{21}^{(b)} + \bar{\sigma} \cdot p_{\bar{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\bar{g}} C_{22}^{(b)} \right. \\
&\left. \left. - \left(\bar{\sigma} \cdot p_{\tilde{\chi}_3^0} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\bar{g}} + \bar{\sigma} \cdot p_{\bar{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} \right) C_{23}^{(b)} + \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] \right) u(p_{\bar{g}}),
\end{aligned} \tag{90}$$

which equivalently could be rewritten as:

$$\tilde{f}^3 \bar{u}(p_{\tilde{\chi}_3^0}) \left[\bar{\sigma} \cdot \mathcal{A} + \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot \mathcal{B}_1 + \bar{\sigma} \cdot p_{g_\mu} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot \mathcal{B}_2 + D_3 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] u(p_{\tilde{g}}), \quad (91)$$

where

$$\begin{aligned} \mathcal{A}^\mu &\equiv \mathcal{V}^{-2.1} \left[p_{\tilde{\chi}_3^0}^\mu \epsilon \cdot p_{\tilde{\chi}_3^0} \left(2C_{11}^{(a)} + C_{21}^{(a)} \right) + p_{g_\mu}^\mu \epsilon \cdot p_{\tilde{\chi}_3^0} \left(C_{12}^{(a)} + C_{23}^{(a)} \right) + \epsilon^\mu C_{24}^{(a)} \right]; \\ \mathcal{B}_1^\mu &\equiv \mathcal{V}^{-3} \left[-p_{g_\mu}^\mu \left(C_{11}^{(b)} + C_{12}^{(b)} + C_{23}^{(b)} - C_{22}^{(b)} \right) + p_{\tilde{\chi}_3^0}^\mu \left(C_{21}^{(b)} + C_{22}^{(b)} - 2C_{23}^{(b)} \right) \right]; \\ \mathcal{B}_2^\mu &\equiv \mathcal{V}^{-3} \left[p_{g_\mu}^\mu \left(C_{12}^{(b)} + C_{22}^{(b)} \right) + p_{\tilde{\chi}_3^0}^\mu \left(C_{22}^{(b)} - C_{23}^{(b)} \right) \right]; \\ D_3 &\equiv \mathcal{V}^{-3} \end{aligned} \quad (92)$$

Replacing $\bar{u}(p_{\tilde{\chi}_3^0}) \bar{\sigma} \cdot p_{\tilde{\chi}_3^0}$ by $m_{\tilde{\chi}_3^0} \bar{u}(p_{\tilde{\chi}_3^0})$ and $\bar{\sigma} \cdot p_{\tilde{g}} u(p_{\tilde{g}})$ by $m_{\tilde{g}}$, and using $\epsilon \cdot p_{\tilde{\chi}_3^0} = 0$, (91) be simplified to:

$$\mathcal{M} \sim \tilde{f}^3 \bar{u}(p_{\tilde{\chi}_3^0}) \left[A \bar{\sigma} \cdot \epsilon + B_1 \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + B_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_1 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + D_3 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] u(p_{\tilde{g}}), \quad (93)$$

where

$$\begin{aligned} A &\equiv \mathcal{V}^{-2.1} C_{24}^{(a)} - m_{\tilde{g}} m_{\tilde{\chi}_3^0} \mathcal{V}^{-3} \left\{ C_{11}^{(b)} + 2C_{12}^{(b)} + C_{23}^{(b)} - C_{22}^{(b)} \right\}, \\ B_1 &\equiv \mathcal{V}^{-3} \left(C_{11}^{(b)} + 2C_{12}^{(b)} + C_{21}^{(b)} \right) m_{\tilde{\chi}_3^0}, \\ B_2 &\equiv \mathcal{V}^{-3} \left(C_{12}^{(b)} + C_{22}^{(b)} \right) m_{\tilde{g}}, \quad D_1 \equiv \mathcal{V}^{-3} \left(-C_{12}^{(b)} - C_{23}^{(b)} \right). \end{aligned} \quad (94)$$

Utilizing values of C's calculated in (89),

$$A \sim \mathcal{O}(10^{-3}), B_1 \sim \mathcal{O}(10^{-14}) GeV^{-2}, B_2 \sim \mathcal{O}(10^{-14}) GeV^{-2}, D_1 \sim \mathcal{O}(10^{-25}) GeV^{-2}, D_3 \sim \mathcal{V}^{-3}; \quad (95)$$

$$\begin{aligned} \sum_{\tilde{g} \text{ and } \tilde{\chi}_3^0 \text{ spins}} |\mathcal{M}|^2 &\sim \tilde{f}^6 Tr \left(\sigma \cdot p_{\tilde{\chi}_3^0} \left[A \bar{\sigma} \cdot \epsilon + B_1 \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + B_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_1 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + D_3 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] \right. \\ &\quad \left. \times \sigma \cdot p_{\tilde{g}} \left[A \bar{\sigma} \cdot \epsilon + B_1 \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + B_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_1 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{\chi}_3^0} + D_3 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right]^\dagger \right), \end{aligned} \quad (96)$$

which at:

$$p_{\tilde{\chi}_3^0}^0 = \sqrt{m_{\tilde{\chi}_3^0}^2 c^4 + \rho^2}, p_{\tilde{\chi}_3^0}^1 = p_{\tilde{\chi}_3^0}^2 = p_{\tilde{\chi}_3^0}^3 = \frac{\rho}{\sqrt{3}} = \frac{1}{\sqrt{3}} \frac{c \left(m_{\tilde{g}}^2 - m_{\tilde{\chi}_3^0}^2 \right)}{2m_{\tilde{g}}}, \quad (97)$$

yields:

$$\frac{\tilde{f}^6}{256} m_{\tilde{g}}^2 \left[6m_{\tilde{g}}^2 (B_1 + D_1 m_{\tilde{g}})^2 + \left\{ 8A_1 + 16D_3 C_{24} + m_{\tilde{g}} \left(\left(5 + \sqrt{3} \right) B_1 + 8B_2 + \left(5 + \sqrt{3} \right) D_1 m_{\tilde{g}} \right) \right\}^2 \right], \quad (98)$$

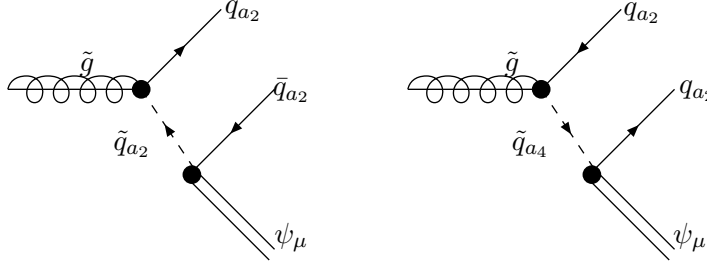


Figure 8: Three-body Gluino decay into Gravitino

in the rest frame of the gluino.

Incorporating results of (94) in equation (98), one gets

$$\sum_{\tilde{g} \text{ and } \tilde{\chi}_3^0 \text{ spins}} |\mathcal{M}|^2 \sim 0.2 \tilde{f}^6 D_1^2 m_{\tilde{g}}^6 \quad (99)$$

Now, using standard two-body decay results (See [29]), the decay width Γ is given by the following expression:

$$\Gamma = \frac{\sum_{\tilde{g} \text{ and } \tilde{\chi}_3^0 \text{ spins}} |\mathcal{M}|^2 (m_{\tilde{g}}^2 - m_{\tilde{\chi}_3^0}^2)}{16\pi \hbar m_{\tilde{g}}^3} \quad (100)$$

Using result of (95); $D_1 \sim O(10^{-25}) GeV^{-2}$ and $m_{\tilde{g}} \sim \mathcal{V}^{-\frac{4}{3}} M_p$ GeV, $m_{\tilde{\chi}_3^0} \sim m_{\tilde{g}} + \frac{\tilde{f}^2 M_p}{\mathcal{V}}$ GeV, two body decay width is given as:

$$\Gamma = \frac{(0.2) \tilde{f}^6 D_1^2 m_{\tilde{g}}^6}{16\pi m_{\tilde{g}}^3} \sim \frac{O(10^{-2}) \tilde{f}^6 \cdot 10^{-54} \cdot m_{\tilde{g}}^6 (2m_{\tilde{g}} \cdot \frac{\tilde{f}^2 M_p}{\mathcal{V}})}{m_{\tilde{g}}^3} \sim 10^{-3} \tilde{f}^6 GeV \quad (101)$$

Considering $\tilde{f}^2 < 10^{-8}$ as calculated above, $\Gamma < 10^{-27}$ GeV. Life time of gluino is given as:

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} Jsec}{10^{-3} \tilde{f}^6 GeV} \sim \frac{10^{-22}}{\tilde{f}^6} sec > 10^2 sec \quad (102)$$

Gluino(\tilde{g}) decays into Goldstino(\tilde{G})

We now consider the three-body decay of the gluino into Goldstino and a quark and anti-quark: $\tilde{g} \rightarrow \tilde{G} + q + \bar{q}$. The Feynman diagrams corresponding to this particular decay are shown in Fig. 8.

The gluino-(anti)-quark-squark vertex will again be given by (148). The gravitino-quark-squark vertex would come from a term of type $\bar{\psi}_\mu \tilde{q}_L q_L$, which in $\mathcal{N} = 1$ gauged supergravity lagrangian of [12] is given by:

$$-g_{I\bar{J}} \left(\partial_\mu \bar{\mathcal{A}}^{\bar{J}} \right) \chi^I \sigma^\nu \bar{\sigma}_\mu \psi_\nu - \frac{i}{2} e^{\frac{K}{2}} (D_I W) \chi^I \sigma^\mu \bar{\psi}_\mu + \text{h.c.} \quad (103)$$

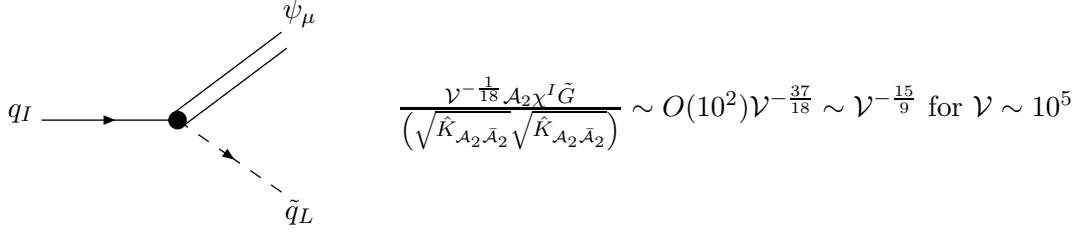


Figure 9: The Goldstino-quark-squark vertex

From [30], the gravitino field can be decomposed into the spin- $\frac{1}{2}$ Goldstino field \tilde{G} via:

$$\psi_\nu = \rho_\nu + \sigma_\nu \tilde{G}, \quad \tilde{G} = -\frac{1}{3} \sigma^\mu \psi_\mu, \quad (104)$$

ρ_ν being a spin- $\frac{3}{2}$ field. Hence, the Goldstino-content of (103), using $\sigma^\nu \bar{\sigma}^\mu \sigma_\nu = -2\bar{\sigma}^\mu$, is given by:

$$2g_{I\bar{J}} \chi^I \left(\partial_\mu \bar{\mathcal{A}}^{\bar{J}} \right) \bar{\sigma}^\mu \tilde{G} + \frac{3i}{2} e^{\frac{K}{2}} (D_I W) \chi^I \tilde{G} + \text{h.c.} \quad (105)$$

Now, utilizing:

$$g_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \sim \mathcal{V}^{-\frac{5}{9}} \text{ and } e^{\frac{K}{2}} D_{\mathcal{A}_2} W \Big|_{\mathcal{A}_2 \rightarrow \mathcal{A}_2 + \mathcal{V}^{-\frac{1}{3}}} \sim \mathcal{V}^{-\frac{37}{18}} \mathcal{A}_2 \quad (106)$$

one obtains:

$$\begin{aligned} & 2g_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \chi^I \left(\bar{\sigma} \cdot \frac{p_{\mathcal{A}_2} + p_{\tilde{G}}}{M_p} \right) \bar{\sigma}^\mu \tilde{G} + \frac{3i}{2} e^{\frac{K}{2}} (D_{\mathcal{A}_2} W) \chi^I \tilde{G} + \text{h.c.} \\ & \sim \mathcal{V}^{-\frac{5}{9}} \chi^I \left(\frac{m_G}{M_p} \right) \mathcal{A}_2 \tilde{G} + \mathcal{V}^{-\frac{5}{9}} \chi^I \left(\frac{m_{q_{\mathcal{A}_2}}}{M_p} \right) \mathcal{A}_2 \tilde{G} + \mathcal{V}^{-\frac{37}{18}} \mathcal{A}_2 \chi^I \tilde{G} \sim \mathcal{V}^{-\frac{37}{18}} \mathcal{A}_2 \chi^I \tilde{G} \end{aligned} \quad (107)$$

for $m_{q_{\mathcal{A}_2}} \sim O(10) MeV, m_G \sim 0$ and $\mathcal{V} \sim 10^5$

The physical Goldstino-quark-squark vertex will be given as:

For this particular case:-

$$\begin{aligned} A_{ij} & \left(Tr \left[G_{\tilde{q}_{\mathcal{A}_2}}^{\tilde{q}_{\mathcal{A}_2}} G_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \right] Tr \left[\tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \right] \right) \sim \tilde{f}^2 \mathcal{V}^{-3} \cdot \mathcal{V}^{-\frac{10}{3}} \sim \tilde{f}^2 \mathcal{V}^{-6}, \\ B_{ij} & \left(Tr \left[G_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} G_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \right] Tr \left[\tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{\tilde{q}_{\mathcal{A}_2}} \tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{\tilde{q}_{\mathcal{A}_2}} \right] \right) \sim \tilde{f}^2 \mathcal{V}^{-3} \cdot \mathcal{V}^{-\frac{10}{3}} \sim \tilde{f}^2 \mathcal{V}^{-6} \\ C & \left(Tr \left[G_{\tilde{q}_{\mathcal{A}_2}}^{\tilde{q}_{\mathcal{A}_2}} G_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \right] Tr \left[\tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{q_{\mathcal{A}_2}} \tilde{G}_{\tilde{q}_{\mathcal{A}_2}}^{\tilde{q}_{\mathcal{A}_2}} \right] \right) \sim \tilde{f}^2 \mathcal{V}^{-3} \cdot \mathcal{V}^{-\frac{10}{3}} \sim \tilde{f}^2 \mathcal{V}^{-6} \end{aligned}$$

Limits of integration as given in [11] are:

$$s_{23 \max} = \left(\mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} - m_q \right)^2, \quad s_{23 \min} = m_q^2,$$

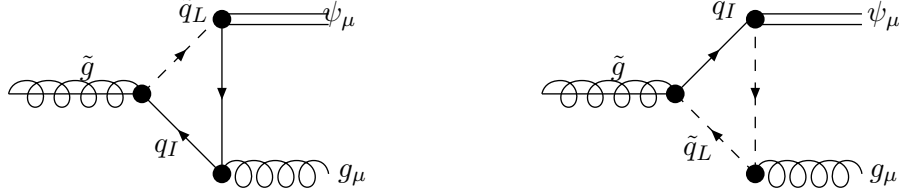


Figure 10: Diagrams contributing to one-loop Gluino decay into Goldstino and gluon

$$s_{13 \max} = \mathcal{V}^{\frac{4}{3}} m_{\frac{3}{2}}^2 - s_{23}, s_{13 \min} = 0$$

where $m_{\tilde{g}} = \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} \sim 10^{11} \text{ GeV}$, $m_{\tilde{G}} = 0$. Utilizing the values of vertex elements calculated above and from (79), we can calculate decay width for Gluino in this particular case.

$$\begin{aligned} \Gamma(\tilde{g} \rightarrow \chi_n^0 q_I \bar{q}_J) &\sim \frac{g_s^2}{256\pi^3 \mathcal{V}^2 m_{\frac{3}{2}}^3} \left[-\tilde{f}^2 \mathcal{V}^{-6} m_{\frac{3}{2}}^4 \mathcal{V}^{\frac{8}{3}} + \tilde{f}^2 \mathcal{V}^{-\frac{37}{6}} \mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4 - \tilde{f}^2 \mathcal{V}^{-6} \mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4 \right] \\ &\sim \frac{g_s^2}{256\pi^3 \mathcal{V}^2 m_{\frac{3}{2}}^3} (\tilde{f}^2 \mathcal{V}^{-6} \mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4) \\ &\sim O(10^{-4}) \mathcal{V}^{-5} m_{\frac{3}{2}} \tilde{f}^2 \text{ GeV} < 10^{-21} \tilde{f}^2 \text{ GeV} < O(10^{-29}) \text{ GeV} \end{aligned} \quad (108)$$

The life time of gluino is given as:

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} \text{ Jsec}}{10^{-21} \tilde{f}^2 \text{ GeV}} \sim \frac{10^{-4}}{\tilde{f}^2} > 10^4 \text{ sec} \quad (109)$$

We now consider the two-body decay of the gluino into a Goldstino and a gluon:

The matrix element for the above will be given by:

$$\begin{aligned} \mathcal{M} &\sim \tilde{f}^2 \int \frac{d^4 k}{(2\pi)^4} \times \mathcal{V}^{-\frac{3}{2}} \left(\frac{i \bar{\sigma} \cdot k}{k^2 - m_q^2 + i\epsilon} \right) \left(\mathcal{V}^{-\frac{15}{9}} \right) \left(\frac{i}{[(k - p_{\tilde{G}})^2 - m_q^2 + i\epsilon]} \right) \\ &\times \left(\mathcal{V}^{\frac{8}{45}} \epsilon \cdot (2k - p_{\tilde{G}} - p_{\tilde{g}}) \right) \left(\frac{i}{[(k - p_{\tilde{g}})^2 - m_q^2 + i\epsilon]} \right) + \\ &\tilde{f}^2 \int \frac{d^4 k}{(2\pi)^4} \times \mathcal{V}^{-\frac{3}{2}} \left(\frac{i}{[(k + p_{\tilde{G}})^2 - m_q^2 + i\epsilon]} \right) \left(\mathcal{V}^{-\frac{15}{9}} \right) \left(\mathcal{V}^{-\frac{23}{30}} \bar{\sigma} \cdot \epsilon \right) \\ &\times \left(\frac{i \bar{\sigma} \cdot (k - p_{g_\mu})}{[(k - p_{g_\mu})^2 - m_q^2 + i\epsilon]} \right) \end{aligned} \quad (110)$$

As discussed in [11], the Wilson coefficients corresponding to Gluino-Goldstino- Gluon coupling do not change much upon RG evolution to EW scale. The matrix amplitude in equation (110) can be

written as:

$$\begin{aligned}
& \tilde{f}^2 \mathcal{V}^{-3} \left[\left\{ \bar{\sigma} \cdot p_{\tilde{G}} C_{11}^{(a)} + \bar{\sigma} \cdot p_{g_\mu} C_{12}^{(a)} \right\} (2\epsilon \cdot p_{\tilde{G}}) + \left\{ \bar{\sigma} \cdot p_{\tilde{G}} \epsilon \cdot p_{\tilde{G}} C_{21}^{(a)} + \bar{\sigma} \cdot p_{g_\mu} \epsilon \cdot p_{\tilde{G}} C_{23}^{(a)} + \bar{\sigma} \cdot \epsilon C_{24}^{(a)} \right\} \right] \\
& + \tilde{f}^2 \mathcal{V}^{-4} \left[- \left\{ \bar{\sigma} \cdot p_{\tilde{G}} C_{11}^{(b)} + \bar{\sigma} \cdot p_{\tilde{g}} C_{12}^{(b)} \right\} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{g_\mu} + \bar{\sigma} \cdot p_{\tilde{G}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{G}} C_{21}^{(b)} + \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{g}} C_{22}^{(b)} \right. \\
& \left. - \left(\bar{\sigma} \cdot p_{\tilde{G}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{g}} + \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{G}} \right) C_{23}^{(b)} + \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right], \tag{111}
\end{aligned}$$

which equivalently could be rewritten as:

$$\tilde{f}^2 \bar{u}(p_{\tilde{G}}) \left[\bar{\sigma} \cdot \mathcal{A} + \bar{\sigma} \cdot p_{\tilde{G}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot \mathcal{B}_1 + \bar{\sigma} \cdot p_{g_\mu} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot \mathcal{B}_2 + D_4 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] u(p_{\tilde{g}}), \tag{112}$$

where

$$\begin{aligned}
\mathcal{A}^\mu &\equiv \mathcal{V}^{-3} \left[p_{\tilde{G}}^\mu \epsilon \cdot p_{\tilde{G}} \left(2C_{11}^{(a)} + C_{21}^{(a)} \right) + p_{g_\mu}^\mu \epsilon \cdot p_{\tilde{G}} \left(C_{12}^{(a)} + C_{23}^{(a)} - C_{22}^{(b)} \right) + \epsilon^\mu C_{24}^{(a)} \right]; \\
\mathcal{B}_1^\mu &\equiv \mathcal{V}^{-4} \left[-p_{g_\mu}^\mu \left(C_{11}^{(b)} + C_{12}^{(b)} + C_{23}^{(b)} - C_{22}^{(b)} \right) + p_{\tilde{G}}^\mu \left(C_{21}^{(b)} + C_{22}^{(b)} - 2C_{23}^{(b)} \right) \right]; \\
\mathcal{B}_2^\mu &\equiv \mathcal{V}^{-4} \left[p_{g_\mu}^\mu \left(C_{12}^{(b)} + C_{22}^{(b)} \right) + p_{\tilde{G}}^\mu \left(C_{22}^{(b)} - C_{23}^{(b)} \right) \right]; \\
D_4 &\equiv \mathcal{V}^{-4}. \tag{113}
\end{aligned}$$

This time around replacing $\bar{u}(p_{\tilde{G}}) \bar{\sigma} \cdot p_{\tilde{G}}$ by 0 and $\bar{\sigma} \cdot p_{\tilde{g}} u(p_{\tilde{g}})$ by $m_{\tilde{g}} u(p_{\tilde{g}})$, (112) can be rewritten as:

$$\tilde{f}^2 \bar{u}(p_{\tilde{G}}) \left(A_2 \bar{\sigma} \cdot \epsilon + B_3 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{G}} + D_4 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right) u(p_{\tilde{g}}), \tag{114}$$

where

$$\begin{aligned}
A_2 &\equiv \mathcal{V}^{-3} C_{24}^{(a)}; B_3 \equiv \mathcal{V}^{-4} M_{\tilde{g}} (C_{12}^{(b)} + C_{22}^{(b)}) \\
D_2 &\equiv \mathcal{V}^{-4} \left(-C_{12}^{(b)} + C_{23}^{(b)} \right); D_4 \equiv \mathcal{V}^{-4}. \tag{115}
\end{aligned}$$

Results of various C's functions required for this particular case similar to the ones given in [11] are:

$$\begin{aligned}
C_{24}^{(a)} &= C_{24}^{(b)} \sim O(1); \\
C_0^{(a)} &\sim O(10) \times 10^{-22} GeV^{-2}; \\
C_{12}^{(b)} &\sim O(10) \times 10^{-22} GeV^{-2}; \\
C_{11}^{(b)} &\sim O(10) \times 10^{-22} GeV^{-2}; \\
C_{22}^{(b)} &= C_{23}^{(b)} \sim O(10) \times 10^{-22} GeV^{-2}; \\
C_0^{(b)} &= O(10) \times 10^{-22} GeV^{-2}. \tag{116}
\end{aligned}$$

Utilizing (116), one gets: $A_2 \equiv O(1) \times 10^{-15}$, $B_3 \equiv O(1) \times 10^{-31} GeV^{-1}$, $D_2 \equiv O(1) \times 10^{-42} GeV^{-2}$, $D_4 \equiv \mathcal{V}^{-4}$

$$\sum_{\tilde{g} \text{ and } \tilde{G} \text{ spins}} |\mathcal{M}|^2 \sim \tilde{f}^4 Tr \left(\sigma \cdot p_{\tilde{G}} \left[A_2 \bar{\sigma} \cdot \epsilon + B_3 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{G}} + D_4 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right] \right. \\ \left. \times \sigma \cdot p_{\tilde{g}} \left[A_2 \bar{\sigma} \cdot \epsilon + B_3 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon + D_2 \bar{\sigma} \cdot p_{\tilde{g}} \bar{\sigma} \cdot \epsilon \bar{\sigma} \cdot p_{\tilde{G}} + D_4 \bar{\sigma}_\mu \bar{\sigma} \cdot \epsilon \bar{\sigma}^\mu C_{24}^{(b)} \right]^\dagger \right), \quad (117)$$

which at:

$$p_{\tilde{G}}^0 = m_{\tilde{g}}/2, p_{\tilde{G}}^1 = p_{\tilde{G}}^2 = p_{\tilde{G}}^3 = \frac{m_{\tilde{g}}}{2\sqrt{3}}, \quad (118)$$

yields:

$$\tilde{f}^4 m_{\tilde{g}}^2 \left[D_2^2 m_{\tilde{g}}^4 + \frac{1}{6} \left(6A_2 + 12D_4 C_{24}^{(b)} + m_{\tilde{g}} \left(6B_3 + (3 + \sqrt{3}) D_2 m_{\tilde{g}} \right) \right)^2 \right] \sim \tilde{f}^4 A_2^2 m_{\tilde{g}}^2. \quad (119)$$

So, using results from [29], the decay width comes out to be equal to:

$$\Gamma = \frac{\sum_{\tilde{g} \text{ and } \tilde{G} \text{ spins}} |\mathcal{M}|^2}{16\pi \hbar m_{\tilde{g}}} \sim O(10^{-1}) \tilde{f}^4 A_2^2 m_{\tilde{g}} \sim 10^{-20} \tilde{f}^4 GeV. \quad (120)$$

since $\tilde{f}^2 < 10^{-8}$ as calculated above, $\Gamma < 10^{-36} \text{ GeV}$. Life time of gluino is given as:

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} Jsec}{10^{-20} \tilde{f}^4 GeV} \sim \frac{10^{-5}}{\tilde{f}^4} sec > 10^{11} sec \quad (121)$$

3.2 R-Parity Violating Neutralino Decays

In the following we will be discussing the decay width of tree level decay diagrams of neutralino mediated by sleptons/squarks, Higgs and gauge Bosons.

Decays mediated via squarks and sleptons

Now, we evaluate all possible neutralino decays which are mediated by squarks/sleptons and also involve R-parity violating vertices as shown in Fig. 12. The life time calculation requires the evaluation of various matrix amplitudes corresponding to various decay channels. In particular, to evaluate the contribution of neutralino-squark-quark/neutralino-slepton-lepton, we will explicitly work out squark-quark-gaugino vertex replacing the gaugino by $-\tilde{\chi}_3^0$ with mass half of that of the gluino and also the squark-quark-Higgsino vertex replacing the Higgsino by $\tilde{f}\tilde{\chi}_3^0$, and then add these contributions. To calculate the contribution of various interaction vertices shown in Fig.12 in the context of $N = 1$ gauged supergravity, one needs to consider the following terms of gauged supergravity action of Wess and Bagger [12].

$$\mathcal{L} = g_{YM} g_{\alpha\bar{J}} X^\alpha \bar{\chi}^{\bar{J}} \lambda_{\tilde{g}} + i g_{i\bar{J}} \bar{\chi}^{\bar{J}} \left[\bar{\sigma} \cdot \partial \chi^i + \Gamma_{Lj}^i \bar{\sigma} \cdot \partial a^L \chi^j + \frac{1}{4} (\partial_{aL} K \bar{\sigma} \cdot a_L - \text{c.c.}) \chi^i \right] \\ + \frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_i D_J W) \chi^i \chi^J + \text{h.c.}, \quad (122)$$

where, X^α corresponds to the components of a killing isometry vector. From here, one notes that $X^\alpha = -6i\kappa_4^2 \mu_7 Q_\alpha$, where $\alpha = B, Q_\alpha = 2\pi\alpha' \int_{T_B} i^* \omega_\alpha \wedge P_- \tilde{f}$ where P_- is a harmonic zero-form on Σ_B

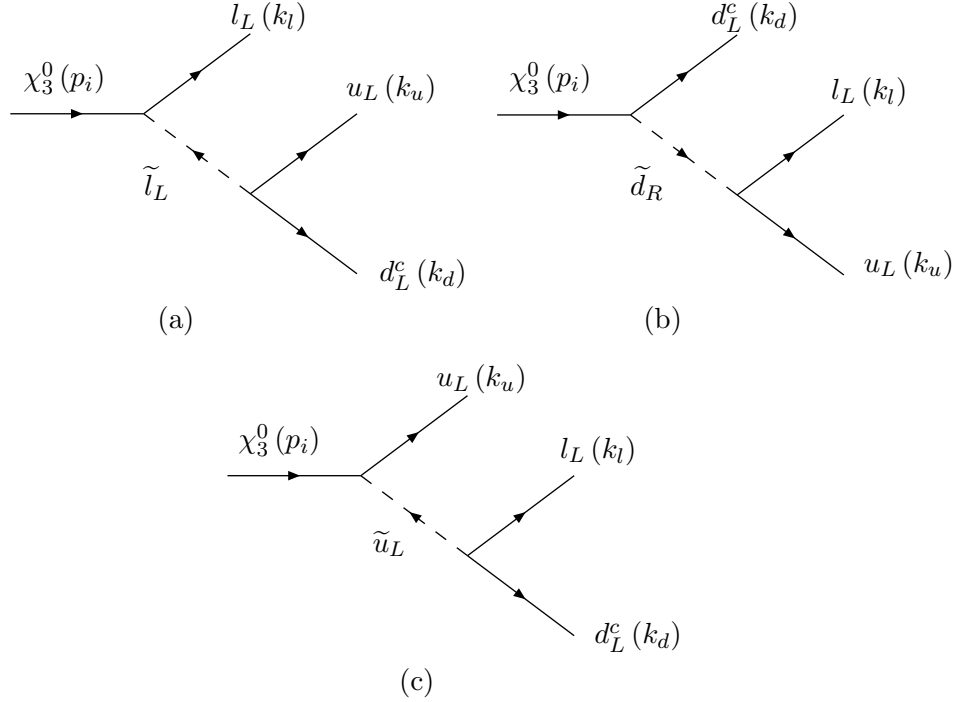


Figure 11: Feynman diagrams for the R-parity violating decays of Neutralino χ_3^0 .

taking value +1 on Σ_B and -1 on $\sigma(\Sigma_B)$ - σ being a holomorphic isometric involution as part of the Calabi-Yau orientifold - and $\tilde{f} \in \tilde{H}_-(\Sigma^B) \equiv \text{coker} \left(H_-^2(CY_3) \xrightarrow{i^*} H_-^2(\Sigma^B) \right)$. Also,

$$\mathcal{D}_i \mathcal{D}_j W = \partial_i \partial_j W + (\partial_i \partial_j K) W + \partial_i K \mathcal{D}_j W + \partial_j K \mathcal{D}_i W - (\partial_i K \partial_j K) W - \Gamma_{ij}^k \mathcal{D}_k W. \quad (123)$$

Considering the the fluctuations $z_i \rightarrow z_1 + \mathcal{V}^{\frac{1}{36}} M_p$, $a_1 \rightarrow a_1 + \mathcal{V}^{-\frac{2}{9}} M_p$, $a_2 \rightarrow a_2 + \mathcal{V}^{-\frac{1}{3}} M_p$, $a_3 \rightarrow a_3 + \mathcal{V}^{-\frac{13}{18}} M_p$, $a_3 \rightarrow a_3 + \mathcal{V}^{-\frac{11}{9}} M_p$, one can show that near (5) along which $|z_1|$ and $|z_2|$ are on the same footing:

$$g_{A\bar{B}} \sim \begin{pmatrix} \frac{1}{\mathcal{V}^{2/3}} & \frac{1}{\mathcal{V}^{5/12}} & \frac{1}{\mathcal{V}^{11/12}} & \frac{12\sqrt{\mathcal{V}}}{\mathcal{V}^{17/18}} & \frac{\mathcal{V}^{7/12}}{\mathcal{V}^{13/9}} \\ \frac{1}{\mathcal{V}^{5/12}} & \frac{1}{\mathcal{V}^{4/9}} & \frac{1}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{13/9}} & \frac{\mathcal{V}^{17/18}}{\mathcal{V}^{35/18}} \\ \frac{1}{\mathcal{V}^{11/12}} & \frac{1}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{5/9}} & \frac{1}{\mathcal{V}^{13/9}} & \frac{\mathcal{V}^{17/18}}{\mathcal{V}^{35/18}} \\ \frac{12\sqrt{\mathcal{V}}}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{13/9}} & \frac{1}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{35/18}} & \frac{\mathcal{V}^{22/9}}{\mathcal{V}^{22/9}} \end{pmatrix} + \begin{pmatrix} \frac{1}{\mathcal{V}} & \frac{1}{\mathcal{V}^{4/9}} & \frac{1}{\mathcal{V}^{17/18}} & \frac{18\sqrt{\mathcal{V}}}{\mathcal{V}^{11/36}} & \frac{\mathcal{V}^{5/9}}{\mathcal{V}^{29/36}} \\ \frac{1}{\mathcal{V}^{4/9}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\mathcal{V}^{25/36}} & \frac{1}{\mathcal{V}^{11/36}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{47/36}} \\ \frac{1}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{25/36}} & \frac{1}{\mathcal{V}^{43/36}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{47/36}} \\ \frac{18\sqrt{\mathcal{V}}}{\mathcal{V}^{11/36}} & \frac{1}{\mathcal{V}^{11/36}} & \frac{1}{\mathcal{V}^{7/36}} & \frac{1}{\mathcal{V}^{47/36}} & \frac{\mathcal{V}^{65/36}}{\mathcal{V}^{65/36}} \end{pmatrix} \delta z_1 +$$

$$\begin{pmatrix} \frac{1}{\mathcal{V}^{4/9}} & \frac{1}{\mathcal{V}^{7/36}} & \frac{1}{\mathcal{V}^{25/36}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{29/36}}{\mathcal{V}^{5/3}} \\ \frac{1}{\mathcal{V}^{7/36}} & \frac{1}{\mathcal{V}^{2/3}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{2/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{7/6}} \\ \frac{1}{\mathcal{V}^{25/36}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\mathcal{V}^{5/6}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{13/6}} \\ \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{29/36}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{8/3}} \end{pmatrix} \delta a_1 + \begin{pmatrix} \frac{1}{\mathcal{V}^{17/18}} & \frac{1}{\mathcal{V}^{25/36}} & \frac{1}{\mathcal{V}^{43/36}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} \\ \frac{1}{\mathcal{V}^{25/36}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\mathcal{V}^{5/6}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{2/3}} \\ \frac{1}{\mathcal{V}^{43/36}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{5/3}} \\ \frac{1}{\mathcal{V}^{7/36}} & \frac{1}{\mathcal{V}^{2/3}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{5/3}} \\ \frac{1}{\mathcal{V}^{11/36}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{13/6}} \end{pmatrix} \delta a_2$$

$$\begin{pmatrix} \frac{18\sqrt{\mathcal{V}}}{\mathcal{V}^{11/36}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} & \frac{1}{\mathcal{V}^{7/36}} & \frac{\mathcal{V}^{29/36}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{47/36}}{\mathcal{V}^{13/6}} \\ \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{5/3}} & \frac{1}{\mathcal{V}^{2/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{8/3}} \\ \frac{1}{\mathcal{V}^{7/36}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{5/3}} & \frac{1}{\sqrt{\mathcal{V}}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{19/6}} \\ \frac{\mathcal{V}^{29/36}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{8/3}} & \frac{\mathcal{V}^{19/6}}{\mathcal{V}^{19/6}} \\ \frac{\mathcal{V}^{47/36}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{8/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{8/3}}{\mathcal{V}^{19/6}} & \frac{\mathcal{V}^{19/6}}{\mathcal{V}^{11/3}} \end{pmatrix} \delta a_3 + \begin{pmatrix} \frac{\mathcal{V}^{5/9}}{\mathcal{V}^{29/36}} & \frac{\mathcal{V}^{29/36}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{47/36}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{65/36}}{\mathcal{V}^{8/3}} \\ \frac{\mathcal{V}^{29/36}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{2/3}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{8/3}}{\mathcal{V}^{13/6}} \\ \frac{\mathcal{V}^{11/36}}{\mathcal{V}^{7/6}} & \frac{\mathcal{V}^{7/6}}{\mathcal{V}^{2/3}} & \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{8/3}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{19/6}} \\ \frac{\mathcal{V}^{47/36}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{5/3}} & \frac{\mathcal{V}^{5/3}}{\mathcal{V}^{8/3}} & \frac{\mathcal{V}^{8/3}}{\mathcal{V}^{19/6}} & \frac{\mathcal{V}^{19/6}}{\mathcal{V}^{11/3}} \\ \frac{\mathcal{V}^{65/36}}{\mathcal{V}^{8/3}} & \frac{\mathcal{V}^{8/3}}{\mathcal{V}^{13/6}} & \frac{\mathcal{V}^{13/6}}{\mathcal{V}^{19/6}} & \frac{\mathcal{V}^{19/6}}{\mathcal{V}^{11/3}} & \frac{\mathcal{V}^{11/3}}{\mathcal{V}^{11/3}} \end{pmatrix} \delta a_4 \quad (124)$$

$$\begin{aligned}
g^{A\bar{B}} \sim & \begin{pmatrix} \nu^{2/3} & \frac{1}{\nu^{7/36}} & \nu^{11/36} & \frac{1}{\nu^{25/36}} & \frac{1}{\nu^{43/36}} \\ \frac{1}{\nu^{7/36}} & \frac{1}{\nu^{4/9}} & \sqrt[18]{\nu} & \frac{1}{\nu^{17/18}} & \frac{1}{\nu^{13/9}} \\ \nu^{11/36} & \frac{1}{\sqrt[18]{\nu}} & \nu^{5/9} & \frac{1}{\nu^{17/18}} & \frac{1}{\nu^{17/18}} \\ \frac{1}{\nu^{25/36}} & \frac{1}{\nu^{17/18}} & \frac{1}{\nu^{4/9}} & \frac{1}{\nu^{13/9}} & \frac{1}{\nu^{35/18}} \\ \frac{1}{\nu^{43/36}} & \frac{1}{\nu^{13/9}} & \frac{1}{\nu^{17/18}} & \frac{1}{\nu^{35/18}} & \frac{1}{\nu^{22/9}} \end{pmatrix} + \begin{pmatrix} \sqrt[3]{\nu} & \frac{1}{\nu^{2/9}} & \nu^{5/18} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} \\ \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{13/12}} & \frac{1}{\nu^{7/12}} & \frac{1}{\nu^{19/12}} & \frac{1}{\nu^{25/12}} \\ \nu^{5/18} & \frac{1}{\nu^{7/12}} & \frac{1}{\sqrt[12]{\nu}} & \frac{1}{\nu^{13/12}} & \frac{1}{\nu^{19/12}} \\ \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{19/12}} & \frac{1}{\nu^{13/12}} & \frac{1}{\nu^{25/12}} & \frac{1}{\nu^{31/12}} \\ \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{25/12}} & \frac{1}{\nu^{19/12}} & \frac{1}{\nu^{31/12}} & \frac{1}{\nu^{37/12}} \end{pmatrix} \delta z_1 + \\
& \begin{pmatrix} \nu^{8/9} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{19/36} & \frac{1}{\nu^{17/36}} & 0 \\ \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{2/9}} & \nu^{5/18} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} \\ \nu^{19/36} & \nu^{5/18} & \nu^{7/9} & \frac{1}{\nu^{2/9}} & 0 \\ \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{31/18}} \\ 0 & \frac{1}{\nu^{11/9}} & 0 & \frac{1}{\nu^{31/18}} & 0 \end{pmatrix} \delta a_1 + \begin{pmatrix} \nu^{7/18} & \frac{1}{\nu^{17/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{35/36}} & \frac{1}{\nu^{53/36}} \\ \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{31/18}} \\ \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{2/9}} & \nu^{5/18} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} \\ \frac{1}{\nu^{35/36}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{31/18}} & \frac{1}{\nu^{20/9}} \\ \frac{1}{\nu^{53/36}} & \frac{1}{\nu^{31/18}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{20/9}} & \frac{1}{\nu^{49/18}} \end{pmatrix} a_2 \\
& \begin{pmatrix} \nu^{25/18} & \nu^{19/36} & \nu^{37/36} & 0 & 0 \\ \nu^{19/36} & \nu^{5/18} & \nu^{7/9} & 0 & 0 \\ \nu^{37/36} & \nu^{7/9} & \nu^{23/18} & \nu^{5/18} & \frac{1}{\nu^{2/9}} \\ 0 & 0 & \nu^{5/18} & \nu^{13/18} & 0 \\ 0 & 0 & \frac{1}{\nu^{2/9}} & 0 & \frac{1}{\nu^{31/18}} \end{pmatrix} \delta a_3 + \begin{pmatrix} \nu^{17/9} & \nu^{37/36} & \nu^{55/36} & \nu^{19/36} & \frac{36\sqrt{\nu}}{\nu^{17/36}} \\ \nu^{37/36} & \nu^{7/9} & \nu^{23/18} & \nu^{5/18} & \frac{1}{\nu^{2/9}} \\ \nu^{55/36} & \nu^{23/18} & \nu^{16/9} & \nu^{7/9} & \nu^{5/18} \\ \nu^{19/36} & \nu^{5/18} & \nu^{7/9} & \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{13/18}} \\ \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{2/9}} & \nu^{5/18} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} \end{pmatrix} a_4 \quad (125)
\end{aligned}$$

$$\begin{aligned}
\partial_{z_1} g_{A\bar{B}} \sim & \begin{pmatrix} \frac{1}{\nu} & \frac{1}{\nu^{4/9}} & \frac{1}{\nu^{17/18}} & \sqrt[18]{\nu} & \nu^{5/9} \\ \frac{1}{\nu^{4/9}} & \frac{1}{\sqrt{\nu}} & \frac{1}{\nu^{25/36}} & 60 & \nu^{29/36} \\ \frac{1}{\nu^{17/18}} & \frac{1}{\sqrt[18]{\nu}} & \frac{1}{\nu^{43/36}} & \frac{1}{\nu^{7/36}} & \nu^{11/36} \\ \sqrt[18]{\nu} & 60 & \frac{1}{\nu^{7/36}} & \sqrt{\nu} & \nu^{47/36} \\ \nu^{5/9} & \nu^{29/36} & \nu^{11/36} & \nu^{47/36} & \nu^{65/36} \end{pmatrix} + \begin{pmatrix} \frac{1}{\nu^{37/36}} & \frac{1}{\nu^{7/9}} & \frac{1}{\nu^{23/18}} & \frac{1}{\nu^{5/18}} & \nu^{2/9} \\ \frac{1}{\nu^{7/9}} & \frac{1}{\nu^{19/36}} & \frac{1}{\nu^{13/18}} & \frac{1}{\sqrt[36]{\nu}} & \nu^{7/9} \\ \frac{1}{\nu^{23/18}} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{2/9}} & \nu^{5/18} \\ \frac{1}{\nu^{5/18}} & \frac{1}{\sqrt[36]{\nu}} & \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{17/36}} & \nu^{23/18} \\ \nu^{2/9} & \nu^{7/9} & \nu^{5/18} & \nu^{23/18} & \nu^{16/9} \end{pmatrix} \delta z_1 \\
& \begin{pmatrix} \frac{1}{\nu^{7/9}} & \frac{1}{\nu^{2/9}} & \frac{1}{\nu^{13/18}} & \nu^{5/18} & \nu^{7/9} \\ \frac{1}{\nu^{2/9}} & \frac{1}{\sqrt[36]{\nu}} & \frac{1}{\nu^{17/36}} & \nu^{19/36} & \nu^{37/36} \\ \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{35/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{19/36} \\ \nu^{5/18} & \nu^{19/36} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{37/36} & \nu^{55/36} \\ \nu^{7/9} & \nu^{37/36} & \nu^{19/36} & \nu^{55/36} & \nu^{73/36} \end{pmatrix} \delta a_1 + \begin{pmatrix} \frac{1}{\nu^{23/18}} & \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{2/9}} & \nu^{5/18} \\ \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{35/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{19/36} \\ \frac{1}{\nu^{11/9}} & \frac{1}{\nu^{35/36}} & \frac{1}{\nu^{53/36}} & \frac{1}{\nu^{17/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} \\ \frac{1}{\nu^{2/9}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{19/36}} & \frac{1}{\nu^{37/36}} & \nu^{37/36} \\ \nu^{5/18} & \nu^{19/36} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{37/36} & \nu^{55/36} \end{pmatrix} \delta a_2 \\
& \begin{pmatrix} \frac{1}{\nu^{5/18}} & \nu^{5/18} & \frac{1}{\nu^{2/9}} & \nu^{7/9} & \nu^{23/18} \\ \nu^{5/18} & \nu^{19/36} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{37/36} & \nu^{55/36} \\ \frac{1}{\nu^{2/9}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{17/36}} & \nu^{19/36} & \nu^{37/36} \\ \nu^{7/9} & \nu^{37/36} & \nu^{19/36} & \nu^{55/36} & \nu^{73/36} \\ \nu^{23/18} & \nu^{55/36} & \nu^{37/36} & \nu^{73/36} & \nu^{91/36} \end{pmatrix} \delta a_3 + \begin{pmatrix} \nu^{2/9} & \nu^{7/9} & \nu^{5/18} & \nu^{23/18} & \nu^{16/9} \\ \nu^{7/9} & \nu^{37/36} & \nu^{19/36} & \nu^{55/36} & \nu^{73/36} \\ \nu^{5/18} & \nu^{19/36} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{37/36} & \nu^{55/36} \\ \nu^{23/18} & \nu^{55/36} & \nu^{37/36} & \nu^{73/36} & \nu^{91/36} \\ \nu^{16/9} & \nu^{73/36} & \nu^{55/36} & \nu^{91/36} & \nu^{109/36} \end{pmatrix} \delta a_4 \quad (126)
\end{aligned}$$

$$\begin{aligned}
\partial_{a_1} g_{A\bar{B}} \sim & \begin{pmatrix} \frac{1}{\nu^{4/9}} & \frac{1}{\sqrt{\nu}} & \frac{1}{\nu^{25/36}} & 81 & \nu^{29/36} \\ \frac{1}{\sqrt{\nu}} & \nu^{2/3} & \sqrt[6]{\nu} & \nu^{7/6} & \nu^{5/3} \\ \frac{1}{\nu^{25/36}} & \sqrt[6]{\nu} & \frac{1}{\sqrt[3]{\nu}} & \nu^{2/3} & \nu^{7/6} \\ 81 & \nu^{7/6} & \nu^{2/3} & \nu^{5/3} & \nu^{13/6} \\ \nu^{29/36} & \nu^{5/3} & \nu^{7/6} & \nu^{13/6} & \nu^{8/3} \end{pmatrix} + \begin{pmatrix} \frac{1}{\nu^{7/9}} & \frac{1}{\nu^{19/36}} & \frac{1}{\nu^{13/18}} & \frac{1}{\sqrt[36]{\nu}} & \nu^{7/9} \\ \frac{1}{\nu^{19/36}} & \frac{1}{\sqrt[36]{\nu}} & \frac{1}{\nu^{17/36}} & \nu^{19/36} & \nu^{37/36} \\ \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{35/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{19/36} \\ \frac{1}{\sqrt[36]{\nu}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{37/36}} & \nu^{37/36} & \nu^{55/36} \\ \nu^{7/9} & \nu^{37/36} & \nu^{19/36} & \nu^{55/36} & \nu^{73/36} \end{pmatrix} \delta z_1 + \\
& \begin{pmatrix} \frac{1}{\nu^{2/9}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \frac{1}{\nu^{17/36}} & \nu^{19/36} & \nu^{37/36} \\ \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{8/9} & \frac{1}{\nu^{7/18}} & \nu^{25/18} & \nu^{25/18} \\ \frac{1}{\nu^{17/36}} & \nu^{7/18} & \frac{1}{\sqrt[3]{\nu}} & \nu^{8/9} & \nu^{25/18} \\ \nu^{19/36} & \nu^{25/18} & \frac{1}{\sqrt[3]{\nu}} & \nu^{8/9} & \nu^{17/9} \\ \nu^{37/36} & \nu^{25/18} & \nu^{25/18} & \nu^{43/18} & \nu^{26/9} \end{pmatrix} \delta a_1 + \begin{pmatrix} \frac{1}{\nu^{13/18}} & \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{17/36}} & \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{19/36} \\ \frac{1}{\nu^{17/36}} & \frac{1}{\nu^{7/18}} & \frac{1}{\sqrt[3]{\nu}} & \frac{1}{\nu^{8/9}} & \nu^{25/18} \\ \frac{1}{\nu^{17/36}} & \frac{1}{\sqrt[3]{\nu}} & \frac{1}{\nu^{11/18}} & \nu^{7/18} & \nu^{8/9} \\ \frac{36\sqrt{\nu}}{\nu^{17/36}} & \nu^{8/9} & \frac{1}{\nu^{7/18}} & \nu^{25/18} & \nu^{17/9} \\ \nu^{19/36} & \nu^{25/18} & \nu^{8/9} & \nu^{17/9} & \nu^{43/18} \end{pmatrix} \delta a_2
\end{aligned}$$

$$\left(\begin{array}{ccccc} \gamma_{5/18} & \gamma_{19/36} & 3\sqrt{\gamma} & \gamma_{37/36} & \gamma_{55/36} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ 3\sqrt{\gamma} & \gamma_{8/9} & \gamma_{7/18} & \gamma_{25/18} & \gamma_{17/9} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \end{array} \right) \delta a_3 + \left(\begin{array}{ccccc} \gamma_{7/9} & \gamma_{37/36} & \gamma_{19/36} & \gamma_{55/36} & \gamma_{73/36} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \\ \gamma_{73/36} & \gamma_{26/9} & \gamma_{43/18} & \text{Null}^{61/18} & \gamma_{35/9} \end{array} \right) \delta a_4 \quad (127)$$

$$\partial_{a2} g_{A\bar{B}} \sim \begin{pmatrix} \frac{1}{\sqrt{17/18}} & \frac{1}{\sqrt[6]{25/36}} & \frac{1}{\sqrt[3]{43/36}} & \frac{1}{\sqrt[7]{36}} & \sqrt[11]{1/36} \\ \frac{1}{\sqrt{25/36}} & \frac{1}{\sqrt[3]{9}} & \frac{1}{\sqrt[3]{9}} & \sqrt[7]{6} & \sqrt[11]{36} \\ \frac{1}{\sqrt[3]{43/36}} & \frac{1}{\sqrt[3]{9}} & \frac{1}{\sqrt[6]{8/6}} & \sqrt[6]{9} & \sqrt[11]{36} \\ \frac{1}{\sqrt[7]{36}} & \sqrt[11]{36} & \sqrt[6]{9} & \sqrt[11]{36} & \sqrt[13]{6} \\ \frac{1}{\sqrt[11]{36}} & \sqrt[11]{36} & \sqrt[11]{36} & \sqrt[13]{6} & \sqrt[13]{6} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{23/18}} & \frac{1}{\sqrt{13/18}} & \frac{1}{\sqrt[11]{9}} & \frac{1}{\sqrt[36]{9}} & \sqrt[5]{18} \\ \frac{1}{\sqrt{13/18}} & \frac{1}{\sqrt{17/36}} & \frac{1}{\sqrt[35]{36}} & \frac{1}{\sqrt[36]{9}} & \sqrt[19]{36} \\ \frac{1}{\sqrt[11]{9}} & \frac{1}{\sqrt[35]{36}} & \frac{1}{\sqrt[53]{36}} & \frac{1}{\sqrt[17]{7/36}} & \sqrt[36]{9} \\ \frac{1}{\sqrt[2]{9}} & \frac{1}{\sqrt[36]{9}} & \frac{1}{\sqrt[17]{7/36}} & \frac{1}{\sqrt[19]{36}} & \sqrt[37]{36} \\ \frac{1}{\sqrt[5]{18}} & \sqrt[19]{36} & \sqrt[36]{9} & \sqrt[37]{36} & \sqrt[55]{36} \end{pmatrix} \delta z_1 +$$

$$\begin{pmatrix} \frac{1}{\sqrt[13]{18}} & \frac{1}{\sqrt[17]{36}} & \frac{\sqrt[35]{36}}{\sqrt[9]{V}} & \frac{\sqrt[36]{V}}{\nu^{8/9}} & \frac{\sqrt[36]{V}}{\nu^{25/18}} \\ \frac{1}{\sqrt[17]{36}} & \frac{1}{\sqrt[9]{V}} & \frac{1}{\sqrt[11]{18}} & \nu^{7/18} & \nu^{8/9} \\ \frac{\sqrt[35]{36}}{\sqrt[36]{V}} & \frac{1}{\sqrt[9]{V}} & \frac{1}{\sqrt[11]{18}} & \nu^{7/18} & \nu^{8/9} \\ \frac{\sqrt[36]{V}}{\nu^{8/9}} & \nu^{8/9} & \nu^{17/18} & \nu^{25/18} & \nu^{17/9} \\ \frac{\sqrt[36]{V}}{\nu^{25/18}} & \nu^{25/18} & \nu^{8/9} & \nu^{17/9} & \nu^{43/18} \end{pmatrix} \delta a_1 + \begin{pmatrix} \frac{1}{\sqrt[11]{9}} & \frac{1}{\sqrt[35]{36}} & \frac{1}{\sqrt[53]{36}} & \frac{1}{\sqrt[17]{36}} & \frac{\sqrt[36]{V}}{\nu^{17/36}} \\ \frac{1}{\sqrt[35]{36}} & \frac{1}{\sqrt[9]{V}} & \frac{1}{\sqrt[11]{18}} & \frac{1}{\sqrt[10]{9}} & \frac{1}{\sqrt[9]{V}} \\ \frac{1}{\sqrt[53]{36}} & \frac{1}{\sqrt[11]{18}} & \frac{1}{\sqrt[10]{9}} & \frac{1}{\sqrt[9]{V}} & \nu^{7/18} \\ \frac{1}{\sqrt[17]{36}} & \nu^{7/18} & \frac{1}{\sqrt[9]{V}} & \nu^{8/9} & \nu^{25/18} \\ \frac{\sqrt[36]{V}}{\nu^{8/9}} & \nu^{8/9} & \nu^{7/18} & \nu^{25/18} & \nu^{17/9} \end{pmatrix} \delta a_2$$

$$\begin{pmatrix} \frac{1}{\sqrt[12]{9}} & \frac{\sqrt[36]{V}}{\nu^{8/9}} & \frac{1}{\sqrt[17]{36}} & \nu^{19/36} & \nu^{37/36} \\ \frac{\sqrt[36]{V}}{\nu^{8/9}} & \nu^{8/9} & \nu^{7/18} & \nu^{25/18} & \nu^{17/9} \\ \frac{1}{\sqrt[17]{36}} & \nu^{7/18} & \frac{1}{\sqrt[9]{V}} & \nu^{8/9} & \nu^{25/18} \\ \nu^{19/36} & \nu^{25/18} & \nu^{8/9} & \nu^{17/9} & \nu^{43/18} \\ \nu^{37/36} & \nu^{17/9} & \nu^{25/18} & \nu^{43/18} & \nu^{26/9} \end{pmatrix} \delta a_3 + \begin{pmatrix} \nu^{5/18} & \nu^{19/36} & \frac{\sqrt[36]{V}}{\nu^{8/9}} & \nu^{37/36} & \nu^{55/36} \\ \nu^{19/36} & \nu^{25/18} & \nu^{8/9} & \nu^{17/9} & \nu^{43/18} \\ \frac{\sqrt[36]{V}}{\nu^{8/9}} & \nu^{8/9} & \nu^{7/18} & \nu^{25/18} & \nu^{17/9} \\ \nu^{37/36} & \nu^{17/9} & \nu^{25/18} & \nu^{43/18} & \nu^{26/9} \\ \nu^{55/36} & \nu^{43/18} & \nu^{17/9} & \nu^{26/9} & \nu^{61/18} \end{pmatrix} \delta a_4 \quad (128)$$

$$\partial_{a_3} g_{AB}(z_1) \sim \begin{pmatrix} \sqrt[18]{V} & 81 & \frac{1}{\sqrt[7]{36}} & \sqrt{V} & \gamma^{47/36} \\ 81 & \gamma^{7/6} & \gamma^{2/3} & \gamma^{5/3} & \gamma^{13/6} \\ \frac{1}{\sqrt[7]{36}} & \gamma^{2/3} & \sqrt[6]{V} & \gamma^{7/6} & \gamma^{5/3} \\ \sqrt[7]{V} & \gamma^{5/3} & \gamma^{7/6} & \gamma^{13/6} & \gamma^{8/3} \\ \gamma^{47/36} & \gamma^{13/6} & \gamma^{5/3} & \gamma^{8/3} & \gamma^{19/6} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt[5]{18}} & \frac{1}{3\sqrt[6]{V}} & \frac{1}{\sqrt[2]{9}} & \gamma^{17/36} & \gamma^{23/18} \\ \frac{1}{3\sqrt[6]{V}} & \gamma^{19/36} & \sqrt[3]{V} & \gamma^{37/36} & \gamma^{55/36} \\ \frac{1}{\sqrt[2]{9}} & \frac{1}{\gamma^{17/36}} & 3\sqrt[6]{V} & \gamma^{19/36} & \gamma^{37/36} \\ \frac{1}{\gamma^{17/36}} & \gamma^{37/36} & \gamma^{19/36} & \gamma^{55/36} & \gamma^{73/36} \\ \gamma^{23/18} & \gamma^{55/36} & \gamma^{37/36} & \gamma^{73/36} & \gamma^{91/36} \end{pmatrix} \delta z_1 +$$

$$\begin{pmatrix} \gamma_{5/18} & \gamma_{19/36} & \frac{36\sqrt{\gamma}}{\gamma_{17/36}} & \gamma_{37/36} & \gamma_{55/36} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \frac{36\sqrt{\gamma}}{\gamma_{17/36}} & \gamma_{8/9} & \gamma_{7/18} & \gamma_{25/18} & \gamma_{17/9} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \end{pmatrix} \delta a_1 + \begin{pmatrix} \gamma_{2/9} & \frac{36\sqrt{\gamma}}{\gamma_{17/36}} & \frac{1}{\gamma_{7/18}} & \gamma_{19/36} & \gamma_{37/36} \\ \frac{36\sqrt{\gamma}}{\gamma_{17/36}} & \gamma_{8/9} & \gamma_{7/18} & \gamma_{25/18} & \gamma_{17/9} \\ \frac{1}{\gamma_{7/18}} & \gamma_{7/18} & \frac{1}{\gamma_{9/18}} & \gamma_{8/9} & \gamma_{25/18} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \end{pmatrix} \delta a_2$$

$$\begin{pmatrix} \gamma_{7/9} & \gamma_{37/36} & \gamma_{19/36} & \gamma_{55/36} & \gamma_{73/36} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \\ \gamma_{73/36} & \gamma_{26/9} & \gamma_{43/18} & \gamma_{61/18} & \gamma_{35/9} \end{pmatrix} \delta a_3 + \begin{pmatrix} \gamma_{23/18} & \gamma_{55/36} & \gamma_{37/36} & \gamma_{73/36} & \gamma_{91/36} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{73/36} & \gamma_{26/9} & \gamma_{43/18} & \gamma_{61/18} & \gamma_{35/9} \\ \gamma_{91/36} & \gamma_{61/18} & \gamma_{26/9} & \gamma_{35/9} & \gamma_{79/18} \end{pmatrix} \delta a_4 \quad (129)$$

$$\partial_{a_4} g_{A\bar{B}} \sim \begin{pmatrix} \gamma^{5/9} & \gamma^{29/36} & \gamma^{11/36} & \gamma^{47/36} & \gamma^{65/36} \\ \gamma^{29/36} & \gamma^{5/3} & \gamma^{7/6} & \gamma^{13/6} & \gamma^{8/3} \\ \gamma^{11/36} & \gamma^{7/6} & \gamma^{2/3} & \gamma^{5/3} & \gamma^{13/6} \\ \gamma^{47/36} & \gamma^{13/6} & \gamma^{5/3} & \gamma^{8/3} & \gamma^{19/6} \\ \gamma^{65/36} & \gamma^{8/3} & \gamma^{13/6} & \gamma^{19/6} & \gamma^{11/3} \end{pmatrix} + \begin{pmatrix} \gamma^{2/9} & \gamma^{7/9} & \gamma^{5/18} & \gamma^{23/18} & \gamma^{16/9} \\ \gamma^{7/9} & \gamma^{37/36} & \gamma^{19/36} & \gamma^{55/36} & \gamma^{73/36} \\ \gamma^{5/18} & \gamma^{19/36} & \sqrt[36]{\gamma} & \gamma^{37/36} & \gamma^{55/36} \\ \gamma^{23/18} & \gamma^{55/36} & \gamma^{37/36} & \gamma^{73/36} & \gamma^{91/36} \\ \gamma^{16/9} & \gamma^{73/36} & \gamma^{55/36} & \gamma^{91/36} & \gamma^{109/36} \end{pmatrix} \delta z_1$$

$$\begin{pmatrix} \gamma_{7/9} & \gamma_{37/36} & \gamma_{19/36} & \gamma_{55/36} & \gamma_{73/36} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \\ \gamma_{73/36} & \gamma_{26/9} & \gamma_{43/18} & \gamma_{61/18} & \gamma_{35/9} \end{pmatrix} \delta a_1 + \begin{pmatrix} \gamma_{5/18} & \gamma_{19/36} & \frac{36}{\sqrt{V}} & \gamma_{37/36} & \gamma_{55/36} \\ \gamma_{19/36} & \gamma_{25/18} & \gamma_{8/9} & \gamma_{17/9} & \gamma_{43/18} \\ \frac{36}{\sqrt{V}} & \gamma_{8/9} & \gamma_{7/18} & \gamma_{25/18} & \gamma_{17/9} \\ \gamma_{37/36} & \gamma_{17/9} & \gamma_{25/18} & \gamma_{43/18} & \gamma_{26/9} \\ \gamma_{55/36} & \gamma_{43/18} & \gamma_{17/9} & \gamma_{26/9} & \gamma_{61/18} \end{pmatrix} \delta a_2$$

$$\begin{pmatrix} \mathcal{V}^{23/18} & \mathcal{V}^{55/36} & \mathcal{V}^{37/36} & \mathcal{V}^{73/36} & \mathcal{V}^{91/36} \\ \mathcal{V}^{55/36} & \mathcal{V}^{43/18} & \mathcal{V}^{17/9} & \mathcal{V}^{26/9} & \mathcal{V}^{61/18} \\ \mathcal{V}^{37/36} & \mathcal{V}^{17/9} & \mathcal{V}^{25/18} & \mathcal{V}^{43/18} & \mathcal{V}^{26/9} \\ \mathcal{V}^{73/36} & \mathcal{V}^{26/9} & \mathcal{V}^{43/18} & \mathcal{V}^{61/18} & \mathcal{V}^{35/9} \\ \mathcal{V}^{91/36} & \mathcal{V}^{61/18} & \mathcal{V}^{26/9} & \mathcal{V}^{35/9} & \mathcal{V}^{79/18} \end{pmatrix} \delta a_3 + \begin{pmatrix} \mathcal{V}^{16/9} & \mathcal{V}^{73/36} & \mathcal{V}^{55/36} & \mathcal{V}^{91/36} & \mathcal{V}^{109/36} \\ \mathcal{V}^{73/36} & \mathcal{V}^{26/9} & \mathcal{V}^{43/18} & \mathcal{V}^{61/18} & \mathcal{V}^{35/9} \\ \mathcal{V}^{55/36} & \mathcal{V}^{43/18} & \mathcal{V}^{17/9} & \mathcal{V}^{26/9} & \mathcal{V}^{61/18} \\ \mathcal{V}^{91/36} & \mathcal{V}^{61/18} & \mathcal{V}^{26/9} & \mathcal{V}^{35/9} & \mathcal{V}^{79/18} \\ \mathcal{V}^{109/36} & \mathcal{V}^{35/9} & \mathcal{V}^{61/18} & \mathcal{V}^{79/18} & \mathcal{V}^{44/9} \end{pmatrix} \delta a_4 \quad (130)$$

These expressions will be useful for all subsequent calculations.

Higgsino-fermion/anitiferfion-sfermion + Higgsino-fermion/anitiferfion-sfermion + Fermion-sfermion-fermion/anitiferfion vertices

- As discussed above, the neutralino-lepton-slepton vertex corresponding to Fig. 11(a), is given by considering the contribution of gaugino-slepton-lepton vertex with a small admixture of higgsino-lepton-slepton vertex as given in equation (66). Since neutralino is of Majorana nature, in two-component notation, the contribution of Higgsino-lepton-slepton vertex in gauged supergravity action of Wess and Bagger [12], is given by

$$\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\mathcal{Z}_1} D_{\tilde{\mathcal{A}}_1} W) \chi^{\mathcal{Z}_i} \chi^{c\mathcal{A}_1} + i g_{\tilde{L}\tilde{\mathcal{A}}_1} \tilde{\chi}^{\mathcal{Z}_i} \left[\bar{\sigma} \cdot \partial \chi^{c\mathcal{A}_1} + \Gamma_{\mathcal{A}_1 \tilde{\mathcal{A}}_1}^{\mathcal{A}_1} \bar{\sigma} \cdot \partial \mathcal{A}_1 \chi^{c\mathcal{A}_1} + \frac{1}{4} (\partial_{\mathcal{A}_3} K \bar{\sigma} \cdot \mathcal{A}_1 - \text{c.c.}) \chi^{c\mathcal{A}_1} \right]$$

$\chi^{\mathcal{Z}}$ is $SU(2)_L$ higgsino, $\chi^{\mathcal{A}_1}$ corresponds to $SU(2)_L$ electron and $\tilde{\mathcal{A}}_1$ corresponds to left-handed squark and $g_{\tilde{L}\tilde{\mathcal{A}}_1} = 0$.

Strictly speaking, $SU(2)$ EW symmetry gets spontaneously broken for Higgsino-lepton-slepton vertex, however the effective Lagrangian respects $SU(2)$ symmetry. Therefore to calculate the contribution of same, basic idea is to generate a term of the type $l_L \tilde{l}_L \tilde{H}_L^c H_L$ wherein χ^{l_L} and H_L are respectively the $SU(2)_L$ quark and Higgs doublets, \tilde{l}_L is also an $SU(2)_L$ doublet and \tilde{H}_L^c is $SU(2)_L$ Higgsino doublet. After spontaneous breaking of the EW symmetry when H^0 in H_L acquires a non-zero vev $\langle H^0 \rangle$, this term generates: $\langle H^0 \rangle \tilde{H}_L^c l_L \tilde{l}_L$. Now, in terms of undiagonalized basis, consider

$$\mathcal{D}_i D_{\tilde{a}_1} W = (\partial_i \partial_{\tilde{a}_1} K) W + \partial_i K D_{\tilde{a}_1} W + \partial_{\tilde{a}_1} K D_i W - (\partial_i K \partial_{\tilde{a}_1} K) W,$$

where a_1, z_i correspond to undiagonalized moduli fields.

1. Considering $a_1 \rightarrow a_1 + \mathcal{V}^{-\frac{2}{9}} M_p$, using equations (29),(33) and further picking up the component linear in z_1 as well as linear in fluctuation $(a_1 - \mathcal{V}^{-\frac{2}{9}} M_p)$, we see that:

$$e^{\frac{K}{2}} ((\partial_i \partial_{a_1} K) W + \partial_i K D_{a_1} W + \partial_{a_1} K D_i W - (\partial_i K \partial_{a_1} K) W) \bar{\chi}^i \bar{\chi}^{a_1} \sim \mathcal{V}^{-\frac{31}{18}} z_i (a_1 - \mathcal{V}^{-\frac{2}{9}} M_p). \quad (131)$$

As shown in (35)

$$e^{\frac{K}{2}} \mathcal{D}_i D_{\tilde{\mathcal{A}}_1} W \sim \mathcal{O}(1) e^{\frac{K}{2}} \mathcal{D}_i D_{\tilde{a}_1} W,$$

Higgsino-lepton-slepton vertex will hence be given as:

$$e^{\frac{K}{2}} \mathcal{D}_i D_{\mathcal{A}_1} W \chi^{\mathcal{Z}_i} \chi^{c\mathcal{A}_1} \sim e^{\frac{K}{2}} \mathcal{D}_i D_{\tilde{a}_1} W \chi^{\mathcal{Z}_i} \chi^{c\mathcal{A}_1} \sim \mathcal{V}^{-\frac{31}{18}} \mathcal{Z}_i \delta \mathcal{A}_1 \chi^{\mathcal{Z}_i} \chi^{c\mathcal{A}_1};$$

\mathcal{A}_1 corresponds to left-handed slepton. The contribution of physical Higgsino-lepton-slepton vertex after giving VEV to \mathcal{Z}_I will be given as :

$$C^{\tilde{H}_L^c l_L \tilde{l}_L} \sim \frac{\mathcal{V}^{-\frac{31}{18}} \langle \mathcal{Z}_i \rangle}{\sqrt{\hat{K}_{\mathcal{Z}_i \mathcal{Z}_i}^2 \hat{K}_{\mathcal{A}_1 \tilde{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_1 \tilde{\mathcal{A}}_1}}} \sim \mathcal{V}^{-\frac{3}{2}}. \quad (132)$$

2. The interaction vertex corresponding to gaugino-lepton-slepton vertex is given by

$$g_{YM} g_{J\bar{T}_B} X^{*B} \bar{\chi}^{\bar{J}} \lambda_{\bar{g}}.$$

Utilizing (124) and $X^B = -6i\kappa_4^2 \mu_7 Q_{T_B}$, $\kappa_4^2 \mu_7 \sim \frac{1}{\mathcal{V}}$, $g_{YM} \sim \mathcal{V}^{-\frac{1}{36}}$, $Q_{T_B} \sim \mathcal{V}^{\frac{1}{3}} (2\pi\alpha')^2 \tilde{f}$, $g_{YM} g_{T_B \bar{a}_1} \rightarrow -\mathcal{V}^{-\frac{2}{9}} a_1$ and

$$g_{YM} g_{T_B \bar{\mathcal{A}}_1} \sim \mathcal{V}^{-\frac{2}{9}} (\mathcal{A}_1 - \mathcal{V}^{-\frac{2}{9}} M_p),$$

for $\mathcal{Z}_i \rightarrow \langle \mathcal{Z}_i \rangle \sim V^{\frac{1}{36}}$ (in $M_p = 1$ units), the dominant contribution to the physical gaugino-lepton-slepton vertex is proportional to :

$$C^{\lambda_{\bar{g}} l_L \tilde{l}_L} \sim \frac{g_{YM} g_{T_B \bar{\mathcal{A}}_1} X^{T_B} \sim \mathcal{V}^{-\frac{11}{12}} \tilde{f}}{\left(\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1}} \right) \left(\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1}} \right)} \sim \tilde{f} \left(10^{-3} \mathcal{V}^{-\frac{11}{12}} \right) \sim \tilde{f} \left(\mathcal{V}^{-\frac{17}{10}} \right). \quad (133)$$

3. Keeping in mind the fact that physical neutralino eigenstate χ_3^0 is largely a gaugino with a small admixture of Higgsino in our set up, by adding the contribution of (132) and (133) as according to (66), the physical *neutralino-lepton-slepton* vertex is given as:

$$C^{\chi_3^0 l_L \tilde{l}_L} : \tilde{f} \left(\mathcal{V}^{-\frac{3}{2}} \right) \delta \mathcal{A}_1 \chi^I \bar{\chi}^{\bar{\mathcal{A}}_1} + \tilde{f} \mathcal{V}^{-\frac{17}{10}} \delta \mathcal{A}_1 \lambda_{\bar{g}} \bar{\chi}^{\bar{\mathcal{A}}_1} \sim \tilde{f} \left(\mathcal{V}^{-\frac{3}{2}} \right) \chi_3^0 l_L \tilde{l}_L \text{ for } \mathcal{V} \sim 10^5. \quad (134)$$

- The *quark-quark-slepton* vertex corresponding to Fig. 11(a) in gauged supergravity action is given by the following term: $\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\bar{\mathcal{A}}_2} D_{\mathcal{A}_4} W) \chi^{\mathcal{A}_2^c} \chi^{\mathcal{A}_4}$. In terms of undiagonalized basis, consider

$$\mathcal{D}_{\bar{a}_2} D_{a_4} W = (\partial_{\bar{a}_2} \partial_{a_4} K) W + \partial_{\bar{a}_2} K D_{a_4} W + \partial_{a_4} K D_{\bar{a}_2} W - (\partial_{\bar{a}_2} K \partial_{a_4} K) W.$$

Using equations (29) and (33):

$$e^{\frac{K}{2}} ((\partial_{\bar{a}_2} \partial_{a_4} K) W + \partial_{\bar{a}_2} K D_{a_4} W + \partial_{a_4} K D_{\bar{a}_2} W - (\partial_{\bar{a}_2} K \partial_{a_4} K) W) \sim \mathcal{V}^{-\frac{5}{9}} + \mathcal{V}^{\frac{1}{3}} \delta a_1. \quad (135)$$

From (135), picking up the component of fluctuations linear in a_1 and from (35)

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_2} D_{\mathcal{A}_4} W \sim O(1) e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_2} D_{a_4} W,$$

one gets the contribution of quark-slepton-quark vertex as :

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_2} D_{\mathcal{A}_4} W \chi^{\mathcal{A}_2^c} \chi^{\mathcal{A}_4} \sim e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_2} D_{a_4} W \chi^{\mathcal{A}_2^c} \chi^{\mathcal{A}_4} \sim \left(\mathcal{V}^{-\frac{1}{3}} \delta \mathcal{A}_1 \right) \chi^{\mathcal{A}_2^c} \chi^{\mathcal{A}_4}. \quad (136)$$

The physical quark-slepton-quark vertex is given by

$$C^{d_L^c \tilde{l}_L u_L} \sim \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{\hat{K}_{\mathcal{A}_2 \mathcal{A}_2^c} \hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \sim (10)^{-\frac{14}{2}} \mathcal{V}^{-\frac{1}{3}} \sim \mathcal{V}^{-\frac{5}{3}} \text{ for } \mathcal{V} \sim 10^5. \quad (137)$$

- The contribution of Higgsino-quark-squark relevant to the *Neutralino-quark-squark* vertex of Fig. 11(b) is given by:

$$\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_i D_{\mathcal{A}_4} W) \bar{\chi}^i \chi^{\mathcal{A}_4} + i g_{i \mathcal{A}_4} \bar{\chi}^I \left[\bar{\sigma} \cdot \partial \chi^{\mathcal{A}_4} + \Gamma_{\mathcal{A}_4 \mathcal{A}_4}^{\mathcal{A}_4} \bar{\sigma} \cdot \partial \mathcal{A}_4 \chi^{\mathcal{A}_4} + \frac{1}{4} (\partial_{\mathcal{A}_4} K \bar{\sigma} \cdot \mathcal{A}_4 - \text{c.c.}) \chi^{\mathcal{A}_4} \right].$$

Working in diagonalized basis, $g_{i\mathcal{A}_4}=0$.

Now consider

$$\mathcal{D}_i D_{a_4} W = \partial_i \partial_{a_4} W + (\partial_i \partial_{a_4} K) W + \partial_i K D_{a_4} W + \partial_{a_4} K D_i W - (\partial_i K \partial_{a_4} K) W - \Gamma_{ia_4}^k D_k W,$$

where a_4, z_i correspond to undiagonalized moduli fields.

1. Considering $a_4 \rightarrow a_4 + \mathcal{V}^{-\frac{11}{9}} M_p$ and utilizing (125), (126) and (130), one gets:

$$\begin{aligned} \Gamma_{z_i a_4}^{z_i} &\sim \mathcal{V}^{\frac{11}{9}} - \mathcal{V}^{\frac{11}{6}} \delta a_4, \Gamma_{z_i a_4}^{a_1} \sim \mathcal{V}^{-\frac{13}{36}} + \mathcal{V}^{\frac{19}{12}} \delta a_4, \Gamma_{z_i a_4}^{a_2} \sim \mathcal{V}^{\frac{31}{36}} + \mathcal{V}^{\frac{25}{12}} \delta a_4, \Gamma_{z_i a_4}^{a_3} \sim \mathcal{V}^{-\frac{5}{36}} + \mathcal{V}^{\frac{13}{12}} \delta a_4, \\ \Gamma_{z_i a_4}^{a_4} &\sim \mathcal{V}^{-\frac{23}{36}} + \mathcal{V}^{\frac{7}{12}} \delta a_4. \end{aligned} \quad (138)$$

Next, using equations (29), (33), (138), one obtains:

$$\frac{e^{\frac{K}{2}}}{2} (\Gamma_{z_i a_4}^{z_i} D_{z_i} W + \Gamma_{z_i a_4}^{a_1} D_{a_1} W + \Gamma_{z_i a_4}^{a_2} D_{a_2} W + \Gamma_{z_i a_4}^{a_3} D_{a_3} W + \Gamma_{z_i a_4}^{a_4} D_{a_4} W) \sim \left(\mathcal{V}^{-\frac{17}{72}} + \mathcal{V}^{\frac{71}{72}} \delta a_4 \right) \quad (139)$$

and

$$e^{\frac{K}{2}} ((\partial_i \partial_{a_4} K) W + \partial_i K D_{a_4} W + \partial_{a_4} K D_i W - (\partial_i K \partial_{a_4} K) W) \chi^i \chi^{a_4} \sim \mathcal{V}^{-\frac{17}{72}} + \mathcal{V}^{\frac{71}{72}} \delta a_4 \chi^i \chi^{a_4}. \quad (140)$$

Utilizing equation (139) and (140), picking up the component linear in fluctuation δa_4 , contribution of Higgsino-quark-squark vertex will be

$$e^{\frac{K}{2}} \mathcal{D}_i D_{\mathcal{A}_4} W \chi^{z_i} \chi^{\mathcal{A}_4} \sim e^{\frac{K}{2}} \mathcal{D}_i D_{a_4} W \chi^i \chi^{\mathcal{A}_4} \sim \mathcal{V}^{\frac{71}{72}} \delta \mathcal{A}_4 \chi^i \chi^{\mathcal{A}_4}$$

and physical Higgsino-quark-squark takes the form as below :

$$C^{H_L^c \bar{d}_L^c \tilde{d}_R} \sim \frac{\mathcal{V}^{\frac{71}{72}}}{\sqrt{\hat{K}_{z_i \bar{z}_i} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \sim \left[(10)^{-\frac{19}{2}} \mathcal{V}^{\frac{71}{72}} \right] \sim \mathcal{V}^{-\frac{11}{12}}. \quad (141)$$

2. The interaction vertex corresponding to gaugino-quark-squark vertex is given by

$$g_{YM} g_{\bar{\mathcal{A}}_4 \bar{T}_B} X^{*B} \chi^{c\mathcal{A}_4} \lambda_{\tilde{g}}.$$

For a Kähler moduli space, $g_{\bar{\mathcal{A}}_4 \bar{T}_B} = 0$ i.e the gaugino-quark-squark vertex does not contribute to this particular vertex.

3. In view of above, the physical *neutralino-quark-squark* vertex as according to equation (66) is given as:

$$C^{\chi^0_3 \bar{d}_L^c \tilde{d}_R} : \tilde{f} [\mathcal{V}^{-\frac{11}{12}} \delta a_4 \bar{\chi}^{\bar{a}_1} \chi^i] \sim \tilde{f} \mathcal{V}^{-\frac{11}{12}} \chi^0_3 \bar{d}_L^c \tilde{d}_R \quad (142)$$

- The *lepton-squark-quark* vertex corresponding to Fig. 11(b) is given by considering the contribution $\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_2} W) \chi^{c\mathcal{A}_1} \chi^{\mathcal{A}_2} + \frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\mathcal{A}_1} D_{\bar{\mathcal{A}}_2} W) \chi^{\mathcal{A}_1} \chi^{c\mathcal{A}_2}$. In terms of undiagonalized basis

$$\mathcal{D}_{\bar{a}_1} D_{a_2} W = (\partial_{\bar{a}_1} \partial_{a_2} K) W + \partial_{\bar{a}_1} K D_{a_2} W + \partial_{a_2} K D_{\bar{a}_1} W - (\partial_{\bar{a}_1} K \partial_{a_2} K) W$$

and

$$\mathcal{D}_{a_1} D_{\bar{a}_2} W = (\partial_{a_1} \partial_{\bar{a}_2} K) W + \partial_{a_1} K D_{\bar{a}_2} W + \partial_{\bar{a}_2} K D_{a_1} W - (\partial_{a_1} K \partial_{\bar{a}_2} K) W$$

Utilizing $a_4 \rightarrow a_4 + \mathcal{V}^{-\frac{11}{9}}$ and equations (29), (33), on solving,

$$\begin{aligned} e^{\frac{K}{2}} (\partial_{\bar{a}_1} \partial_{a_2} K) W + \partial_{\bar{a}_1} K D_{a_2} W + \partial_{a_2} K D_{\bar{a}_1} W - (\partial_{\bar{a}_1} K \partial_{a_2} K) W \chi^{\bar{a}_1} \chi^{a_2} &\sim \left(\mathcal{V}^{-\frac{14}{9}} + \mathcal{V}^{-\frac{1}{3}} \delta a_4 \right) \chi^{a_1^c} \chi^{a_2} \\ e^{\frac{K}{2}} (\partial_{a_1} \partial_{\bar{a}_2} K) W + \partial_{a_1} K D_{\bar{a}_2} W + \partial_{\bar{a}_2} K D_{a_1} W - (\partial_{a_1} K \partial_{\bar{a}_2} K) W \chi^{a_1} \chi^{\bar{a}_2} &\sim \left(\mathcal{V}^{-\frac{14}{9}} + \mathcal{V}^{-\frac{1}{3}} \delta a_4 \right) \chi^{a_1} \chi^{a_2^c}. \end{aligned} \quad (143)$$

From (143), one gets the contribution of lepton-squark-quark vertex as :

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_1} D_{a_2} W \chi^{a_1^c} \chi^{a_2} + e^{\frac{K}{2}} \mathcal{D}_{a_1} D_{\bar{a}_2} W \chi^{a_1} \chi^{a_2^c} \sim \left(\mathcal{V}^{-\frac{1}{3}} \delta a_4 \right) \chi^{a_1^c} \chi^{a_2} + \left(\mathcal{V}^{-\frac{1}{3}} \delta a_4 \right) \chi^{a_1} \chi^{a_2^c} \quad (144)$$

and the physical lepton-squark-quark vertex is given by

$$C^{l_L \bar{d}_R u_L} \sim \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} + \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \sim \mathcal{V}^{-\frac{5}{3}}. \quad (145)$$

- The Neutralino-quark-squark vertex of Fig. 11(c) is given as considering the contribution of

$$\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_i D_{\bar{\mathcal{A}}_2} W) \chi^{\mathcal{Z}_i} \chi^{c^{\mathcal{A}_2}} + i g_{\bar{\mathcal{Z}}_i \bar{\mathcal{A}}_2} \bar{\chi}^{\mathcal{Z}_i} \left[\bar{\sigma} \cdot \partial + \Gamma_{\mathcal{A}_2 \bar{\mathcal{A}}_2}^{\mathcal{A}_2} \bar{\sigma} \cdot \partial \mathcal{A}_2 \chi^{c^{\mathcal{A}_2}} + \frac{1}{4} (\partial_{\mathcal{A}_2} K \bar{\sigma} \cdot \mathcal{A}_2 - \text{c.c.}) \chi^{c^{\mathcal{A}_2}} \right]$$

χ^I is $SU(2)_L$ higgsino, $\chi^{\mathcal{A}_2}$ corresponds to $SU(2)_L$ quark and \mathcal{A}_2 corresponds to left-handed squark. and $g_{\bar{I} \bar{\mathcal{A}}_2} = 0$.

In terms of undiagonalized basis, consider

$$\mathcal{D}_i D_{\bar{a}_2} W = (\partial_i \partial_{\bar{a}_2} K) W + \partial_i K D_{\bar{a}_2} W + \partial_{\bar{a}_2} K D_i W - (\partial_i K \partial_{\bar{a}_2} K) W,$$

where a_2, z_i correspond to undiagonalized moduli fields.

1. Considering $a_2 \rightarrow a_2 + \mathcal{V}^{-\frac{1}{3}} M_p$, using equations (29),(33) and picking up the component linear in z_1 as well as fluctuations of a_2 i.e $(a_2 - \mathcal{V}^{-\frac{1}{3}} M_p)$, one gets

$$e^{\frac{K}{2}} ((\partial_i \partial_{\bar{a}_2} K) W + \partial_i K D_{\bar{a}_2} W + \partial_{\bar{a}_2} K D_i W - (\partial_i K \partial_{\bar{a}_2} K) W) \sim (\mathcal{V}^{-\frac{20}{9}}) z_i \delta a_2. \quad (146)$$

Using (35):

$$e^{\frac{K}{2}} \mathcal{D}_i D_{\bar{\mathcal{A}}_2} W \sim O(1) e^{\frac{K}{2}} \mathcal{D}_i D_{\bar{a}_2} W,$$

the Higgsino-quark-squark vertex will be given as

$$e^{\frac{K}{2}} \mathcal{D}_i D_{\mathcal{A}_2} W \chi^{\mathcal{Z}_i} \chi^{c^{\mathcal{A}_2}} \sim e^{\frac{K}{2}} \mathcal{D}_i D_{\bar{a}_2} W \chi^{\mathcal{Z}_i} \chi^{c^{\mathcal{A}_2}} \sim \mathcal{V}^{-\frac{20}{9}} \mathcal{Z}_i \delta \mathcal{A}_2 \chi^{\mathcal{Z}_i} \chi^{c^{\mathcal{A}_2}};$$

\mathcal{A}_2 correspond to left- handed squark and the contribution of physical Higgsino-quark-squark vertex will be given as :

$$C^{\tilde{H}_L^c u_L \bar{u}_L} \sim \frac{\mathcal{V}^{-\frac{20}{9}} < \mathcal{Z}_i >}{\sqrt{\hat{K}_{\mathcal{Z}_i \bar{\mathcal{Z}}_i}^2 \hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2}}} \sim \mathcal{V}^{-\frac{4}{5}}. \quad (147)$$

2. As discussed for gaugino lepton-slepton vertex in Figure (a), the gaugino- quark-squark vertex is given by

$$g_{YM} g_{JB^*} X^{*B} \bar{\chi}^{\bar{J}} \lambda_{\tilde{g}}$$

. Utilizing 124 and $X^B = -6i\kappa_4^2 \mu_7 Q_B, \kappa_4^2 \mu_7 \sim \frac{1}{\mathcal{V}}, g_{YM} \sim \mathcal{V}^{-\frac{1}{36}}, Q_B \sim \mathcal{V}^{\frac{1}{3}} (2\pi\alpha')^2 \tilde{f}$, This time,

$$g_{YM} g_{B\bar{a}_2} \rightarrow -\mathcal{V}^{-\frac{5}{4}} \delta a_2.$$

Now,

$$g_{YM} g_{T_B \bar{\mathcal{A}}_2} \sim O(1) g_{YM} g_{T_B \bar{a}_2} \rightarrow \mathcal{V}^{-\frac{5}{4}} \delta \mathcal{A}_2.$$

The dominant contribution to the physical gaugino-quark-squark vertex is proportional to :

$$C^{\lambda_{\tilde{g}} u_L \tilde{u}_L} \sim \frac{\mathcal{V}^{-\frac{23}{12}} \tilde{f}}{\left(\sqrt{\hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2}}\right) \left(\sqrt{\hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2}}\right)} \sim \tilde{f} \left(\mathcal{V}^{-\frac{3}{2}}\right). \quad (148)$$

Following equation no (66) , the neutralino-quark squark vertex will be given by :

$$C^{\chi_3^0 u_L \tilde{u}_L} \sim C^{\lambda_{\tilde{g}} u_L \tilde{u}_L} + \tilde{f} C^{H_L^c u_L \tilde{u}_L} \sim \tilde{f} \left(\mathcal{V}^{-\frac{4}{5}}\right) \delta \mathcal{A}_2 \chi^I \chi^{c\mathcal{A}_2} + \tilde{f} \left(\mathcal{V}^{-\frac{3}{2}}\right) \delta \mathcal{A}_2 \bar{\chi}^{\bar{\mathcal{A}}_1} \lambda_{\tilde{g}} \sim \tilde{f} \mathcal{V}^{-\frac{4}{5}} \chi_3^0 u_L \tilde{u}_L. \quad (149)$$

- The lepton-squark-quark corresponding to Fig. 11(c) is given by considering the contribution of $\frac{e\frac{K}{2}}{2} (\mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_4} W) \chi^{c\mathcal{A}_1} \chi^{\mathcal{A}_4}$ in gauged supergravity action. In terms of undiagonalized basis,

$$\mathcal{D}_{\bar{a}_1} D_{a_4} W = (\partial_{\bar{a}_1} \partial_{a_4} K) W + \partial_{\bar{a}_1} K D_{a_4} W + \partial_{a_4} K D_{\bar{a}_1} W - (\partial_{\bar{a}_1} K \partial_{a_4} K) W.$$

Utilizing equations (29) and (33), one obtains:

$$e^{\frac{K}{2}} ((\partial_{\bar{a}_1} \partial_{a_4} K) W + \partial_{\bar{a}_1} K D_{a_4} W + \partial_{a_4} K D_{\bar{a}_1} W - (\partial_{\bar{a}_1} K \partial_{a_4} K) W) \sim \mathcal{V}^{-\frac{1}{18}} + \mathcal{V}^{-\frac{1}{3}} \delta a_2. \quad (150)$$

From (150), picking up the component of fluctuations linear in a_2 and from (35) using:

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_4} W \sim O(1) e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_1} D_{a_4} W,$$

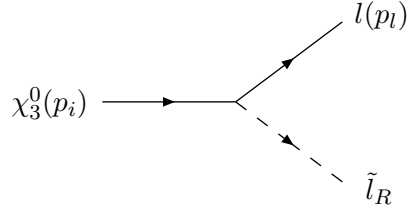
one gets the contribution of lepton-squark-quark vertex as :

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_4} W \chi^{c\mathcal{A}_1} \bar{\chi}^{\bar{\mathcal{A}}_4} \sim e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_1} D_{a_4} W \chi^{c\mathcal{A}_1} \chi^{\mathcal{A}_4} \sim \left(\mathcal{V}^{-\frac{1}{3}} \delta \mathcal{A}_2\right) \chi^{c\mathcal{A}_1} \chi^{\mathcal{A}_4}. \quad (151)$$

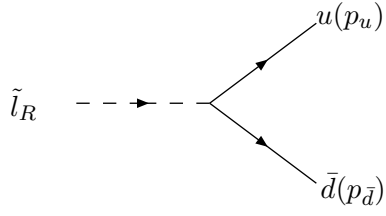
The physical lepton-squark-quark vertex is given by

$$C^{d_L^c \tilde{l}_L l_L} \sim \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{\hat{K}_{\mathcal{A}_2 \bar{\mathcal{A}}_2} \hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \sim (10)^{-\frac{14}{2}} \mathcal{V}^{-\frac{1}{3}} \sim \mathcal{V}^{-\frac{5}{3}} \text{ for } \mathcal{V} \sim 10^5. \quad (152)$$

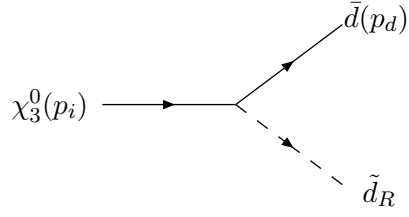
Now, working in two component notation, in order to calculate the decay width, we will be using the decay width formula as given by H.Dreiner et al. in [27]. Because of Majorana nature of neutralino, we are considering both right handed as well as left handed incoming neutralino though the dominant contribution occurs in case of vertices corresponding to right handed neutralino. Henceforth, we will be using incoming right handed neutralino in the matrix amplitude calculation.



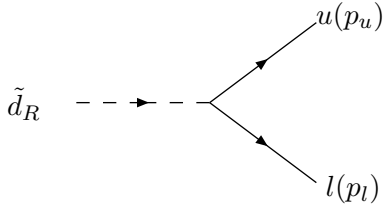
$$\tilde{f}\mathcal{V}^{-\frac{3}{2}}\chi_3^0 l_L \tilde{l}_L$$



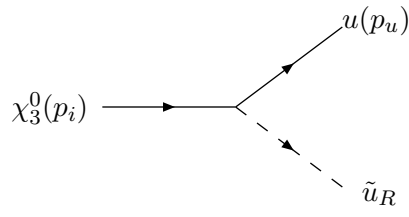
$$\mathcal{V}^{-\frac{5}{3}}d_L^c \tilde{l}_L u_L$$



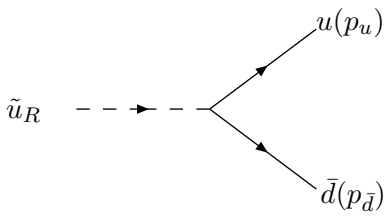
$$\tilde{f}\mathcal{V}^{-\frac{11}{12}}\chi_3^0 \bar{d}_L^c \tilde{d}_R$$



$$\mathcal{V}^{-\frac{5}{3}}l_L \tilde{d}_R u_L$$



$$\tilde{f}\mathcal{V}^{-\frac{4}{5}}\chi_3^0 u_L \tilde{u}_L$$



$$\mathcal{V}^{-\frac{5}{3}}\tilde{l}_L \chi^{d_L^c} \chi^{l_L}$$

Guided by their notations, right handed incoming and left handed outgoing are denoted by wave function $y_i^\dagger \equiv y^\dagger(\vec{p}_i, \lambda_i)$, $x_l^\dagger \equiv x^\dagger(\vec{p}_l, \lambda_l)$, $x_u^\dagger \equiv x^\dagger(\vec{p}_u, \lambda_u)$, and $x_d^\dagger \equiv x^\dagger(\vec{p}_d, \lambda_d)$, respectively. Using the numerical estimate of vertices as calculated in set of equations (134),(137),(142), (145), (149),(152) for all three feynman diagrams, the corresponding contributions to the decay amplitude is:

$$\mathcal{M}_1 = \tilde{f}\mathcal{V}^{-\frac{3}{2}}\mathcal{V}^{-\frac{5}{3}} \left(\frac{i}{(p_i - k_l)^2 - m_{l_L}^2} \right) y_i^\dagger x_l^\dagger x_u^\dagger x_d^\dagger \quad (153)$$

$$\begin{aligned} i\mathcal{M}_2 &= \tilde{f}\mathcal{V}^{-\frac{11}{12}}\mathcal{V}^{-\frac{5}{3}} \left(\frac{i}{(p_i - k_d)^2 - m_{d_R}^2} \right) y_i^\dagger x_d^\dagger x_l^\dagger x_u^\dagger \\ i\mathcal{M}_3 &= \tilde{f}\mathcal{V}^{-\frac{4}{5}}\mathcal{V}^{-\frac{5}{3}} \left(\frac{i}{(p_i - k_u)^2 - m_{u_L}^2} \right) y_i^\dagger x_u^\dagger x_d^\dagger x_l^\dagger. \end{aligned} \quad (154)$$

Neglecting all of the final state fermion masses, one can express kinematic variables in the form as given below:

$$z_l \equiv 2p_i \cdot k_l / m_{\chi_3^0}^2 = 2E_l / m_{\chi_3^0}, z_d \equiv 2p_i \cdot k_d / m_{\chi_3^0}^2 = 2E_d / m_{\chi_3^0}, z_u \equiv 2p_i \cdot k_u / m_{\chi_3^0}^2 = 2E_u / m_{\chi_3^0} \quad (155)$$

and the total matrix amplitude can be rewritten as:

$$\mathcal{M} = c_1 y_i^\dagger x_l^\dagger x_u^\dagger x_d^\dagger + c_2 y_i^\dagger x_d^\dagger x_l^\dagger x_u^\dagger + c_3 y_i^\dagger x_u^\dagger x_d^\dagger x_l^\dagger, \quad (156)$$

where for $m_{\chi_3^0} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}}$ and $m_{l_L} \sim m_{\bar{d}_R} \sim m_{\bar{u}_L} \sim \mathcal{V}^{\frac{1}{2}} m_{\frac{3}{2}}$

$$\begin{aligned} c_1 &\equiv \tilde{f}\mathcal{V}^{-\frac{3}{2}}\mathcal{V}^{-\frac{5}{3}}/[m_{l_L}^2 - m_{\chi_3^0}^2(1 - z_l)] = -\tilde{f}\mathcal{V}^{-\frac{3}{2}}\mathcal{V}^{-\frac{5}{3}}/m_{\frac{3}{2}}^2[\mathcal{V} - \mathcal{V}^{\frac{4}{3}}(1 - z_l)] \\ c_2 &\equiv \tilde{f}\mathcal{V}^{-\frac{11}{12}}\mathcal{V}^{-\frac{5}{3}}/[m_{d_R}^2 - m_{\chi_3^0}^2(1 - z_d)] = -\tilde{f}\mathcal{V}^{-\frac{11}{12}}\mathcal{V}^{-\frac{5}{3}}/m_{\frac{3}{2}}^2[\mathcal{V} - \mathcal{V}^{\frac{4}{3}}(1 - z_d)] \\ c_3 &\equiv \tilde{f}\mathcal{V}^{-\frac{4}{5}}\mathcal{V}^{-\frac{5}{3}}/[m_{u_L}^2 - m_{\chi_3^0}^2(1 - z_u)] = -\tilde{f}\mathcal{V}^{-\frac{4}{5}}\mathcal{V}^{-\frac{5}{3}}/m_{\frac{3}{2}}^2[\mathcal{V} - \mathcal{V}^{\frac{4}{3}}(1 - z_u)] \end{aligned} \quad (157)$$

As explained in [27], by applying Fierz identity, one can reduce the number of terms:

$$\mathcal{M} = (c_1 - c_3) y_i^\dagger x_l^\dagger x_u^\dagger x_d^\dagger + (c_2 - c_3) y_i^\dagger x_d^\dagger x_l^\dagger x_u^\dagger. \quad (158)$$

and

$$\begin{aligned} |\mathcal{M}|^2 &= |c_1 - c_3|^2 y_i^\dagger x_l^\dagger x_u^\dagger x_d^\dagger x_l x_u x_d x_i + |c_2 - c_3|^2 y_i^\dagger x_d^\dagger x_l^\dagger x_u^\dagger x_d x_l x_u x_i \\ &\quad - 2\text{Re}[(c_1 - c_3)(c_2^* - c_3^*) y_i^\dagger x_l^\dagger x_l x_u x_u^\dagger x_d^\dagger x_d x_i], \end{aligned} \quad (159)$$

Summing over the fermion spins by using following identities,

$$\begin{aligned} \sum_s x_\alpha(\vec{p}, s) x_\beta^\dagger(\vec{p}, s) &= +p \cdot \sigma_{\alpha\dot{\beta}}, \sum_s x^{\dagger\dot{\alpha}}(\vec{p}, s) x^\beta(\vec{p}, s) = +p \cdot \bar{\sigma}^{\dot{\alpha}\beta}, \sum_s y^{\dagger\dot{\alpha}}(\vec{p}, s) y^\beta(\vec{p}, s) = +p \cdot \bar{\sigma}^{\dot{\alpha}\beta} \\ \sum_s y_\alpha(\vec{p}, s) y_\beta^\dagger(\vec{p}, s) &= +p \cdot \sigma_{\alpha\dot{\beta}} \end{aligned} \quad (160)$$

one obtain:

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}|^2 &= |c_1 - c_3|^2 \text{Tr}[k_l \cdot \bar{\sigma} p_i \cdot \sigma] \text{Tr}[k_d \cdot \bar{\sigma} k_u \cdot \sigma] + |c_2 - c_3|^2 \text{Tr}[k_d \cdot \bar{\sigma} p_i \cdot \sigma] \text{Tr}[k_u \cdot \bar{\sigma} k_l \cdot \sigma] \\ &\quad - 2\text{Re}[(c_1 - c_3)(c_2^* - c_3^*) \text{Tr}[k_l \cdot \bar{\sigma} k_u \cdot \sigma k_d \cdot \bar{\sigma} p_i \cdot \sigma]]. \end{aligned} \quad (161)$$

Applying the trace formulae and using

$$+ 2k_l \cdot k_d = (1 - z_u)m_{\chi_3^0}^2, \quad + 2k_l \cdot k_u = (1 - z_d)m_{\chi_3^0}^2, \quad + 2k_d \cdot k_u = (1 - z_l)m_{\chi_3^0}^2. \quad (162)$$

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}|^2 &= 4|c_1 - c_3|^2 p_i \cdot k_l k_d \cdot k_u + 4|c_2 - c_3|^2 p_i \cdot k_d k_l \cdot k_u \\ &\quad - 4\text{Re}[(c_1 - c_3)(c_2^* - c_3^*)](k_l \cdot k_u p_i \cdot k_d + p_i \cdot k_l k_d \cdot k_u - k_l \cdot k_d p_i \cdot k_u) \\ &= m_{\chi_3^0}^4 \left[|c_1|^2 z_l(1 - z_l) + |c_2|^2 z_d(1 - z_d) + |c_3|^2 z_u(1 - z_u) \right. \\ &\quad - 2\text{Re}[c_1 c_2^*](1 - z_l)(1 - z_d) - 2\text{Re}[c_1 c_3^*](1 - z_l)(1 - z_u) \\ &\quad \left. - 2\text{Re}[c_2 c_3^*](1 - z_d)(1 - z_u) \right], \end{aligned} \quad (163)$$

The differential decay rate follows:

$$\frac{d^2\Gamma}{dz_l dz_d} = \frac{N_c m_{\chi_3^0}}{2^8 \pi^3} \left(\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right), \quad (164)$$

where $N_c = 3$ and the kinematic limits are

$$0 < z_l < 1, 1 - z_l < z_d < 1. \quad (165)$$

Adopting the technique as used in [27], the total decay width for the above Feynman diagrams is given as:

$$\Gamma = \frac{N_c m_{\chi_3^0}^5}{2^8 \cdot 3\pi^3} (|c_1'^*|^2 + |c_2'^*|^2 + |c_3'^*|^2 - \text{Re}[c_1'^* c_2'^* + c_1'^* c_3'^* + c_2'^* c_3'^*]), \quad (166)$$

where $c_i'^*, i = 1, 2, 3$ are obtained from c_i 's by integrating z_l, z_d in the aforementioned limits. Therefore, on simplifying, the result comes out to be

$$\Gamma \sim \frac{3 \cdot \mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^5 \tilde{f}^2 \cdot \mathcal{V}^{-\frac{74}{15}}}{2^8 \cdot 3 \cdot \pi^3 \mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4} \sim 10^{-4} \mathcal{V}^{\frac{2}{3} - \frac{74}{15}} m_{\frac{3}{2}} \tilde{f}^2 \sim 10^{-18} \tilde{f}^2 \quad (167)$$

In dilute flux approximation, considering $\tilde{f}^2 < 10^{-8}$, life time of neutralino:

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} \text{Jsec}}{10^{-18} \tilde{f}^2} \sim \frac{10^{-34} \text{Jsec}}{10^{-26} \text{GeV}} > O(10^1) \text{sec} \quad (168)$$

3.3 Two-Body Slepton/Squark Decays

The decay width for $\tilde{l}/\tilde{q} \rightarrow l/q + \psi_\mu$ in our set up is given by:

$$\Gamma(\tilde{l}/\tilde{q} \rightarrow l/q + \psi_\mu) \sim \frac{m_{\tilde{l}/\tilde{q}}^5}{m_{3/2}^2 M_p^2} \sim \mathcal{V}^{-\frac{7}{2}} M_p, \quad (169)$$

implying a lifetime of around $10^{-25.5} \text{s}$, satisfying the BBN constraints.

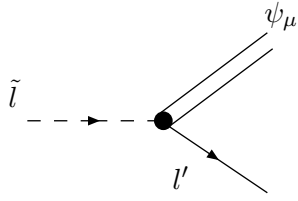


Figure 13: Two-Body Slepton/Squark Decay

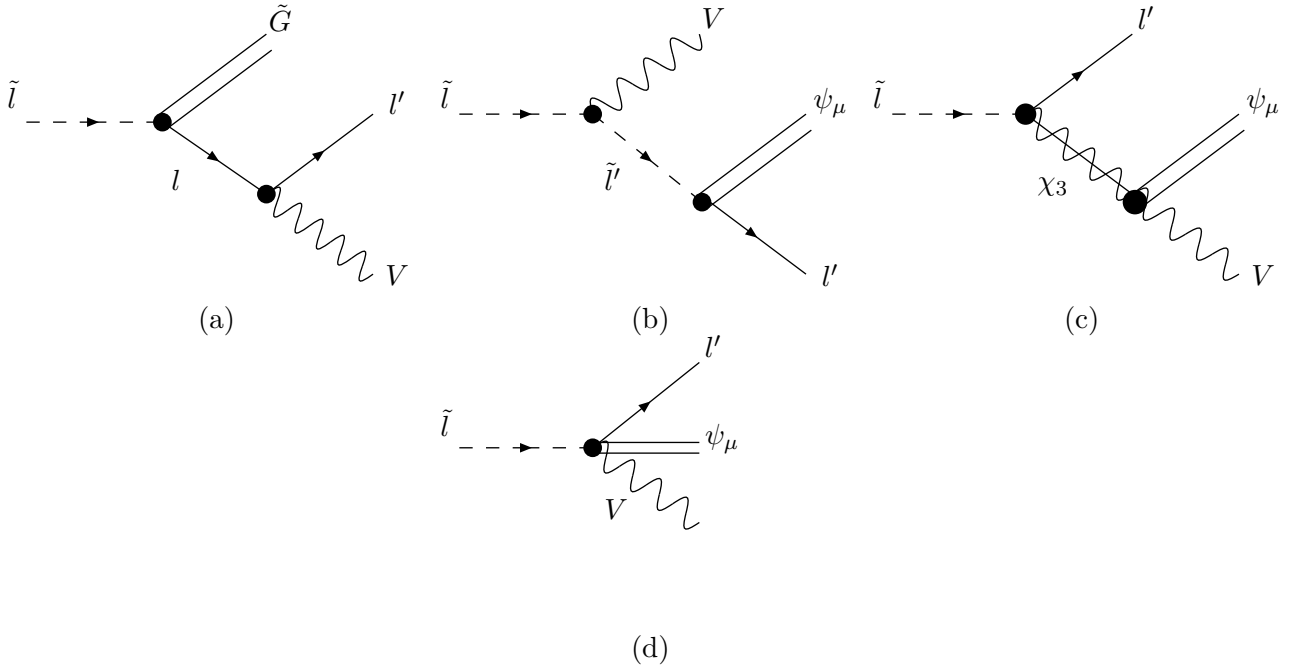


Figure 14: Three-body slepton decays

3.4 Three-Body Slepton Decays

As explained in [31], the following set of effective operators are relevant to three-body slepton decays:

$$\begin{aligned}
\mathcal{O}_1 &= \bar{l}'_h p_{\tilde{l}} \cdot \tilde{G}^c p_{l'} \cdot \epsilon^* \\
\mathcal{O}_2 &= \bar{l}'_h p_l \cdot \tilde{G}^c p_{\tilde{l}} \cdot \epsilon^* \\
\mathcal{O}_3 &= \bar{l}'_h p_{\tilde{G}} \cdot \tilde{G}^c p_{\tilde{l}} \cdot \epsilon^* \\
\mathcal{O}_4 &= i \bar{l}'_h \gamma \cdot \epsilon^* p_V \cdot \tilde{G}^c \\
\mathcal{O}_5 &= \bar{l}'_h \gamma \cdot p_V \gamma \cdot \epsilon^* p_{\tilde{l}} \cdot \tilde{G}^c \\
\mathcal{O}_6 &= i \bar{l}'_h \gamma \cdot p_V \gamma \cdot \epsilon^* p_V \cdot \tilde{G}^c \\
\mathcal{O}_7 &= i \bar{l}'_h \gamma \cdot p_{\tilde{G}} \gamma \cdot \epsilon^* p_V \cdot \tilde{G}^c \\
\mathcal{O}_8 &= i \bar{l}'_h \epsilon^* \cdot \tilde{G}^c \\
\mathcal{O}_9 &= i \bar{l}'_h \gamma \cdot p_V \epsilon^* \cdot \tilde{G}^c \\
\mathcal{O}_{10} &= i \bar{l}'_h \gamma \cdot p_{\tilde{G}} \gamma \cdot p_V \epsilon^* \cdot \tilde{G}^c ,
\end{aligned} \tag{170}$$

where ϵ_μ is the polarization of gauge boson V . Notice that for an on-shell gravitino in the final state, $\mathcal{O}_3 = 0$ by using the gravitino equation of motion.

Using the following volume-suppression factors in various vertices in Fig. 14(a)-(d) above:

$$\begin{aligned}
\tilde{l} - \tilde{G} - l \text{ vertex} : & \frac{g_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}{K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}} = 1; \\
l - l' - V \text{ vertex} : & \frac{g_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \tilde{f} \mathcal{V}^{-\frac{2}{3}} \ln \mathcal{V}}{K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}; \\
\tilde{l} - \tilde{l} - V \text{ vertex} : & \frac{\left(\frac{\kappa_4^2 \mu_7 Q^B G^{T^B \bar{T}^B}}{\mathcal{V}} \right) \kappa_4^2 \mu_7 C_{a_1 \bar{a}_1}}{\kappa_4^2 K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}} \sim \frac{\tilde{f} \mathcal{V}^{-2}}{10^4}; \\
\tilde{l} - l' - \tilde{G} \text{ vertex} : & \frac{g_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}{K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}} = 1,
\end{aligned} \tag{171}$$

and the operators of (170), the matrix elements for three-body slepton decay [diagrams (a)- (d)] are

$$\begin{aligned}
\mathcal{M}_V^a &\sim \frac{\tilde{f} \mathcal{V}^{-\frac{11}{18}}}{M_p} \frac{1}{s_{23}} (2\mathcal{O}_1 - \mathcal{O}_5) \\
\mathcal{M}_V^b &\sim \frac{\tilde{f} \mathcal{V}^{-2}}{10^4 M_p} \frac{1}{m_{\tilde{G}}^2 + m_V^2 - s_{13} - s_{23}} (\mathcal{O}_2 + \mathcal{O}_3) \\
\mathcal{M}_V^c &\sim \sum_i \frac{10 \tilde{f} \mathcal{V}^{-\frac{3}{2}}}{M_p} \frac{1}{s_{13} - s_{\chi_3}} [m_{\chi_3} (\mathcal{O}_4 - \mathcal{O}_9) - 4 (\mathcal{O}_6 + \mathcal{O}_7 - \mathcal{O}_{10}) + m_V^2 \mathcal{O}_8] \\
\mathcal{M}_V^d &\sim i \frac{\tilde{f} \mathcal{V}^{-\frac{11}{9}}}{10^4 M_p} \mathcal{O}_8 ,
\end{aligned} \tag{172}$$

where χ_3 denotes the lightest neutralino in our set up, and

$$s_{12} = (p_{\tilde{G}} + p_{l'})^2 , \quad s_{13} = (p_{\tilde{G}} + p_V)^2 , \quad s_{23} = (p_{l'} + p_V)^2 . \tag{173}$$

Notice that $s_{12} + s_{13} + s_{23} = m_{\tilde{l}}^2 + m_{\tilde{G}}^2 + m_Z^2$.

The differential decay width is

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32m_{\tilde{l}}^3} |\mathcal{M}|^2 dm_{13}^2 dm_{23}^2, \quad (174)$$

where $\mathcal{M} = \mathcal{M}_V^a + \mathcal{M}_V^b + \mathcal{M}_V^c + \mathcal{M}_V^d$.

The sum of the matrix elements can be written as

$$\mathcal{M}(\tilde{l}_h \rightarrow l' V \tilde{G}) = \mathcal{M}_V^a + \mathcal{M}_V^b + \mathcal{M}_V^c + \mathcal{M}_V^d = \sum_{i=1\dots 10} \mathcal{M}_i \mathcal{O}_i, \quad (175)$$

where the \mathcal{M}_i can be read off from Eqs. (172) above as:

$$\begin{aligned} \mathcal{M}_{1,5} &\sim \frac{\tilde{f}\mathcal{V}^{-\frac{11}{18}}}{M_p s_{23}}; \quad \mathcal{M}_{2,3} \sim \frac{\left(\frac{\tilde{f}\mathcal{V}^{-2}}{10^4}\right)}{M_p(m_{3/2}^2 - s_{13} - s_{23})}; \\ \mathcal{M}_{4,9} &\sim \frac{m_\chi \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}}\right)}{M_p(s_{13} - m_{\chi_3}^2)}; \quad \mathcal{M}_{6,7,10} \sim \frac{M_{4,9}}{m_{\chi_3}}; \\ \mathcal{M}_8 &\sim \frac{1}{M_p} \left(\frac{m_V^2 \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}}\right)}{s_{13} - m_{\chi_3}^2} + \frac{\tilde{f}\mathcal{V}^{-\frac{11}{9}}}{10^4} \right). \end{aligned} \quad (176)$$

The squared matrix element is

$$\begin{aligned} \left| \mathcal{M}(\tilde{l}_h \rightarrow l' V \tilde{G}) \right|^2 &= \sum_{i=1\dots 10} |\mathcal{M}_i|^2 \mathcal{O}_{i,i} \\ &+ \sum_{i,j=1\dots 10}^{i<j} \text{Re}(\mathcal{M}_i \mathcal{M}_j^*) \mathcal{O}_{i,j}^{\text{re}} + \sum_{i,j=1\dots 10}^{i<j} \Im(\mathcal{M}_i \mathcal{M}_j^*) \mathcal{O}_{i,j}^{\text{im}}. \end{aligned} \quad (177)$$

All the \mathcal{M}_i are real except for \mathcal{M}_8 , which has both real and imaginary components. The only non-zero contributions to the last term in Eq. (177) come from $\Im(\mathcal{M}_i \mathcal{M}_8^*)$ ($i < 8$) and $\Im(\mathcal{M}_8 \mathcal{M}_j^*)$ ($j > 8$). Expressions for $\mathcal{O}_{i,i}$, $\mathcal{O}_{i,j}^{\text{re}}$, $\mathcal{O}_{i,8}^{\text{im}}$ ($i < 8$) and $\mathcal{O}_{8,j}^{\text{im}}$ ($j > 8$), and the corresponding contributions

to $\int \int ds_{13} ds_{23} \frac{|\mathcal{M}|^2}{m_i^3}$ are given as under: are:

$$\begin{aligned}
\mathcal{O}_{1,1} &= -\frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 m_{\tilde{l}}^2 - (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \right] \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right] \\
&\quad \times \left[m_{\tilde{G}}^2 m_V^2 + m_{\tilde{l}}^2 m_V^2 - 2m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} - (p_{\tilde{G}} \cdot p_V)^2 + 2p_{\tilde{G}} \cdot p_V p_{\tilde{l}} \cdot p_V - (p_{\tilde{l}} \cdot p_V)^2 \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_1|^2 \mathcal{O}_{1,1}}{m_{\tilde{l}}^3} \sim \left(\tilde{f} \mathcal{V}^{-\frac{11}{18}} \right)^2 \mathcal{V}^{-\frac{1}{2}} M_p \sim 10^{-16.5} M_p; \\
\mathcal{O}_{2,2} &= -\frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right] \left[m_{\tilde{l}}^2 m_V^2 - (p_{\tilde{l}} \cdot p_V)^2 \right] \\
&\quad \times \left[- (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 + 2p_{\tilde{G}} \cdot p_{\tilde{l}} p_{\tilde{G}} \cdot p_V - (p_{\tilde{G}} \cdot p_V)^2 + m_{\tilde{G}}^2 (m_{\tilde{l}}^2 + m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_2|^2 \mathcal{O}_{2,2}}{m_{\tilde{l}}^3} \sim \left(\frac{\tilde{f} \mathcal{V}^{-2}}{10^4} \right)^2 \mathcal{V}^{-\frac{1}{2}} M_p \sim 10^{-21} M_p; \\
\mathcal{O}_{3,3} &= 0; \\
\mathcal{O}_{4,4} &= \frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 m_V^2 - (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\quad \times \left[m_{\tilde{G}}^2 m_V^2 - m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} + 2(p_{\tilde{G}} \cdot p_V)^2 + p_{\tilde{G}} \cdot p_V (3m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_4|^2 \mathcal{O}_{4,4}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{5}{9}} M_p \sim 10^{-18} M_p;
\end{aligned} \tag{178}$$

$$\begin{aligned}
\mathcal{O}_{5,5} &= -\frac{4}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 m_{\tilde{l}}^2 - (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \right] \\
&\times \left[m_{\tilde{G}}^2 m_V^2 - m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} - 4 (p_{\tilde{G}} \cdot p_V)^2 + p_{\tilde{G}} \cdot p_V (-3m_Z^2 + 4p_{\tilde{l}} \cdot p_V) \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_5|^2 \mathcal{O}_{5,5}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{11}{18}} \right)^2 \mathcal{V}^{-\frac{1}{2}} M_p \sim 10^{-16.5} M_p; \\
\mathcal{O}_{6,6} &= -\frac{4}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 m_Z^2 - (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\times \left[m_{\tilde{G}}^2 m_V^2 - m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} - 4 (p_{\tilde{G}} \cdot p_V)^2 + p_{\tilde{G}} \cdot p_V (-3m_V^2 + 4p_{\tilde{l}} \cdot p_V) \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_6|^2 \mathcal{O}_{6,6}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{25}{9}} M_p \sim 10^{-35} M_p; \\
\mathcal{O}_{7,7} &= \frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 m_V^2 - (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\left\{ m_{\tilde{G}}^4 m_V^2 + 2m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^2 + 4p_{\tilde{G}} \cdot p_V^3 - p_{\tilde{G}} \cdot p_{\tilde{l}} \left[m_{\tilde{G}}^2 m_V^2 + 4(p_{\tilde{G}} \cdot p_V)^2 \right] \right. \\
&\left. - m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V (m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right\}, \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_7|^2 \mathcal{O}_{7,7}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{8}{9}} M_p \sim 10^{-16.6} M_p; \\
\mathcal{O}_{8,8} &= -\frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right] \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_8|^2 \mathcal{O}_{8,8}}{m_{\tilde{l}}^3} \sim 10^{-27.5} M_p; \\
\mathcal{O}_{9,9} &= \frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\times \left[m_{\tilde{G}}^2 m_V^2 - m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} - 2(p_{\tilde{G}} \cdot p_V)^2 - p_{\tilde{G}} \cdot p_V (m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_9|^2 \mathcal{O}_{9,9}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{5}{9}} M_p \sim 10^{-18} M_p; \\
\mathcal{O}_{10,10} &= \frac{4}{3m_{\tilde{G}}^2 m_V^2} \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\left\{ m_{\tilde{G}}^4 m_V^2 - 2m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^2 - 4(p_{\tilde{G}} \cdot p_V)^3 \right. \\
&\left. + p_{\tilde{G}} \cdot p_{\tilde{l}} \left[-m_{\tilde{G}}^2 m_V^2 + 4(p_{\tilde{G}} \cdot p_V)^2 \right] + m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V (3m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right\}, \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{|\mathcal{M}_{10}|^2 \mathcal{O}_{10,10}}{m_{\tilde{l}}^3} \sim \left(10 \tilde{f} \mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{8}{9}} M_p \sim 10^{-16.6} M_p; \tag{179}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{1,2}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right]; \\
&\times \left[- (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 + p_{\tilde{G}} \cdot p_{\tilde{l}} p_{\tilde{G}} \cdot p_V + m_{\tilde{G}}^2 (m_{\tilde{l}}^2 - p_{\tilde{l}} \cdot p_V) \right] \\
&\times \left[-m_{\tilde{l}}^2 m_V^2 + m_Z^2 p_{\tilde{G}} \cdot p_{\tilde{l}} - p_{\tilde{G}} \cdot p_V p_{\tilde{l}} \cdot p_V + (p_{\tilde{l}} \cdot p_V)^2 \right], \\
&\int_{m_{3/2}^2 + m_V^2}^{2m_{\tilde{l}}^2 + m_V^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_1 \mathcal{M}_2) \mathcal{O}_{1,2}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-26} M_p; \\
\mathcal{O}_{1,3}^{\text{re}} &= \mathcal{O}_{1,4}^{\text{re}} = 0
\end{aligned} \tag{180}$$

$$\begin{aligned}
\mathcal{O}_{1,5}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 m_{\tilde{l}}^2 - (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \right] \left[- (p_{\tilde{G}} \cdot p_V)^2 + m_{\tilde{G}}^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) \right. \\
&\quad \left. + p_{\tilde{G}} \cdot p_V (-m_{\tilde{l}}^2 + p_{\tilde{l}} \cdot p_V) + p_{\tilde{G}} \cdot p_{\tilde{l}} (-m_V^2 + p_{\tilde{G}} \cdot p_V + p_{\tilde{l}} \cdot p_V) \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_1 \mathcal{M}_5) \mathcal{O}_{1,5}^{\text{re}}}{m_{\tilde{l}}^3} \sim \tilde{f}^2 \mathcal{V}^{-\frac{11}{9} - \frac{1}{2}} \sim 10^{-16.5} M_p; \\
\mathcal{O}_{1,6}^{\text{re}} &= \mathcal{O}_{1,7}^{\text{re}} = \mathcal{O}_{1,8}^{\text{re}} = \mathcal{O}_{1,9}^{\text{re}} = \mathcal{O}_{1,10}^{\text{re}} = 0 \\
\mathcal{O}_{2,3}^{\text{re}} &= \mathcal{O}_{2,4}^{\text{re}} = 0
\end{aligned} \tag{181}$$

$$\begin{aligned}
\mathcal{O}_{2,5}^{\text{re}} &= \frac{4}{3m_{\tilde{G}}^2} \left\{ (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \left[m_{\tilde{G}}^2 m_V^2 + 2 (m_{\tilde{l}}^2 - m_V^2) p_{\tilde{G}} \cdot p_V \right] \right. \\
&\quad + 2 (p_{\tilde{G}} \cdot p_{\tilde{l}})^3 (m_V^2 - p_{\tilde{l}} \cdot p_V) - 2 p_{\tilde{G}} \cdot p_{\tilde{l}} \left[(p_{\tilde{G}} \cdot p_V)^2 (m_{\tilde{l}}^2 - p_{\tilde{l}} \cdot p_V) \right. \\
&\quad \left. + m_{\tilde{G}}^2 (m_{\tilde{l}}^2 - p_{\tilde{l}} \cdot p_V) (m_V^2 - p_{\tilde{l}} \cdot p_V) + m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V p_{\tilde{l}} \cdot p_V \right] \\
&\quad \left. + m_{\tilde{G}}^2 \left[m_{\tilde{l}}^2 (p_{\tilde{G}} \cdot p_V)^2 - 2 p_{\tilde{G}} \cdot p_V (m_{\tilde{l}}^2 - p_{\tilde{l}} \cdot p_V)^2 \right. \right. \\
&\quad \left. \left. + m_{\tilde{G}}^2 (-m_{\tilde{l}}^2 m_V^2 + (p_{\tilde{l}} \cdot p_V)^2) \right] \right\}, \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_2 \mathcal{M}_5) \mathcal{O}_{2,5}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-37} M_p;
\end{aligned}$$

$$\mathcal{O}_{2,6}^{\text{re}} = \mathcal{O}_{2,7}^{\text{re}} = \mathcal{O}_{2,8}^{\text{re}} = \mathcal{O}_{2,9}^{\text{re}} = \mathcal{O}_{2,10}^{\text{re}} = 0 \tag{182}$$

$$\mathcal{O}_{3,4}^{\text{re}} = \mathcal{O}_{3,5}^{\text{re}} = \mathcal{O}_{3,6}^{\text{re}} = \mathcal{O}_{3,7}^{\text{re}} = \mathcal{O}_{3,8}^{\text{re}} = \mathcal{O}_{3,9}^{\text{re}} = \mathcal{O}_{3,10}^{\text{re}} = 0 \tag{183}$$

$$\mathcal{O}_{4,5}^{\text{re}} = 0;$$

$$\begin{aligned}
\mathcal{O}_{4,6}^{\text{re}} &= \frac{8}{m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 m_V^2 - (p_{\tilde{G}} \cdot p_V)^2 \right] \left[m_V^2 + p_{\tilde{G}} \cdot p_V - p_{\tilde{l}} \cdot p_V \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_4 \mathcal{M}_6) \mathcal{O}_{4,6}^{\text{re}}}{m_{\tilde{l}}^3} \sim \tilde{f}^2 \mathcal{V}^{-\frac{65}{18}} M_p \sim 10^{-42} M_p; \\
\mathcal{O}_{4,7}^{\text{re}} &= \frac{8}{m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right] \left[m_{\tilde{G}}^2 m_V^2 - (p_{\tilde{G}} \cdot p_V)^2 \right], \\
&\int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_4 \mathcal{M}_7) \mathcal{O}_{4,7}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(\tilde{f} \mathcal{V}^{-\frac{3}{2}} \right) \mathcal{V}^{-\frac{28}{9}} M_p \sim 10^{-36} M_p;
\end{aligned} \tag{184}$$

$$\begin{aligned}
\mathcal{O}_{4,8}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}m_V^2} \left[-m_V^2 \left(m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) p_{\tilde{G}} \cdot p_V + p_{\tilde{G}} \cdot p_V^3 \right. \\
&\quad \left. + m_{\tilde{G}}^2 m_V^2 \left(m_V^2 - p_{\tilde{l}} \cdot p_V \right) - \left(p_{\tilde{G}} \cdot p_V \right)^2 \left(m_V^2 + p_{\tilde{l}} \cdot p_V \right) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_4 \mathcal{M}_8) \mathcal{O}_{4,8}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-15} M_p; \\
\mathcal{O}_{4,9}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^4 m_V^4 + m_{\tilde{G}}^2 m_V^4 p_{\tilde{G}} \cdot p_V + m_{\tilde{G}}^2 m_V^2 \left(p_{\tilde{G}} \cdot p_V \right)^2 - 2 \left(p_{\tilde{G}} \cdot p_V \right)^4 \right. \\
&\quad \left. - m_V^2 p_{\tilde{G}} \cdot p_{\tilde{l}} \left(m_{\tilde{G}}^2 m_V^2 + \left(p_{\tilde{G}} \cdot p_V \right)^2 \right) - \left(p_{\tilde{G}} \cdot p_V \right)^3 \left(m_V^2 - 2p_{\tilde{l}} \cdot p_V \right) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_4 \mathcal{M}_9) \mathcal{O}_{4,9}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{2}{9}} M_p \sim 10^{-20} M_p; \\
\mathcal{O}_{4,10}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}} \left\{ m_{\tilde{G}}^4 m_V^2 - m_{\tilde{G}}^2 \left(p_{\tilde{G}} \cdot p_V \right)^2 - 3p_{\tilde{G}} \cdot p_V^3 + p_{\tilde{G}} \cdot p_{\tilde{l}} \left[-m_{\tilde{G}}^2 m_V^2 + 3 \left(p_{\tilde{G}} \cdot p_V \right)^2 \right] \right. \\
&\quad \left. + m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V \left(3m_V^2 - 2p_{\tilde{l}} \cdot p_V \right) \right\}, \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_4 \mathcal{M}_{10}) \mathcal{O}_{4,10}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{28}{9}} M_p \sim 10^{-24} M_p; \\
\mathcal{O}_{5,6}^{\text{re}} &= \mathcal{O}_{5,7}^{\text{re}} = \mathcal{O}_{5,8}^{\text{re}} = \mathcal{O}_{5,9}^{\text{re}} = \mathcal{O}_{5,10}^{\text{re}} = 0 \\
\mathcal{O}_{6,7}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 m_V^2 - \left(p_{\tilde{G}} \cdot p_V \right)^2 \right] \\
&\quad \times \left[\left(3m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) p_{\tilde{G}} \cdot p_V + 2 \left(p_{\tilde{G}} \cdot p_V \right)^2 + m_{\tilde{G}}^2 \left(m_V^2 - p_{\tilde{l}} \cdot p_V \right) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_6 \mathcal{M}_7) \mathcal{O}_{6,7}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{22}{9}} M_p \sim 10^{-33} M_p; \\
\mathcal{O}_{6,8}^{\text{re}} &= -\frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^4 m_V^2 - m_{\tilde{G}}^2 \left(p_{\tilde{G}} \cdot p_V \right)^2 + \left(p_{\tilde{G}} \cdot p_V \right)^3 \right. \\
&\quad \left. - p_{\tilde{G}} \cdot p_{\tilde{l}} \left(m_{\tilde{G}}^2 m_V^2 + \left(p_{\tilde{G}} \cdot p_V \right)^2 \right) - m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V \left(m_V^2 - 2p_{\tilde{l}} \cdot p_V \right) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_6 \mathcal{M}_8) \mathcal{O}_{6,8}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-27} M_p; \\
\mathcal{O}_{6,9}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}} \left[m_V^2 \left(3m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) p_{\tilde{G}} \cdot p_V - 3 \left(p_{\tilde{G}} \cdot p_V \right)^3 \right. \\
&\quad \left. - \left(p_{\tilde{G}} \cdot p_V \right)^2 \left(m_V^2 - 3p_{\tilde{l}} \cdot p_V \right) + m_{\tilde{G}}^2 m_V^2 \left(m_V^2 - p_{\tilde{l}} \cdot p_V \right) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_6 \mathcal{M}_9) \mathcal{O}_{6,9}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{31}{9}} M_p \sim 10^{-38} M_p; \quad (185)
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{6,10}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^4 m_V^2 p_{\tilde{G}} \cdot p_V - \left(m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) (p_{\tilde{G}} \cdot p_V)^3 - 2(p_{\tilde{G}} \cdot p_V)^4 \right. \\
&\quad \left. + m_{\tilde{G}}^4 m_V^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) + m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_6 \mathcal{M}_{10}) \mathcal{O}_{6,10}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{22}{9}} M_p \sim 10^{-33} M_p; \\
\mathcal{O}_{7,8}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_V^2} \left[m_{\tilde{G}}^4 m_V^2 p_{\tilde{G}} \cdot p_V - \left(m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) (p_{\tilde{G}} \cdot p_V)^3 - 2(p_{\tilde{G}} \cdot p_V)^4 \right. \\
&\quad \left. + m_{\tilde{G}}^4 m_V^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) + m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) \right] \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_7 \mathcal{M}_8) \mathcal{O}_{7,8}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-17.5} M_p; \\
\mathcal{O}_{7,9}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^4 m_V^2 - m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^2 + (p_{\tilde{G}} \cdot p_V)^3 \right. \\
&\quad \left. - p_{\tilde{G}} \cdot p_{\tilde{l}} \left(m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right) - m_{\tilde{G}}^2 p_{\tilde{G}} \cdot p_V (m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_7 \mathcal{M}_9) \mathcal{O}_{7,9}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{28}{9}} M_p \sim 10^{-24} M_p; \\
\mathcal{O}_{7,10}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_Z^2} \left\{ m_{\tilde{G}}^6 m_V^4 + m_{\tilde{G}}^4 m_V^4 p_{\tilde{G}} \cdot p_V + m_{\tilde{G}}^4 m_V^2 (p_{\tilde{G}} \cdot p_V)^2 \right. \\
&\quad \left. - 2m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^4 - 4(p_{\tilde{G}} \cdot p_V)^5 \right. \\
&\quad \left. - p_{\tilde{G}} \cdot p_{\tilde{l}} \left[m_{\tilde{G}}^4 m_V^4 + m_{\tilde{G}}^2 m_V^2 (p_{\tilde{G}} \cdot p_V)^2 - 4(p_{\tilde{G}} \cdot p_V)^4 \right] \right. \\
&\quad \left. + m_{\tilde{G}}^2 (p_{\tilde{G}} \cdot p_V)^3 (3m_V^2 - 2p_{\tilde{l}} \cdot p_V) \right\}, \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_7 \mathcal{M}_{10}) \mathcal{O}_{7,10}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{\frac{8}{9}} M_p \sim 10^{-16.6} M_p; \\
\mathcal{O}_{8,9}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_V^2} \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right] [m_V^2 + p_{\tilde{G}} \cdot p_V - p_{\tilde{l}} \cdot p_V], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_8 \mathcal{M}_9) \mathcal{O}_{8,9}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-25.5} M_p; \\
\mathcal{O}_{8,10}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2 m_V^2} \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right] \\
&\quad \times \left[- \left(m_{\tilde{G}}^2 - 2p_{\tilde{G}} \cdot p_{\tilde{l}} \right) p_{\tilde{G}} \cdot p_V - 2(p_{\tilde{G}} \cdot p_V)^2 + m_{\tilde{G}}^2 (m_V^2 - p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_8 \mathcal{M}_{10}) \mathcal{O}_{8,10}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-18.5} M_p; \\
\mathcal{O}_{9,10}^{\text{re}} &= \frac{8}{3m_{\tilde{G}}^2} \left[m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V \right] \left[2m_{\tilde{G}}^2 m_V^2 + (p_{\tilde{G}} \cdot p_V)^2 \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re}(\mathcal{M}_9 \mathcal{M}_{10}) \mathcal{O}_{9,10}^{\text{re}}}{m_{\tilde{l}}^3} \sim \left(10\tilde{f}\mathcal{V}^{-\frac{3}{2}} \right)^2 \mathcal{V}^{-\frac{28}{9}} M_p \sim 10^{-24} M_p;
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{1,8}^{\text{im}} &= -\frac{8}{3m_{\tilde{G}}^2 m_V^2} (m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V) \left\{ \left[m_{\tilde{l}}^2 m_V^2 - (p_{\tilde{l}} \cdot p_V)^2 + (p_{\tilde{G}} \cdot p_V)(p_{\tilde{l}} \cdot p_V) \right] m_{\tilde{G}}^2 \right. \\
&\quad \left. - m_V^2 (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 + (p_{\tilde{G}} \cdot p_{\tilde{l}})(p_{\tilde{G}} \cdot p_V)(p_{\tilde{l}} \cdot p_V - p_{\tilde{G}} \cdot p_V) \right\}, \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\Im m (\mathcal{M}_1 \mathcal{M}_8^*) \mathcal{O}_{1,8}^{\text{im}}}{m_{\tilde{l}}^3} \sim 10^{-44} M_p; \\
\mathcal{O}_{2,8}^{\text{im}} &= -\frac{8}{3m_{\tilde{G}}^2 m_V^2} (m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} + p_{\tilde{G}} \cdot p_V) \left[(m_{\tilde{l}}^2 m_V^2 - (p_{\tilde{l}} \cdot p_V)^2) m_{\tilde{G}}^2 - m_V^2 (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \right. \\
&\quad \left. - (p_{\tilde{G}} \cdot p_V)^2 p_{\tilde{l}} \cdot p_V + (p_{\tilde{G}} \cdot p_{\tilde{l}})(p_{\tilde{G}} \cdot p_V)(m_V^2 + p_{\tilde{l}} \cdot p_V) \right], \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\Im m (\mathcal{M}_2 \mathcal{M}_8^*) \mathcal{O}_{1,8}^{\text{im}}}{m_{\tilde{l}}^3} \sim 10^{-33} M_p; \\
\mathcal{O}_{3,8}^{\text{im}} &= \mathcal{O}_{4,8}^{\text{im}} = 0 \\
\mathcal{O}_{5,8}^{\text{im}} &= \frac{8}{3m_{\tilde{G}}^2} \left\{ (p_{\tilde{G}} \cdot p_{\tilde{l}} - m_{\tilde{G}}^2) p_{\tilde{l}} \cdot p_V m_{\tilde{G}}^2 - p_{\tilde{G}} \cdot p_{\tilde{l}} (p_{\tilde{G}} \cdot p_V)^2 \right. \\
&\quad \left. + p_{\tilde{G}} \cdot p_V \left[p_{\tilde{G}} \cdot p_{\tilde{l}} m_{\tilde{G}}^2 + (p_{\tilde{l}} \cdot p_V - 2m_{\tilde{l}}^2) m_{\tilde{G}}^2 + (p_{\tilde{G}} \cdot p_{\tilde{l}})^2 \right] \right\}, \\
&\quad \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\Im m (\mathcal{M}_5 \mathcal{M}_8^*) \mathcal{O}_{5,8}^{\text{im}}}{m_{\tilde{l}}^3} \sim 10^{-38} M_p; \\
\mathcal{O}_{6,8}^{\text{im}} &= \mathcal{O}_{7,8}^{\text{im}} = \mathcal{O}_{8,9}^{\text{im}} = \mathcal{O}_{8,10}^{\text{im}} = 0. \tag{187}
\end{aligned}$$

Hence,

$$\Gamma \left(\tilde{l} \rightarrow l' + \tilde{G} + V \right) \sim \int_{m_{3/2}^2}^{2m_{\tilde{l}}^2} ds_{13} \int_{m_V^2}^{m_{\tilde{l}}^2} ds_{23} \frac{\text{Re} (\mathcal{M}_4 \mathcal{M}_8^*) \mathcal{O}_{4,8}^{\text{re}}}{m_{\tilde{l}}^3} \sim 10^{-15} M_p, \tag{188}$$

implying that the associated life-time of this decay is about 10^{-28} s, which respects the BBN constraints.

4 Gravitino Decays

In this section, we discuss two-body and three-body gravitino decays and show that the corresponding lifetimes are either of the order or more than the age of the universe

4.1 Two-Body Gravitino Decays

We discuss the decays of the gravitino into neutrino and gauge bosons as well as the light Higgs and neutrinos.

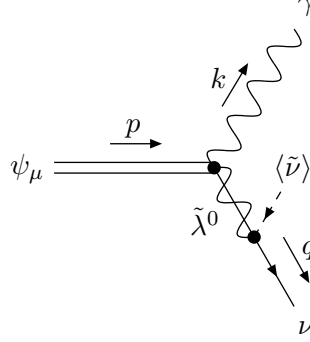


Figure 15: Two-body gravitino decay: $\psi_\mu \rightarrow \nu + \gamma$

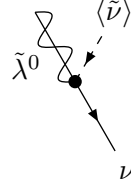


Figure 16: Gaugino- \langle sneutrino \rangle -neutrino vertex

In $\mathcal{N} = 1$ SUGRA the gaugino- \langle sneutrino \rangle -neutrino vertex, Fig. 16, is given by:

$$\mathcal{O}(a_1) - \text{term in } \frac{g_{YM} g_{T^B \bar{a}_1} X^{T^B}}{(\sqrt{K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}})^2} \sim \frac{\mathcal{V}^{-\frac{11}{9}}}{10^4}. \quad (189)$$

Similarly the gravitino-gauge-boson- $\langle \tilde{\nu} \rangle - \nu$ -vertex in the $\mathcal{N} = 1$ SUGRA Lagrangian is given by the $\mathcal{O}(a_1)$ -term in $g_{YM} g_{a_1 \bar{T}^B} X^{T^B} Z^0 / A_\mu \bar{\nu} \gamma^\nu \gamma^\mu \psi_\nu$, which is: $\frac{g_{YM} g_{a_1 \bar{T}^B} X^{T^B}}{(\sqrt{K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}})^2} \sim \frac{\mathcal{V}^{-\frac{11}{9}}}{10^4}$. Using [32], one sees that the lifetime of the decay $\psi_\mu \rightarrow \gamma + \nu_e$ is given by:

$$\begin{aligned} \Gamma(\psi_\mu \rightarrow \gamma + \nu_e) &= \frac{1}{64\pi} \left(\frac{\langle \mathcal{A}_1 \rangle}{\langle \mathcal{Z}_i \rangle} \right)^2 \frac{m_{3/2}^3}{M_p^2} |U_{\tilde{\gamma} \tilde{Z}}|^2 \left(\frac{V^{-\frac{11}{9}}}{10^4} \right)^2 \\ &\sim \frac{1}{64\pi} \left(\frac{\langle \mathcal{V}^{-\frac{2}{9}} \rangle}{\mathcal{V}^{\frac{1}{36}}} \right)^2 \frac{m_{3/2}^2}{M_p^2} m_Z^2 \left(\frac{M_{\lambda_1} - M_{\lambda_2}}{M_{\lambda_1} M_{\lambda_2}} \right)^2 \sin^2 \theta_W \cos^2 \theta_W \left(\frac{V^{-\frac{11}{9}}}{10^4} \right)^2. \end{aligned} \quad (190)$$

We have a gaugino mass degeneracy (up to $\mathcal{O}(1)$ factors) implying: $M_{\lambda_1} \sim M_{\lambda_2}$. One hence gets a very small decay width and an extremely enhanced lifetime.

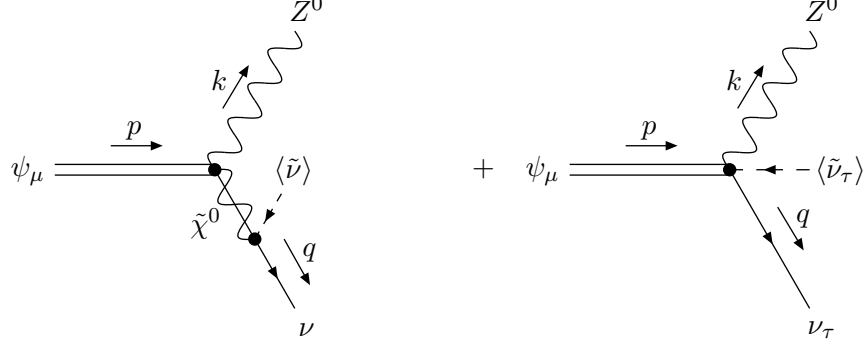


Figure 17: Two-body gravitino decay: $\psi_\mu \rightarrow Z^0 + \nu$

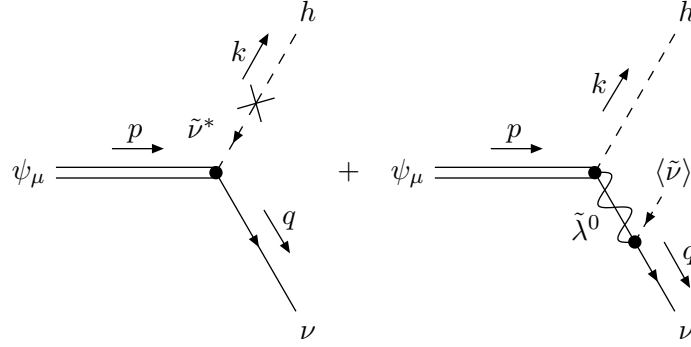


Figure 18: Two-body gravitino decay: $\psi_\mu \rightarrow h + \nu$

Again, using (189) and [32], the decay width for $\psi_\mu \rightarrow Z + \nu$ is given by:

$$\begin{aligned}
 \Gamma(\psi_\mu \rightarrow Z + \nu) &\sim \frac{1}{64\pi} \left(\frac{\langle \mathcal{A}_1 \rangle}{\langle \mathcal{Z}_i \rangle} \right)^2 \frac{m_{3/2}^3}{M_p^2} \left(1 - \frac{M_Z^2}{m_{3/2}^2} \right)^2 \left(\frac{V^{-\frac{11}{9}}}{10^4} \right)^2 \\
 &\times \left[\left(\frac{M_Z}{M_\lambda} \right)^2 \times \mathcal{O}(1) + \mathcal{O}(1) \left| 1 + \sin\beta \cos\beta \left(\frac{M_Z^2}{M_\lambda \hat{\mu}_{Z_1 Z_2}} \right) \right|^2 + \left(\frac{M_Z}{m_{3/2}} \right) \left(\frac{m_Z}{M_\lambda} \right) \left(1 + \sin\beta \cos\beta \left(\frac{M_Z^2}{M_\lambda \hat{\mu}_{Z_1 Z_2}} \right) \right) \times \mathcal{O}(1) \right] \\
 &\sim \frac{\mathcal{V}^{-\frac{1}{2}-6-\frac{22}{9}}}{64\pi \times 10^8} \times \mathcal{O}(1) M_p.
 \end{aligned} \tag{191}$$

This, for $\mathcal{V} \sim 10^6$ yields $\tau \sim 10^{21}s$. The mass-like insertion is given by $m_{\tilde{\nu}h}^2$, which is the coefficient of $\tilde{\nu}h$ in the $\mathcal{N} = 1$ SUGRA action. To estimate this, we note that near $|z_1| \sim |z_2| \sim 0.7\mathcal{V}^{\frac{1}{36}}, |a_1| \sim$

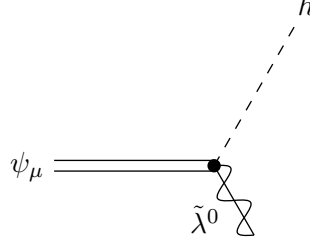


Figure 19: Gravitino-gaugino-Higgs vertex

$\mathcal{V}^{-\frac{2}{9}}, |a_2| \sim \mathcal{V}^{-\frac{1}{9}}, |a_3| \sim \mathcal{V}^{-\frac{13}{18}}, |a_4| \sim \mathcal{V}^{-\frac{11}{9}}$, the potential can be approximated by

$$V \sim e^K G^{T_S \bar{T}_S} |D_{T_S} W|^2$$

$$z^{72} e^{-2n^s(\text{vol}(\Sigma_S) + \mu_3 z^2)} \sqrt{\mathcal{V}^{\frac{1}{18}} + \mu_3 z^2}$$

$$\mathcal{V} + \left(\mathcal{V}^{\frac{2}{3}} + (\alpha_{12} \mathcal{V}^{\frac{5}{18}} + \alpha_{13} \mathcal{V}^{\frac{8}{9}} + \alpha_{14} \mathcal{V}^{\frac{8}{9}}) a_1 + (\alpha_{11} \mathcal{V}^{\frac{10}{9}} a_1 + \alpha_{12} \mathcal{V}^{\frac{5}{18}} + \alpha_{13} \mathcal{V}^{\frac{8}{9}} + \alpha_{14} \mathcal{V}^{\frac{8}{9}}) |a_1| + \mu_3 z^2 \right)^{\frac{3}{2}} - \left(\mathcal{V}^{\frac{1}{18}} + \mu_3 z^2 \right)^{\frac{3}{2}}. \quad (192)$$

The coefficient of the $\mathcal{O}\left((z - 0.6\mathcal{V}^{\frac{1}{36}})(a_1 - \mathcal{V}^{-\frac{2}{9}})\right) / \sqrt{K_{Z_1 \bar{Z}_1} K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}$ in (192) turns out to be:

$$m_{\tilde{\nu}h}^2 \sim \mathcal{V}^{\frac{23}{18}} m_{3/2}^2 \times 10^{-16}. \quad (193)$$

The gravitino-gaugino-Higgs vertex: will be given by the following term in the $\mathcal{N} = 1$ SUGRA action:

$$g_{YM} D^{T^B} \bar{\psi}_\mu \gamma^\mu \lambda = 0, \text{ on shell gravitino.} \quad (194)$$

Hence, the second diagram involving a gaugino NLSP, does not contribute. However, if the gaugino is replaced by a neutralino, then the Higgsino-component of this neutralino will contribute via the term:

$$g_{Z_1 \bar{Z}_1} \partial_\nu Z_1 \bar{\psi}_{\mu R} \gamma^\nu \gamma^\mu \tilde{H}_L^0 + \text{h.c.} \quad (195)$$

in $\mathcal{N} = 1$ SUGRA action. This yields a vertex:

$$\tilde{f} \frac{g_{Z_1 \bar{Z}_1} \gamma_\mu \not{k}}{(\sqrt{K_{Z_1 \bar{Z}_1}})^2} \sim \tilde{f} \gamma_\mu \not{k}. \quad (196)$$

Further, the Higgsino- $\langle \tilde{\nu}_e \rangle$ - ν_e vertex will be given by $\mathcal{O}(a_1 - z_i)$ term in $e^{\frac{K}{2}} (\mathcal{D}_{\bar{a}_1} D_{z_i} \bar{W}) \bar{\nu}_e \not{L} \tilde{H}_R^0$, which after assigning $\langle z_i \rangle$ a value of $\sim \mathcal{V}^{\frac{1}{36}}$ yields:

$$\tilde{f} \mathcal{V}^{-\frac{61}{36}} \langle \nu_e \rangle. \quad (197)$$

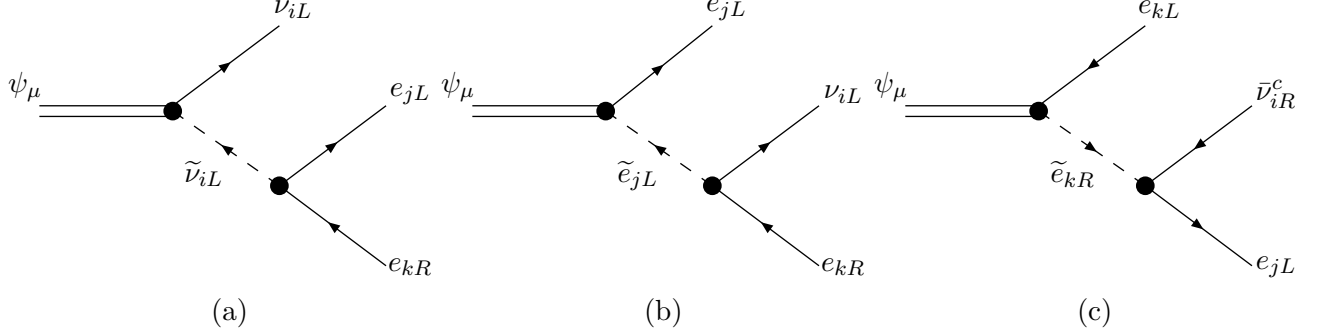


Figure 20: Three-body gravitino decays involving $\mathbb{R}_p \lambda_{ijk}$ coupling

Hence, using (196), (197) and [32]:

$$\begin{aligned} \Gamma(\psi_\mu \rightarrow h + \nu_e) &\sim \frac{1}{384\pi} \left(\frac{m_{3/2}^3}{M_p^2} \right) \left| \frac{m_{\tilde{\nu}_e h}^2}{m_h^2 - m_{\tilde{\nu}_e}^2} + \sin\beta \cos\beta \frac{m_Z^2}{M_{\tilde{g}\hat{U}_{Z_1\bar{Z}_2}}} \frac{\langle \nu_e \rangle}{\langle Z_i \rangle} \tilde{f}^2 \mathcal{V}^{-\frac{61}{36}} \right|^2 \\ &\sim 10^{-3} \mathcal{V}^{-6+\frac{5}{9}} \times 10^{-32} M_p \Big|_{\mathcal{V} \sim 10^5} \sim 10^{-60} M_p, \end{aligned} \quad (198)$$

which yields a lifetime of around $10^{17} s$.

4.2 Three-Body R-parity Violating Gravitino Decays

In this subsection we will be considering gravitino decays involving three types of R-parity violating vertices that appear in R-parity violating superpotentials [33]:

$$W_{\mathbb{R}_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i H L_i.$$

Decays involving λ_{ijk} coupling

• Using the same approach as discussed to calculate R-parity violating interaction term in the previous (sub)section(s), the $\tilde{\nu}_{iL} - e_{jL} - e_{kR}$ vertex corresponding to Fig. 20(a) is given by considering the contribution of following term:

$$\mathcal{L} = \frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\mathcal{A}_1} \mathcal{D}_{\mathcal{A}_3} W) \bar{\chi}^{\mathcal{A}_1} \chi^{\mathcal{A}_3},$$

where

$$\mathcal{D}_{a_1} \mathcal{D}_{a_3} W = \partial_{a_1} \partial_{a_3} W + (\partial_{a_1} \partial_{a_3} K) W + \partial_{a_1} K \mathcal{D}_{a_3} W + \partial_{a_3} K \mathcal{D}_{a_1} W - (\partial_{a_1} K \partial_{a_3} K) W - \Gamma_{a_1 a_3}^k \mathcal{D}_k W,$$

where a_3, z_i correspond to undiagonalized moduli fields. By expanding above in the fluctuations linear in $a_1 \rightarrow a_1 + \mathcal{V}^{-\frac{2}{9}} M_p$, on simplifying

$$\frac{e^{\frac{K}{2}}}{2} \mathcal{D}_{a_1} \mathcal{D}_{a_3} W \sim \mathcal{V}^{-\frac{19}{72}} \delta a_1$$

Utilizing above, the $\tilde{\nu}_{iL} - e_{jL} - e_{kR}$ vertex will be given by

$$e^{\frac{K}{2}} \mathcal{D}_{\mathcal{A}_1} \mathcal{D}_{\mathcal{A}_3} W \chi^{\mathcal{A}_1} \chi^{\mathcal{A}_3} \sim e^{\frac{K}{2}} \mathcal{D}_{a_1} \mathcal{D}_{a_3} W \chi^{\mathcal{A}_1} \chi^{\mathcal{A}_3} \sim \mathcal{V}^{-\frac{19}{72}} \delta \mathcal{A}_1 \chi^{\mathcal{A}_1} \chi^{\mathcal{A}_3}$$

and physical vertex will be given as

$$C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \sim \frac{\mathcal{V}^{-\frac{19}{72}}}{\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_3 \bar{\mathcal{A}}_3}}} \sim \frac{\mathcal{V}^{-\frac{19}{72}}}{\sqrt{10^{15}}} \sim \mathcal{V}^{-\frac{7}{4}}, \text{ for } \mathcal{V} \sim 10^5. \quad (199)$$

•

Similarly, $\tilde{e}_{jL} - \nu_{iL} - e_{kR}$ vertex of Fig. 20(b) is same as $\tilde{\nu}_{iL} - e_{jL} - e_{kR}$ vertex corresponding to Fig. 20(a) and given as:

$$C^{\tilde{e}_{jL} \nu_{iL} e_{kR}} \sim C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \sim \mathcal{V}^{-\frac{7}{4}}. \quad (200)$$

•

The $\tilde{e}_{kR} - \bar{\nu}_{iR}^c - e_{jL}$ vertex corresponding to Fig. 20(c) comes from:

$$\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_1} W) \chi^{\mathcal{A}_1^c} \chi^{\mathcal{A}_1}.$$

In terms of undiagonalized basis,

$$\mathcal{D}_{\bar{a}_1} D_{a_1} W = (\partial_{\bar{a}_1} \partial_{a_1} K) W + \partial_{\bar{a}_1} K D_{a_1} W + \partial_{a_1} K D_{\bar{a}_1} W - (\partial_{\bar{a}_1} K \partial_{a_1} K) W.$$

By expanding above in the fluctuations linear in $a_3 \rightarrow a_3 + \mathcal{V}^{-\frac{13}{18}} M_p$ by utilizing equations (29) and (33), on simplifying

$$\frac{e^{\frac{K}{2}}}{2} \mathcal{D}_{a_1} D_{\bar{a}_1} W \sim \mathcal{V}^{-\frac{1}{3}} \delta a_3,$$

implying that the lepton-squark-quark vertex is given by:

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_1} W \bar{\chi}^{\mathcal{A}_1^c} \bar{\chi}^{\mathcal{A}_1} \sim e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_1} D_{a_1} W \chi^{\mathcal{A}_1^c} \chi^{\mathcal{A}_1} \sim \left(\mathcal{V}^{-\frac{1}{3}} \delta \mathcal{A}_3 \right) \chi^{\mathcal{A}_1^c} \chi^{\mathcal{A}_1}, \quad (201)$$

and physical vertex will be given as:

$$C^{\tilde{e}_{kR} \bar{\nu}_{iL}^c e_{jL}} \sim \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \hat{K}_{\mathcal{A}_3 \bar{\mathcal{A}}_3}}} \sim \frac{\mathcal{V}^{-\frac{1}{3}}}{\sqrt{10^{15}}} \sim \mathcal{V}^{-\frac{7}{4}}, \text{ for } \mathcal{V} \sim 10^5. \quad (202)$$

Now, the matrix amplitude for all three Feynman diagrams corresponding to Fig. 20 will be given as:

$$|M|^2 = |M_a + M_b + M_c|^2 \quad (203)$$

The analytical results for the full matrix amplitude summed over spins in terms of pure and cross terms are given in [33]. Utilizing their results, we will estimate matrix amplitude for all for both pure and cross terms to calculate decay width for the process $\psi_\mu \rightarrow \nu_i e_j \bar{e}_k$ in our set up. Strictly speaking, we will be neglecting fermion mass as compared to gravitino and squark masses. Introducing kinematic variables

$$2p(\nu_i) \cdot p(e_j) = (1 - z_{e_k}) m_{3/2}^2, \quad 2p(e_j) \cdot p(e_k) = (1 - z_{\nu_i}) m_{3/2}^2, \quad 2p(\nu_i) \cdot p(e_k) = (1 - z_{e_j}) m_{3/2}^2. \quad (204)$$

In view of above, we can express the following definitions in terms of kinematic variables as given below:

$$\begin{aligned}
m_{ij}^2 &= (p(\nu_i) + p(e_j))^2 \sim 2p(\nu_i).p(e_j) = (1 - z_{e_k})m_{3/2}^2 \\
m_{jk}^2 &= (p(e_j) + p(e_k))^2 \sim 2p(e_j).p(e_k) = (1 - z_{\nu_i})m_{3/2}^2 \\
m_{ik}^2 &= (p(\nu_i) + p(e_k))^2 \sim 2p(\nu_i).p(e_k) = (1 - z_{e_j})m_{3/2}^2,
\end{aligned} \tag{205}$$

utilizing the form of expressions given in the appendix of [33]

$$\begin{aligned}
|M_a|^2 &= \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2}{M_{pl}^2 (m_{jk}^2 - m_{\tilde{\nu}_{iL}}^2)^2} (m_{3/2}^2 - m_{jk}^2 + m_{\nu_i}^2) (m_{jk}^2 - m_{e_j}^2 - m_{e_k}^2) \\
&\quad \left(\frac{(m_{3/2}^2 + m_{jk}^2 - m_{\nu_i}^2)^2}{4m_{3/2}^2} - m_{jk}^2 \right),
\end{aligned} \tag{206}$$

After simplifying using (230), we have

$$\begin{aligned}
|M_a|^2 &= \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2}{M_{pl}^2 ((1 - z_{e_i})m_{3/2}^2 - m_{\tilde{\nu}_{iL}}^2)^2} [z_{\nu_i}(1 - z_{\nu_i})\left(\frac{z_{\nu_i}^2}{4}\right)], \\
&\sim \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^4} [z_{\nu_i}(1 - z_{\nu_i})\left(\frac{z_{\nu_i}^2}{4}\right)]
\end{aligned} \tag{207}$$

Here we have neglected gravitino mass as compared to sfermion mass. With same steps of similar procedure, we have

$$\begin{aligned}
|M_b|^2 &= \frac{1}{3} \frac{(C^{\tilde{e}_{L\nu_L} e_R})^2}{M_{pl}^2 (m_{ik}^2 - m_{\tilde{e}_{jL}}^2)^2} (m_{3/2}^2 - m_{ik}^2 + m_{e_j}^2) (m_{ik}^2 - m_{\nu_i}^2 - m_{e_k}^2) \\
&\quad \left(\frac{(m_{3/2}^2 + m_{ik}^2 - m_{e_j}^2)^2}{4m_{3/2}^2} - m_{ik}^2 \right) \\
&\sim \frac{1}{3} \frac{(C^{\tilde{e}_{L\nu_L} e_R})^2}{M_{pl}^2 (m_{\tilde{e}_{jL}}^2)^2} [z_{e_j}(1 - z_{e_j})\left(\frac{z_{e_j}^2}{4}\right)]
\end{aligned} \tag{208}$$

$$\begin{aligned}
|M_c|^2 &= \frac{1}{3} \frac{(C^{\tilde{e}_R \tilde{\nu}_R^c e_L})^2}{M_{pl}^2 (m_{ij}^2 - m_{\tilde{e}_{kR}}^2)^2} (m_{3/2}^2 - m_{ij}^2 + m_{e_k}^2) (m_{ij}^2 - m_{\nu_i}^2 - m_{e_j}^2) \\
&\quad \left(\frac{(m_{3/2}^2 + m_{ij}^2 - m_{e_k}^2)^2}{4m_{3/2}^2} - m_{ij}^2 \right) \\
&\sim \frac{1}{3} \frac{(C^{\tilde{e}_R \tilde{\nu}_R^c e_L})^2}{M_{pl}^2 (m_{\tilde{e}_{kR}}^2)^2} [z_{e_k}(1 - z_{e_k})\left(\frac{z_{e_k}^2}{4}\right)]
\end{aligned} \tag{209}$$

$$\begin{aligned}
2Re(M_a M_b^\dagger) &= \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}}.C^{\tilde{e}_{L\nu_L} e_R})}{M_{pl}^2 (m_{jk}^2 - m_{\tilde{\nu}_{iL}}^2)(m_{ik}^2 - m_{\tilde{e}_{jL}}^2)} \left[(m_{ik}^2 m_{jk}^2 - m_{3/2}^2 m_{e_k}^2 - m_{\nu_i}^2 m_{e_j}^2) \right. \\
&\quad \left. \left((m_{3/2}^2 + m_{e_k}^2 - m_{\nu_i}^2 - m_{e_j}^2) - \frac{1}{2m_{3/2}^2} (m_{3/2}^2 + m_{jk}^2 - m_{\nu_i}^2) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \left(m_{3/2}^2 + m_{ik}^2 - m_{ej}^2 \right) \right) + \frac{1}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2) (m_{jk}^2 - m_{ej}^2 - m_{ek}^2) \\
& \left(m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2 \right) - \frac{m_{\nu_i}^2}{2} (m_{jk}^2 - m_{ej}^2 - m_{ek}^2)^2 - \frac{m_{ej}^2}{2} (m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2)^2 \\
& - \frac{m_{ek}^2}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2)^2 + 2m_{\nu_i}^2 m_{ej}^2 m_{ek}^2 \Big] \\
& \sim \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_L \nu_L e_R})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{ej})(-1 - z_{ek} + 2z_{\nu_i} + 2z_{ej} - z_{\nu_i} \cdot z_{ek})
\end{aligned} \tag{210}$$

$$\begin{aligned}
2Re(M_b M_c^\dagger) &= \frac{1}{3} \frac{(C^{\tilde{e}_L \nu_L e_R} \cdot C^{\tilde{e}_R \bar{\nu}_R^c e_L})}{M_{pl}^2 (m_{ik}^2 - m_{\tilde{e}_{jL}}^2) (m_{ij}^2 - m_{\tilde{e}_{kR}}^2)} \left[(m_{ij}^2 m_{ik}^2 - m_{3/2}^2 m_{\nu_i}^2 - m_{ej}^2 m_{ek}^2) \right. \\
& \left((m_{3/2}^2 + m_{\nu_i}^2 - m_{ej}^2 - m_{ek}^2) - \frac{1}{2m_{3/2}^2} (m_{3/2}^2 + m_{ik}^2 - m_{ej}^2) \right. \\
& \left. \left. (m_{3/2}^2 + m_{ij}^2 - m_{ek}^2) \right) + \frac{1}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2) (m_{jk}^2 - m_{ej}^2 - m_{ek}^2) \right. \\
& \left. (m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2) - \frac{m_{\nu_i}^2}{2} (m_{jk}^2 - m_{ej}^2 - m_{ek}^2)^2 - \frac{m_{ej}^2}{2} (m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2)^2 \right. \\
& \left. - \frac{m_{ek}^2}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2)^2 + 2m_{\nu_i}^2 m_{ej}^2 m_{ek}^2 \right] \\
& \sim \frac{2}{3} \frac{(C^{\tilde{e}_L \nu_L e_R} \cdot C^{\tilde{e}_R \bar{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{ej})(1 - z_{ek})(-1 - z_{\nu_i} + 2z_{ej} + 2z_{ek} - z_{ej} \cdot z_{ek}),
\end{aligned} \tag{211}$$

$$\begin{aligned}
2Re(M_a M_c^\dagger) &= \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_R \bar{\nu}_R^c e_L})}{M_{pl}^2 (m_{jk}^2 - m_{\tilde{\nu}_{iL}}^2) (m_{ij}^2 - m_{\tilde{e}_{kR}}^2)} \left[(m_{ij}^2 m_{jk}^2 - m_{3/2}^2 m_{ej}^2 - m_{\nu_i}^2 m_{ek}^2) \right. \\
& \left((m_{3/2}^2 + m_{ej}^2 - m_{\nu_i}^2 - m_{ek}^2) - \frac{1}{2m_{3/2}^2} (m_{3/2}^2 + m_{jk}^2 - m_{\nu_i}^2) \right. \\
& \left. \left. (m_{3/2}^2 + m_{ij}^2 - m_{ek}^2) \right) + \frac{1}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2) (m_{jk}^2 - m_{ej}^2 - m_{ek}^2) \right. \\
& \left. (m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2) - \frac{m_{\nu_i}^2}{2} (m_{jk}^2 - m_{ej}^2 - m_{ek}^2)^2 - \frac{m_{ej}^2}{2} (m_{ik}^2 - m_{\nu_i}^2 - m_{ek}^2)^2 \right. \\
& \left. - \frac{m_{ek}^2}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{ej}^2)^2 + 2m_{\nu_i}^2 m_{ej}^2 m_{ek}^2 \right] \\
& \sim \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_R \bar{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{ek})(-1 - z_{ej} + 2z_{\nu_i} + 2z_{ek} - z_{\nu_i} \cdot z_{ek}),
\end{aligned} \tag{212}$$

Utilizing the results from (206)- (212), (203) one gets the following form:

$$\begin{aligned}
|M|^2 = & \frac{1}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^4} [z_{\nu_i}(1 - z_{\nu_i}) (\frac{z_{\nu_i}^2}{4})] + \frac{1}{3} \frac{(C^{\tilde{e}_L \nu_L e_R})^2}{M_{pl}^2 (m_{\tilde{e}_{jL}}^2)^2} [z_{e_j}(1 - z_{e_j}) (\frac{z_{e_j}^2}{4})] \\
& + \frac{1}{3} \frac{(C^{\tilde{e}_R \tilde{\nu}_R^c e_L})^2}{M_{pl}^2 (m_{\tilde{e}_{kR}}^2)^2} [z_{e_k}(1 - z_{e_k}) (\frac{z_{e_k}^2}{4})] \\
& + \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_L \nu_L e_R})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{e_j})(-1 - z_{e_k} + 2z_{\nu_i} + 2z_{e_j} - z_{\nu_i} \cdot z_{e_k}) \\
& + \frac{2}{3} \frac{(C^{\tilde{e}_L \nu_L e_R} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{e_j})(1 - z_{e_k})(-1 - z_{\nu_i} + 2z_{e_j} + 2z_{e_k} - z_{e_j} \cdot z_{e_k}) + \\
& \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{e_k})(-1 - z_{e_j} + 2z_{\nu_i} + 2z_{e_k} - z_{\nu_i} \cdot z_{e_k}). \tag{213}
\end{aligned}$$

The differential decay rate follows:

$$\frac{d^2\Gamma}{dz_{e_j} dz_{e_k}} = \frac{N_c m_{3/2}}{2^8 \pi^3} \left(\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right) \tag{214}$$

Putting the result of $|M|^2$ from above,

$$\begin{aligned}
\frac{d^2\Gamma}{dz_{e_j} dz_{e_k}} \sim & \frac{N_c m_{3/2}}{2^9 \pi^3} \frac{1}{3} \left[\frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^4} [z_{\nu_i}(1 - z_{\nu_i}) (\frac{z_{\nu_i}^2}{4})] + \frac{1}{3} \frac{(C^{\tilde{e}_L \nu_L e_R})^2}{M_{pl}^2 (m_{\tilde{e}_{jL}}^2)^2} [z_{e_j}(1 - z_{e_j}) (\frac{z_{e_j}^2}{4})] \right. \\
& + \frac{1}{3} \frac{(C^{\tilde{e}_R \tilde{\nu}_R^c e_L})^2}{M_{pl}^2 (m_{\tilde{e}_{kR}}^2)^2} [z_{e_k}(1 - z_{e_k}) (\frac{z_{e_k}^2}{4})] \\
& + \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_L \nu_L e_R})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{e_j})(-1 - z_{e_k} + 2z_{\nu_i} + 2z_{e_j} - z_{\nu_i} \cdot z_{e_k}) \\
& + \frac{2}{3} \frac{(C^{\tilde{e}_L \nu_L e_R} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{e_j})(1 - z_{e_k})(-1 - z_{\nu_i} + 2z_{e_j} + 2z_{e_k} - z_{e_j} \cdot z_{e_k}) + \\
& \left. \frac{2}{3} \frac{(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L})}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{\nu_i})(1 - z_{e_k})(-1 - z_{e_j} + 2z_{\nu_i} + 2z_{e_k} - z_{\nu_i} \cdot z_{e_k}) \right] \tag{215}
\end{aligned}$$

Using the same approach as given in (165),

$$0 < z_{e_j} < 1, 1 - z_j < z_{e_k} < 1. \tag{216}$$

and using the numerical estimates of masses, $m_{3/2} \sim \mathcal{V}^{-2} M_p, m_{\tilde{\nu}_{iL}}^2 = m_{\tilde{e}_{jL}}^2 = m_{\tilde{e}_{kR}}^2 \sim \mathcal{V}^{\frac{1}{2}} m_{3/2}$ in our set up, after integrating, decay width reduces to

$$\begin{aligned}
\Gamma \sim & \frac{N_c m_{3/2}^7}{(2^9 \cdot 3 \cdot 120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} \left[(C^{\tilde{\nu}_{iL} e_{jL} e_{kR}})^2 + (C^{\tilde{e}_L \nu_L e_R})^2 + (C^{\tilde{e}_R \tilde{\nu}_R^c e_L})^2 \right. \\
& + \frac{3}{4} \left((C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_L \nu_L e_R}) + (C^{\tilde{e}_L \nu_L e_R} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L}) + (C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \cdot C^{\tilde{e}_R \tilde{\nu}_R^c e_L}) \right) \left. \right]. \tag{217}
\end{aligned}$$

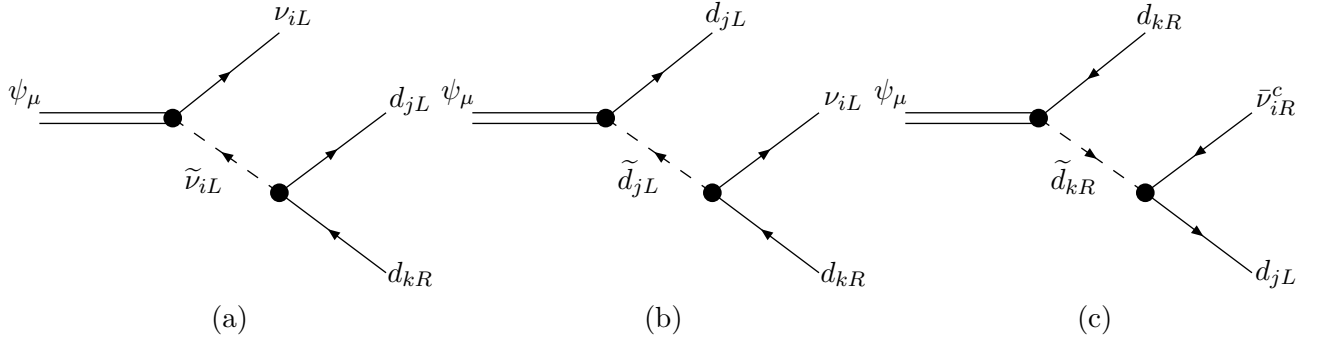


Figure 21: Three-body gravitino decays involving $\mathbb{R}_p \lambda'_{ijk}$ coupling

Utilizing the set of results given in equation no (199) - (202), decay width simplifies to

$$\begin{aligned} \Gamma &\sim \frac{N_c m_{3/2}^7}{(2^9 \cdot 3 \cdot 120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} (\mathcal{V}^{-\frac{7}{2}}) \\ &\sim \frac{1}{10^6} \frac{\mathcal{V}^{-\frac{11}{2}} \cdot m_{3/2}^3}{M_{pl}^2} \sim 10^{-45.5} \text{ GeV}; \text{ for } \mathcal{V} \sim 10^5. \end{aligned} \quad (218)$$

Life time will be given as

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} \text{ Jsec}}{10^{-45.5} \text{ GeV}} \sim O(10^{21}) \text{ sec}. \quad (219)$$

Decays involving λ'_{ijk} coupling

- The $\tilde{\nu}_L - d_{jL} - d_{kR}$ vertex corresponding to figure 3(a) and $\tilde{l}_L - u_L - d_L^c$ vertex calculated in section 4 get identified with same set of moduli space superfields and given by similar interaction vertex. Therefore contribution of $\tilde{\nu}_L - d_{jL} - d_{kR}$ vertex is same as $\tilde{l}_L - u_L - d_L^c$ given in (137).

$$C^{\tilde{\nu}_L d_{jL} d_{kR}} \sim \mathcal{V}^{-\frac{5}{3}} \tilde{\nu}_L d_{jL} d_{kR}, \text{ for } \mathcal{V} \sim 10^5 \quad (220)$$

- The $\tilde{d}_{jL} - \nu_{iL} - d_{kR}$ vertex corresponding to figure 3(b) and $\tilde{u}_L - l_L - d_L^c$ vertex calculated in section 4 get identified with same set of moduli space superfields and given by similar interaction vertex. Therefore contribution of $\tilde{d}_{jL} - \nu_{iL} - d_{kR}$ vertex is same as $\tilde{u}_L - l_L - d_L^c$ given in (152)

$$C^{\tilde{d}_{jL} \nu_{iL} d_{kR}} \sim \mathcal{V}^{-\frac{5}{3}}, \text{ for } \mathcal{V} \sim 10^5 \quad (221)$$

- This time, $\tilde{d}_{kR} - \bar{\nu}_{iR}^c - d_{jL}$ vertex corresponding to Fig. 21(c) and $\tilde{d}_R - l_L - u_L$ vertex calculated in section 4 get identified with same set of moduli space superfields and given by similar interaction vertex. Hence, contribution of $\tilde{d}_{kR} - \bar{\nu}_{iR}^c - d_{jL}$ vertex is a same as $\tilde{d}_R - l_L - u_L$ given in (145)

$$C^{\tilde{d}_{kR} \bar{\nu}_{iR}^c d_{jL}} \sim \mathcal{V}^{-\frac{5}{3}}, \text{ for } \mathcal{V} \sim 10^5. \quad (222)$$

The analytical result of matrix amplitude for all three Feynman diagrams corresponding to Fig. 21 is same as calculated in Fig. 20 by replacing $m_{\tilde{\nu}_{iL}} \rightarrow m_{\tilde{\nu}_{iL}} (m_{\tilde{e}_{iL}})$, $m_{\tilde{e}_{jL}} \rightarrow m_{\tilde{d}_{jL}} (m_{\tilde{u}_{jL}})$ and $m_{\tilde{e}_{kR}} \rightarrow m_{\tilde{d}_{kR}} (m_{\tilde{d}_{kR}})$ and $C^{\tilde{\nu}_{iL} e_{jL} e_{kR}} \rightarrow C^{\tilde{\nu}_{iL} d_{jL} d_{kR}}$, $C^{\tilde{e}_{jL} \nu_{iL} e_{kR}} \rightarrow C^{\tilde{d}_{jL} \nu_{iL} d_{kR}}$, $C^{\tilde{e}_{kR} \bar{\nu}_{iR}^c e_{jL}} \rightarrow C^{\tilde{d}_{kR} \bar{\nu}_{iR}^c d_{jL}}$. Doing

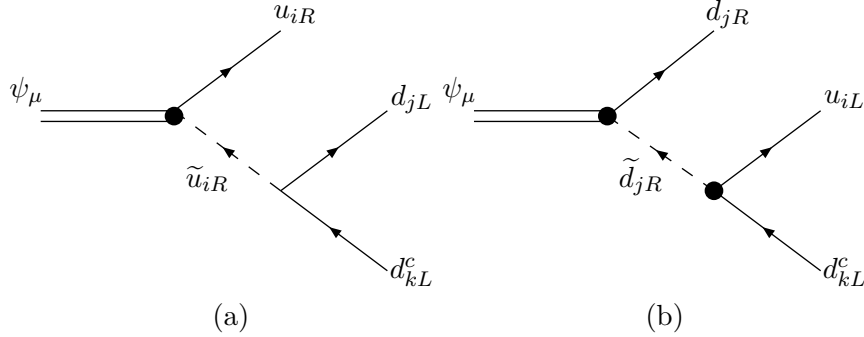


Figure 22: Three-body gravitino decays involving $\mathcal{R}_p \lambda''_{ijk}$ coupling

so and using same numerical estimates of masses, $m_{3/2} \sim \mathcal{V}^{-2} M_p$, $m_{\tilde{\nu}_{iL}}^2 = m_{\tilde{d}_{jL}}^2 = m_{\tilde{d}_{kR}}^2 \sim \mathcal{V}^{\frac{1}{2}} m_{3/2}$ in our set up, after integrating, decay width reduces to

$$\Gamma \sim \frac{N_c m_{3/2}^7}{(2^9 \cdot 3.120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} \left[(C^{\tilde{\nu}_{iL} d_{jL} d_{kR}})^2 + (C^{\tilde{d}_{jL} \nu_{iL} d_{kR}})^2 + (C^{\tilde{d}_{kR} \tilde{\nu}_{iL} d_{jL}})^2 \right. \\ \left. + \frac{3}{4} \left((C^{\tilde{\nu}_{iL} d_{jL} d_{kR}} \cdot C^{\tilde{d}_{jL} \nu_{iL} d_{kR}}) + (C^{\tilde{d}_{jL} \nu_{iL} d_{kR}} \cdot C^{\tilde{d}_{kR} \tilde{\nu}_{iL} d_{jL}}) + (C^{\tilde{\nu}_{iL} d_{jL} d_{kR}} \cdot C^{\tilde{d}_{kR} \tilde{\nu}_{iL} d_{jL}}) \right) \right]. \quad (223)$$

Utilizing the set of results given in equation no (199) - (202), decay width simplifies to

$$\Gamma \sim \frac{N_c m_{3/2}^7}{(2^9 \cdot 3.120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} (\mathcal{V}^{-\frac{10}{3}}) \\ \sim \frac{1}{10^6} \frac{\mathcal{V}^{-\frac{16}{3}} \cdot m_{3/2}^3}{M_{pl}^2} \sim 10^{-44} GeV; \text{ for } \mathcal{V} \sim 10^5 \quad (224)$$

Life time will be given as

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} Jsec}{10^{-44} GeV} \sim O(10^{20}) sec \quad (225)$$

Decays mediated via λ''_{ijk} coupling

- The $\tilde{u}_{iR} - d_{jR} - d_{kL}^c$ vertex corresponding to Fig. 22(a) arises from:

$$\frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{\tilde{\mathcal{A}}_4} D_{\mathcal{A}_4} W) \chi^{\mathcal{A}_4^c} \chi^{\mathcal{A}_4}.$$

In terms of undiagonalized basis,

$$\mathcal{D}_{\tilde{a}_4} D_{a_4} W = (\partial_{\tilde{a}_4} \partial_{a_4} K) W + \partial_{\tilde{a}_4} K D_{a_4} W + \partial_{a_4} K D_{\tilde{a}_4} W - (\partial_{\tilde{a}_4} K \partial_{a_4} K) W.$$

By expanding above in the fluctuations linear in $a_4 \rightarrow a_4 + \mathcal{V}^{-\frac{11}{9}} M_p$ by utilizing equations (29) and (33), on simplifying one obtains:

$$\frac{e^{\frac{K}{2}}}{2} \mathcal{D}_{a_4} D_{\tilde{a}_4} W \sim \mathcal{V}^{\frac{13}{6}} \delta a_4,$$

implying that the contribution of quark-squark-quark vertex is given as under:

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_4} D_{\mathcal{A}_4} W \bar{\chi}^{\mathcal{A}_4^c} \bar{\chi}^{\mathcal{A}_4} \sim e^{\frac{K}{2}} \mathcal{D}_{\bar{a}_4} D_{a_4} W \chi^{\mathcal{A}_4^c} \chi^{\mathcal{A}_4} \sim \left(\mathcal{V}^{\frac{13}{6}} \mathcal{A}_4 \right) \chi^{\mathcal{A}_4^c} \chi^{\mathcal{A}_4}, \quad (226)$$

and the physical vertex will be given as following:

$$C^{\tilde{u}_{iR} d_{jR} d_{kL}^c} \sim \frac{\mathcal{V}^{\frac{13}{6}}}{\sqrt{\hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4} \hat{K}_{\mathcal{A}_4 \bar{\mathcal{A}}_4}}} \sim \frac{\mathcal{V}^{\frac{13}{6}}}{\sqrt{10^{36}}} \sim \mathcal{V}^{-\frac{43}{30}} \tilde{u}_{iR} d_{jR} d_{kL}^c, \text{ for } \mathcal{V} \sim 10^5. \quad (227)$$

• Similarly, the $\tilde{d}_{jR} - d_{kL}^c - u_{iR}$ vertex of Fig. 21(b) is same as $\tilde{u}_{iR} - d_{jR} - d_{kL}^c$ vertex corresponding to Fig. 21(a) and is given as under:

$$C^{\tilde{d}_{jR} d_{kL}^c u_{iR}} \sim C^{\tilde{u}_{iR} d_{jL} d_{kL}^c} \sim \mathcal{V}^{-\frac{43}{30}} \tilde{d}_{jR} d_{kL}^c u_{iR}, \text{ for } \mathcal{V} \sim 10^5. \quad (228)$$

Again, introducing kinematic variable

$$2p(u_i) \cdot p(d_j) = (1 - z_{d_k}) m_{3/2}^2, \quad 2p(d_j) \cdot p(d_k) = (1 - z_{u_i}) m_{3/2}^2, \quad 2p(u_i) \cdot p(d_k) = (1 - z_{d_j}) m_{3/2}^2. \quad (229)$$

and

$$\begin{aligned} m_{ij}^2 &= (p(u_i) + p(d_j))^2 \sim 2p(u_i) \cdot p(d_j) = (1 - z_{d_k}) m_{3/2}^2 \\ m_{jk}^2 &= (p(d_j) + p(d_k))^2 \sim 2p(d_j) \cdot p(d_k) = (1 - z_{u_i}) m_{3/2}^2 \\ m_{ik}^2 &= (p(u_i) + p(d_k))^2 \sim 2p(u_i) \cdot p(d_k) = (1 - z_{d_j}) m_{3/2}^2, \end{aligned} \quad (230)$$

Here also, utilizing the form of expressions given in the appendix of [33] for λ_{ijk}'' coupling, the full analytical result of the squared amplitude summed over the spins for the gravitino decay reaction $\psi_\mu \xrightarrow{\lambda_{ijk}''} u_i d_j d_k$ is the sum of the following squared amplitude and interference teramas:

$$\begin{aligned} |M_a|^2 &= \frac{N_c!}{3} \frac{(C^{\tilde{G} u_{iR} u_{iR}} C^{\tilde{u}_{iR} d_{jR} d_{kR}^c})^2}{M_\star^2 (m_{jk}^2 - m_{\tilde{u}_{iR}}^2)^2} (m_{3/2}^2 - m_{jk}^2 + m_{u_i}^2) (m_{jk}^2 - m_{d_j}^2 - m_{d_k}^2) \\ &\quad \left(\frac{(m_{3/2}^2 + m_{jk}^2 - m_{u_i}^2)^2}{4m_{3/2}^2} - m_{jk}^2 \right) \sim \frac{1}{3} \frac{(C^{\tilde{u}_{iR} d_{jR} d_{kR}^c})^2}{M_{pl}^2 m_{\tilde{\nu}_{iL}}^4} [z_{u_i} (1 - z_{u_i}) \left(\frac{z_{u_i}^2}{4} \right)] \end{aligned} \quad (231)$$

$$\begin{aligned} |M_b|^2 &= \frac{4N_c!}{3} \frac{(C^{\tilde{d}_{jR} u_{iR} d_{kR}^c})^2}{M_\star^2 (m_{ik}^2 - m_{\tilde{d}_{jR}}^2)^2} (m_{3/2}^2 - m_{ik}^2 + m_{d_j}^2) (m_{ik}^2 - m_{u_i}^2 - m_{d_k}^2) \\ &\quad \frac{1}{3} \frac{(C^{\tilde{d}_{jR} u_{iR} d_{kR}^c})^2}{M_{pl}^2 (m_{\tilde{e}_{jL}}^2)^2} [z_{d_j} (1 - z_{d_j}) \left(\frac{z_{d_j}^2}{4} \right)], \end{aligned} \quad (232)$$

$$2Re(M_a M_b^\dagger) = \frac{2N_c!}{3} \frac{(C^{\tilde{u}_{iR} d_{jR} d_{kR}^c} \cdot C^{\tilde{d}_{jR} u_{iR} d_{kR}^c})}{M_\star^2 (m_{jk}^2 - m_{\tilde{u}_{iR}}^2) (m_{ik}^2 - m_{\tilde{d}_{jR}}^2)} \left[(m_{ik}^2 m_{jk}^2 - m_{3/2}^2 m_{d_k}^2 - m_{u_i}^2 m_{d_j}^2) \right]$$

$$\begin{aligned}
& \left((m_{3/2}^2 + m_{d_k}^2 - m_{u_i}^2 - m_{d_j}^2) - \frac{1}{2m_{3/2}^2} (m_{3/2}^2 + m_{j_k}^2 - m_{u_i}^2) \right. \\
& \left. (m_{3/2}^2 + m_{i_k}^2 - m_{d_j}^2) \right) + \frac{1}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{e_j}^2) (m_{j_k}^2 - m_{e_j}^2 - m_{e_k}^2) \\
& (m_{i_k}^2 - m_{\nu_i}^2 - m_{e_k}^2) - \frac{m_{\nu_i}^2}{2} (m_{j_k}^2 - m_{e_j}^2 - m_{e_k}^2)^2 - \frac{m_{e_j}^2}{2} (m_{i_k}^2 - m_{\nu_i}^2 - m_{e_k}^2)^2 \\
& - \frac{m_{e_k}^2}{2} (m_{ij}^2 - m_{\nu_i}^2 - m_{e_j}^2)^2 + 2m_{\nu_i}^2 m_{e_j}^2 m_{e_k}^2 \Big] \\
& \sim \frac{2}{3} \frac{(C^{\tilde{u}_i R d_{jR} d_{kR}^c} \cdot C^{\tilde{d}_{jR} u_{iR} d_{kR}^c})}{M_{pl}^2 m_{\nu_{iL}}^2 m_{\tilde{e}_{jL}}^2} (1 - z_{u_i})(1 - z_{d_j})(-1 - z_{d_k} + 2z_{u_i} + 2z_{d_j} - z_{u_i} \cdot z_{d_k}). \quad (233)
\end{aligned}$$

Decay width will be given as:

$$\Gamma \sim \frac{N_c m_{3/2}^7}{(2^9 \cdot 3 \cdot 120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} \left[(C^{\tilde{u}_i R d_{jR} d_{kR}^c})^2 + (C^{\tilde{d}_{jR} u_{iR} d_{kR}^c})^2 + \frac{3}{4} \left(C^{\tilde{u}_i R d_{jR} d_{kR}^c} \cdot C^{\tilde{d}_{jR} u_{iR} d_{kR}^c} \right) \right] \quad (234)$$

Utilizing the set of results given in equation no (227) - (228), decay width simplifies to

$$\begin{aligned}
\Gamma & \sim \frac{N_c m_{3/2}^7}{(2^9 \cdot 3 \cdot 120) \pi^3 \cdot M_{pl}^2 \cdot \mathcal{V}^2 m_{3/2}^4} (\mathcal{V}^{-\frac{43}{15}}) \\
& \sim \frac{1}{10^6} \frac{\mathcal{V}^{-\frac{73}{15}} \cdot m_{3/2}^3}{M_{pl}^2} \sim 10^{-42} GeV; \text{ for } \mathcal{V} \sim 10^5, \quad (235)
\end{aligned}$$

and therefore the life time will be given by:

$$\tau = \frac{\hbar}{\Gamma} \sim \frac{10^{-34} Jsec}{10^{-42} GeV} \sim O(10^{18}) sec. \quad (236)$$

5 Relic abundance of Gravitino dark matter candidate

As discussed in section 4, explicit calculations yielding the gravitino-decay life time of the same order or greater than age of the universe, justifies taking Gravitino to be a viable dark matter candidate in our setup. Keeping in mind the fact that relic density of dark matter particle should be within the limits provided by recent WMAP observations and other direct and indirect experiments, this section is devoted to calculating the relic-abundance of the dark matter particle. Assuming that re-heating temperature will be low enough to produce an appropriate amount of dark matter in case of heavy gravitino, we focus on the case of gravitinos produced in the decays of co-NLSPs and show that the same are produced in sufficient numbers to constitute all of non-baryonic dark matter. The mass scales and life time estimates of sleptons and neutralino discussed in section 2 and 3 manifestly indicate the same to be valid co-NLSP's which freeze out with appropriate thermal relic density before decaying and then eventually decay into the gravitino. Therefore, gravitino then inherits much of the relic density of the neutralino/slepton.

5.1 Neutralino density calculation

The number density of cold relic density (CDM) from the early universe depends sensitively on the annihilation cross section of such particles. Amongst the various approaches to calculate the thermal

cross section given in the literature, we rely on the partial-wave expansion approach used in [13] to calculate the annihilation cross section for each possible process. Since the μ -split SUSY set up discussed in section 2 includes only neutral light Higgs boson; heavy Higgs boson and similarly superpartners of neutral particles (i.e only Neutralino), we consider possible annihilation channels which proceed via these particles only. The important channels that we will be discussing are: $\chi_3^0 \chi_3^0 \rightarrow hh$, $\chi_3^0 \chi_3^0 \rightarrow ZZ$, $\chi_3^0 \chi_3^0 \rightarrow ff$.

The general procedure which is being followed in partial wave expansion approach, is given as follows. As given in [34], in QFT, the general cross-section is given as

$$\sigma v_{\text{M}\phi\text{l}} = \frac{1}{4E_1 E_2} \int dLIPS |\mathcal{M}|^2 \quad (237)$$

where

$$dLIPS = (2\pi)^4 \delta^4(p_1 + p_2 - \sum p_j) \prod \int \frac{d^3 p_i}{(2\pi)^3 2\pi_0}. \quad (238)$$

The thermally-averaged product of the neutralino pair-annihilation cross section and their relative velocity $\langle \sigma v_{\text{M}\phi\text{l}} \rangle$ is given as [13, 34]

$$\langle \sigma v_{\text{M}\phi\text{l}} \rangle(T) = \frac{\int d^3 p_1 d^3 p_2 \sigma v_{\text{M}\phi\text{l}} e^{-E_1/T} e^{-E_2/T}}{\int d^3 p_1 d^3 p_2 e^{-E_1/T} e^{-E_2/T}}, \quad (239)$$

where $p_1 = (E_1, \mathbf{p}_1)$ and $p_2 = (E_2, \mathbf{p}_2)$ are the 4-momenta of the two colliding particles, and T is the temperature of the bath. Given the complexity of the general cross section, it is difficult to solve analytically. Alternatively, one finds a way to get the solution by expressing $\langle \sigma v_{\text{M}\phi\text{l}} \rangle$ in terms of $x = \frac{T}{M}$. For this, one defines:

$$w(s) = \frac{1}{4} \int dLIPS |\mathcal{M}|^2 = E_1 E_2 \sigma v_{\text{M}\phi\text{l}}. \quad (240)$$

Incorporating the value of $\sigma v_{\text{M}\phi\text{l}}$ in equation (239),

$$\langle \sigma v_{\text{M}\phi\text{l}} \rangle(T) = \frac{\int d^3 p_1 d^3 p_2 w(s) e^{-E_1/T} e^{-E_2/T}}{\int d^3 p_1 d^3 p_2 E_1 E_2 e^{-E_1/T} e^{-E_2/T}}. \quad (241)$$

By defining change of variables discussed in [34] i.e expressing momentum and energy in terms of x , carrying over the integration in terms of x , $\langle \sigma v_{\text{M}\phi\text{l}} \rangle$ takes the form given below:

$$\langle \sigma v_{\text{M}\phi\text{l}} \rangle = \frac{1}{m_\chi^2} \left[w - \frac{3}{2} (2w - w') x + \mathcal{O}(x^2) \right]_{s=4m_\chi^2} \equiv a + bx + \mathcal{O}(x^2). \quad (242)$$

The coefficients a and b summed over all possible final states $f_1 f_2$ are defined in [13], and given as:

$$a = \sum_{f_1 f_2} c \theta (4M_\chi^2 - (m_{f_1} + m_{f_2})^2) v_{f_1 f_2} \tilde{a}_{f_1 f_2},$$

$$b = \sum_{f_1 f_2} c \theta (4M_\chi^2 - (m_{f_1} + m_{f_2})^2) v_{f_1 f_2} \left\{ \tilde{b}_{f_1 f_2} + \tilde{a}_{f_1 f_2} \left[-3 + \frac{3}{4} v_{f_1 f_2}^{-2} \left(\frac{m_{f_1}^2 + m_{f_2}^2}{2M_\chi^2} + \frac{(m_{f_1}^2 - m_{f_2}^2)^2}{8M_\chi^4} \right) \right] \right\}.$$

The analytic expressions of a and b are given for the s, t and u channels of various possible annihilation processes are given in [13]. For the paper to be self-contained, we directly quote the

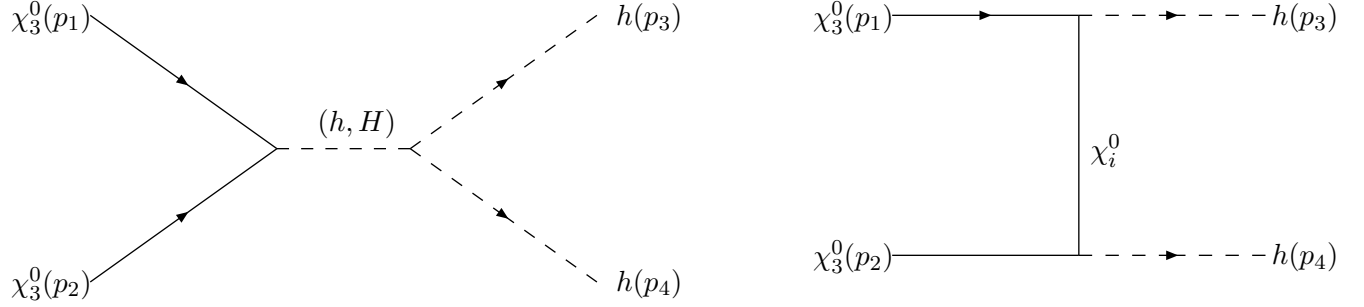


Figure 23: Feynman diagrams for $\chi_3^0 \chi_3^0 \rightarrow hh$ via s -channel Higgs exchange and t -channel χ_i^0 exchange.

forms of analytical expressions and utilizing the same, we calculate the numerical estimates of a and b for all kinematically possible annihilation processes in our set up.

The idea is to first calculate the required vertices corresponding to different annihilation processes in the context of $\mathcal{N} = 1$ gauged supergravity action and then use their estimates to calculate a and b coefficients. Following the same formalism as used in section 3 and 4, utilizing the gauged supergravity action of Wess and Bagger, we obtain the numerical estimates of required vertices.

The physical eigenstates of neutralino mass matrix in the context of gauged supergravity action are given as:

$$\begin{aligned}
\tilde{\chi}_1^0 &\sim \frac{-\tilde{H}_1^0 + \tilde{H}_2^0}{\sqrt{2}}; \text{ mass } \sim \mathcal{V}^{-\frac{35}{36}} M_p > m_{\frac{3}{2}}, \\
\tilde{\chi}_2^0 &\sim \tilde{f}\lambda^0 + \frac{\tilde{H}_1^0 + \tilde{H}_2^0}{\sqrt{2}}; \text{ mass } \sim \mathcal{V}^{-\frac{35}{36}} M_p > m_{\frac{3}{2}}; CP : -, \\
\tilde{\chi}_3^0 &\sim -\lambda^0 + \tilde{f}(\tilde{H}_1^0 + \tilde{H}_2^0); \text{ mass } \sim \mathcal{V}^{-\frac{4}{3}} M_p > m_{\frac{3}{2}}.
\end{aligned} \tag{243}$$

Utilizing (243),

$$\begin{aligned}
C\chi_3^0\chi_1^0h &= C^{\lambda^0\tilde{H}^0h} + \tilde{f}C^{\tilde{H}^0\tilde{H}^0h}, \\
C\chi_3^0\chi_2^0h &= \tilde{f}C^{\lambda^0\lambda^0h} + \tilde{f}^2C^{\lambda^0\tilde{H}^0h} + \tilde{f}C^{\tilde{H}^0\tilde{H}^0h}, \\
C\chi_3^0\chi_3^0h &= C^{\lambda^0\lambda^0h} + \tilde{f}C^{\lambda^0\tilde{H}^0h} + \tilde{f}^2C^{\tilde{H}^0\tilde{H}^0h}
\end{aligned} \tag{244}$$

where $\tilde{H}^0 \sim \frac{\tilde{H}_1^0 + \tilde{H}_2^0}{\sqrt{2}}$ is physical Higgsino and the physical light Higgs is defined as $h = \frac{H_1^0 - H_2^0}{\sqrt{2}}$.

⁴Working in the sublocus, where position moduli z_1 and z_2 are considered to be equivalent, for notational simplification, we will write Higgsino superfield $\chi^{\frac{1}{\sqrt{2}}(Z_1+Z_2)} \sim \chi^{Z_i}$.

Higgsino-higgsino- Higgs vertex

$$\mathcal{L} = \frac{e^{\frac{K}{2}}}{2} (\mathcal{D}_{Z_1} D_{Z_1} W) \chi_L^{Z_1} \bar{\chi}_R^{Z_1} + i g_{Z_1 \bar{Z}_1} \bar{\chi}_L^{\bar{Z}_1} \left[\gamma \cdot \partial \chi_L^{Z_1} + \Gamma_{Z_1 Z_1}^{Z_1} \gamma \cdot \partial Z_1 \chi_L^{Z_1} + \frac{1}{4} (\partial_{Z_1} K \gamma \cdot Z_1 - \text{c.c.}) \chi_L^{Z_1} \right];$$

$\chi_L^{Z_i} / \bar{\chi}_R^{Z_i}$ corresponds to left-/right-handed components of the Higgsino. In terms of undiagonalized basis,

$$\mathcal{D}_{Z_1} D_{Z_1} W = (\partial_{\bar{Z}_1} \partial_{Z_1} W) + (\partial_{\bar{Z}_1} \partial_{Z_1} K) W + \partial_{\bar{Z}_1} K D_{Z_1} W + \partial_{Z_1} K D_{\bar{Z}_1} W - (\partial_{\bar{Z}_1} K \partial_{Z_1} K) W + \Gamma_{Z_1 Z_1}^K D_K W.$$

Since $SU(2)_L$ symmetry gets spontaneously broken for Higgsino-Higgsino-Higgs vertex, therefore the idea is to expand above term quadratic in z_i such that one of the two z_i s acquires a VEV. Utilizing $z_i \rightarrow z_i + \mathcal{V}^{\frac{1}{36}} M_p$ and thereafter solving with the help of equations (29) and (33), one has

$$\frac{e^{\frac{K}{2}}}{2} \mathcal{D}_{Z_i} D_{\bar{Z}_i} W \sim \mathcal{V}^{-\frac{16}{9}} \langle z_i \rangle \delta z_i$$

Following (35), $e^{\frac{K}{2}} \mathcal{D}_{\bar{Z}_i} D_{Z_i} W \sim O(1) e^{\frac{K}{2}} \mathcal{D}_{\bar{Z}_i} D_{Z_i} W$, implying:

$$e^{\frac{K}{2}} \mathcal{D}_{\bar{Z}_i} D_{Z_i} W \chi_L^{Z_i} \bar{\chi}_R^{Z_i} \sim \left(\mathcal{V}^{-\frac{16}{9}} \langle Z_i \rangle \right) \delta Z_i \chi_L^{Z_i} \bar{\chi}_R^{Z_i} \sim \left(\mathcal{V}^{-\frac{7}{4}} \langle Z_i \rangle \right) \delta Z_i \chi_L^{Z_i} \bar{\chi}_R^{Z_i}. \quad (245)$$

Using $\chi^{Z_i} \sim \mathcal{V} m_{3/2}$ for the Higgsino mass and $m_{3/2} = \mathcal{V}^{-2} M_p$, one obtains:

$$\begin{aligned} g_{Z_i \bar{Z}_i} \bar{\chi}_L^{\bar{Z}_i} \gamma \cdot \partial \chi_L^{Z_i} &\sim O(1) g_{Z_1 \bar{Z}_1} \bar{\chi}_L^{\bar{Z}_1} \gamma \cdot \partial \chi_L^{Z_1} \rightarrow \frac{\mathcal{V}^{-\frac{37}{36}} \langle Z_i \rangle}{M_p} \delta Z_i \bar{\chi}_L^{\bar{Z}_i} \gamma \cdot p_{\chi^{Z_i}} \chi_L^{Z_i} \sim \mathcal{V}^{-2} \bar{\chi}_L^{\bar{Z}_i} \delta Z_i \chi_L^{Z_i}; \\ g_{Z_i \bar{Z}_i} \bar{\chi}_L^{\bar{Z}_i} \Gamma_{Z_i Z_i}^{Z_i} \gamma \cdot \partial Z_i \chi_L^{Z_i} &\sim O(1) g_{Z_i \bar{Z}_i} \Gamma_{Z_i Z_i}^{Z_i} \bar{\chi}_L^{\bar{Z}_i} \gamma \cdot \partial Z_i \chi_L^{Z_i} \rightarrow \frac{\mathcal{V}^{-\frac{25}{36}} \langle Z_i \rangle}{M_p} \delta Z_i \bar{\chi}_L^{\bar{Z}_i} \gamma \cdot (p_{\chi^{Z_i}} + p_{\bar{\chi}^{Z_i}}) \chi_L^{Z_i} \\ &\sim \mathcal{V}^{-\frac{5}{3}} \bar{\chi}_L^{\bar{Z}_i} \delta Z_i \chi_L^{Z_i}; \\ g_{Z_i \bar{Z}_i} \bar{\chi}_L^{\bar{Z}_i} \frac{1}{4} (\partial_{Z_i} K \gamma \cdot Z_i - \text{c.c.}) \chi_L^{Z_i} &\sim O(1) g_{Z_1 \bar{Z}_1} \partial_{Z_1} K \bar{\chi}_L^{\bar{Z}_1} \gamma \cdot \partial Z_1 \chi_L^{Z_1} \rightarrow \frac{\mathcal{V}^{-\frac{4}{3}} \langle Z_i \rangle}{M_p} \delta Z_i \bar{\chi}_L^{\bar{Z}_i} \gamma \cdot (p_{\chi^{Z_i}} + p_{\bar{\chi}^{Z_i}}) \chi_L^{Z_i} \\ &\sim \mathcal{V}^{-\frac{83}{36}} \bar{\chi}_L^{\bar{Z}_i} \delta Z_i \chi_L^{Z_i}. \end{aligned} \quad (246)$$

Incorporating results of (245) and (246), the physical Higgsino-Higgsino- Higgs vertex will be given as

$$C^{\tilde{H}^0 \tilde{H}^0 h} = \frac{1}{\sqrt{(\hat{K}_{Z_i \bar{Z}_i})^4}} \left[(\mathcal{V}^{-\frac{7}{4}}) + (\mathcal{V}^{-2} + \mathcal{V}^{-\frac{5}{3}} + \mathcal{V}^{-\frac{83}{36}}) \right] \sim \mathcal{V}^{\frac{1}{4}}. \quad (247)$$

Gaugino-Higgsino-Higgs vertex

$$\mathcal{L} = g_{YMG} g_{B\bar{Z}_i} X^B \bar{\chi}_L^{\bar{Z}_i} \lambda_L + \text{c.c.} \quad (248)$$

The physical Gaugino-Higgsino-Higgs vertex works out to yield:

$$C^{h \tilde{H}^0 \lambda_L} = \frac{\mathcal{V}^{-\frac{5}{2}} \tilde{f}}{\left(\sqrt{\hat{K}_{Z_i \bar{Z}_i}} \right)^2} \sim \tilde{f} \left(10^5 \mathcal{V}^{-\frac{5}{2}} \right) \sim \tilde{f} \mathcal{V}^{-\frac{3}{2}}. \quad (249)$$

Gaugino-Gaugino-Higgs vertex

$$\mathcal{L} = i\bar{\lambda}_L \gamma^m \frac{1}{4} (K_{\mathcal{Z}_i} \partial_m \mathcal{Z}_i - c.c.) \lambda_L, \quad (250)$$

where λ_L corresponds to gaugino. Here also, the aforementioned vertex does not preserve $SU(2)_L$ symmetry - one has to obtain the term linear in $\langle z_i \rangle$. In terms of undiagonalized basis, $\partial_{z_i} K \sim \mathcal{V}^{-\frac{2}{3}} \langle z_i \rangle$, and using $\partial_{\mathcal{Z}_i} K \sim \mathcal{O}(1) \partial_{z_i} K$, we have: $\partial_{\mathcal{Z}_i} K \sim \mathcal{V}^{-\frac{2}{3}} \langle \mathcal{Z}_i \rangle$, incorporating the same

$$\begin{aligned} C^{h\bar{\lambda}_L \lambda_L} : & \frac{\mathcal{V}^{-\frac{2}{3}} \langle \mathcal{Z}_i \rangle \bar{\lambda}_L \frac{\not{\partial} \mathcal{Z}_i}{M_p} \lambda_L}{\sqrt{(\hat{K}_{\mathcal{Z}_1 \bar{\mathcal{Z}}_1})^2}} \sim 10^5 \mathcal{V}^{-\frac{23}{36}} h \bar{\lambda}_L \frac{\not{p}_h}{M_p} \lambda_L \sim \mathcal{V}^{-\frac{23}{36}} h \bar{\lambda}_L \frac{\gamma \cdot (p_{\bar{\lambda}_L} + p_{\lambda_L})}{M_p} \lambda_L \\ & \sim 10^5 \mathcal{V}^{-\frac{23}{36}} h \bar{\lambda}_L \frac{m_{\tilde{g}}}{M_p} \lambda_L \sim \mathcal{V}^{-\frac{35}{36}} h \bar{\lambda}_L \lambda_L \end{aligned} \quad (251)$$

Higgs-Higgs-Higgs vertex

Expanding the effective supergravity potential $V = e^K G^{TsTs} |D_{Ts} W|^2$ in fluctuations: $\mathcal{Z}_I = \delta \mathcal{Z}_I + \mathcal{V}^{\frac{1}{36}} M_p$, the contribution of the term cubic in Higgses is of the order $\mathcal{V}^{-\frac{107}{36}} < z_I >$. Utilizing the same the physical Higgs-Higgs-Higgs vertex will be given as:

$$C^{hhh} : \frac{1}{\sqrt{(\hat{K}_{\mathcal{Z}_i \bar{\mathcal{Z}}_i})^4}} \left[\mathcal{V}^{-\frac{107}{36}} \langle \mathcal{Z}_i \rangle \delta \mathcal{Z}^i \delta \mathcal{Z}^j \delta \mathcal{Z}^k \right] \sim \mathcal{O}(\mathcal{V}^{-1}) \delta \mathcal{Z}^i \delta \mathcal{Z}^j \delta \mathcal{Z}^k.$$

Now, using the set of results given in equation no (247), (249), (251), (330), the contribution of vertices appearing in equation (244) are as follows:

$$\begin{aligned} C^{\chi_3^0 \chi_1^0 h} &= \tilde{f} \mathcal{V}^{-\frac{3}{2}} + \tilde{f} \mathcal{V}^{\frac{1}{4}} \sim \tilde{f} \mathcal{V}^{\frac{1}{4}} \\ C^{\chi_3^0 \chi_2^0 h} &= \tilde{f} \mathcal{V}^{-\frac{35}{36}} + \tilde{f}^3 \mathcal{V}^{-\frac{3}{2}} + \tilde{f} \mathcal{V}^{\frac{1}{4}} \sim \tilde{f} \mathcal{V}^{\frac{1}{4}}, \\ C^{\chi_3^0 \chi_3^0 h} &= \mathcal{V}^{-\frac{35}{36}} + \tilde{f}^2 \mathcal{V}^{-\frac{3}{2}} + \tilde{f}^2 \mathcal{V}^{\frac{1}{4}} \sim \mathcal{V}^{-\frac{35}{36}}. \end{aligned} \quad (252)$$

Since now we have got the estimates of coupling required to calculate partial wave coefficients for $\chi_3^0 \chi_3^0 \rightarrow hh$ annihilation process, we are in position to calculate the contribution of partial wave a and b coefficients in our set up just by using (and quoting verbatim below) the form of analytical results provided in [13].

s-channel Higgs-boson (h, H) exchange:

The analytic expressions of \tilde{a}_{hh} and \tilde{b}_{hh} corresponding to Fig. 23 are given as:

$$\tilde{a}_{hh}^{(h,H)} = 0, \quad (253)$$

$$\tilde{b}_{hh}^{(h,H)} = \frac{3}{64\pi} \left| \sum_{r=h,H} \frac{C^{hhr} C^{\chi_3^0 \chi_3^0 r}}{4M_{\tilde{\chi}}^2 - m_r^2 + i\Gamma_r m_r} \right|^2. \quad (254)$$

Expanding the summation

$$\tilde{b}_{hh}^{(h,H)} = \frac{3}{64\pi} \left| \frac{C^{hhh} C^{\chi_3^0 \chi_3^0 h}}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} + \frac{C^{hhH} C^{\chi_3^0 \chi_3^0 H}}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_H m_H} \right|^2 M_p^2; \quad (255)$$

Utilizing the value of mass $m_h = 125\text{GeV}$, $m_H \sim \mathcal{V}^{\frac{59}{72}} m_{\frac{3}{2}}$, $m_{\chi_3^0} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} \sim \mathcal{V}^{-\frac{4}{3}} M_p$ and $C^{hhh} \sim 10^{\frac{15}{2}} (\mathcal{V}^{-1})$, $C\chi_3^0\chi_3^0h \sim C\chi_3^0\chi_3^0H \sim \mathcal{V}^{-\frac{35}{36}}$ from above, after simplifying, we have

$$\begin{aligned} \tilde{b}_{hh}^{(h,H)} &\sim \frac{3}{64\pi} \left| \frac{\mathcal{V}^{-1} \cdot \mathcal{V}^{-\frac{35}{36}}}{4 \cdot \mathcal{V}^{-\frac{8}{3}} M_p^2} + \frac{\mathcal{V}^{-1} \cdot \mathcal{V}^{-\frac{35}{36}}}{\mathcal{V}^{-\frac{85}{36}} m_{pl}^2} \right|^2 M_p^2 \sim \frac{3}{64\pi} \left(\frac{\mathcal{V}^{\frac{25}{36}}}{m_{pl}^2} \right)^2 M_p^2 \\ &\sim \frac{3}{64\pi} \times O(10)^{-29} \text{GeV}^{-2} \text{ for } \mathcal{V} \sim 10^5. \end{aligned} \quad (256)$$

Here we assume that $\Gamma_{h,H} < m_{h,H}$ in our set up.

• Neutralino (χ_i^0) exchange:

$$\tilde{a}_{hh}^{(\chi^0)} = 0, \quad (257)$$

$$\begin{aligned} \tilde{b}_{hh}^{(\chi^0)} &= \frac{1}{16\pi} \sum_{i,j=1}^3 (C\chi_i^0\chi_3^0,h)^2 (C\chi_j^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{hi}^2 \Delta_{hj}^2} \\ &\times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_i^0}) \Delta_{hi} \right. \\ &\left. + 3(m_{\chi_3^0} + m_{\chi_i^0}) (m_{\chi_3^0} + m_{\chi_j^0}) \Delta_{hi} \Delta_{hj} \right], \end{aligned} \quad (258)$$

where $\Delta_{hi} \equiv m_h^2 - m_{\chi_3^0}^2 - m_{\chi_i^0}^2$. Utilizing the values of masses given above, $\Delta_{h1} \equiv m_h^2 - m_{\chi_3^0}^2 - m_{\chi_1^0}^2 \sim \mathcal{V}^2 m_{\frac{3}{2}}^2$, $\Delta_{h2} \equiv m_h^2 - m_{\chi_3^0}^2 - m_{\chi_2^0}^2 \sim \mathcal{V}^2 m_{\frac{3}{2}}^2$, $\Delta_{h3} \equiv m_h^2 - m_{\chi_3^0}^2 - m_{\chi_3^0}^2 \sim \mathcal{V}^{\frac{4}{3}} m_{\frac{3}{2}}^2$, and

$$\begin{aligned} \tilde{b}_{hh}^{(\chi^0)} &= \frac{1}{16\pi} (C\chi_1^0\chi_3^0,h)^2 (C\chi_1^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h1}^2 \Delta_{h1}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_1^0}) \Delta_{h1} \right. \\ &\left. + 3(m_{\chi_3^0} + m_{\chi_1^0}) (m_{\chi_3^0} + m_{\chi_j^0}) \Delta_{h1} \Delta_{h1} \right] + (C\chi_1^0\chi_3^0,h)^2 (C\chi_2^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h1}^2 \Delta_{h2}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 \right. \\ &\left. + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_1^0}) \Delta_{h1} + 3(m_{\chi_3^0} + m_{\chi_1^0}) (m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h1} \Delta_{h2} \right] + \\ &(C\chi_1^0\chi_3^0,h)^2 (C\chi_3^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h1}^2 \Delta_{h3}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_1^0}) \Delta_{h1} \right. \\ &\left. + 3(m_{\chi_3^0} + m_{\chi_1^0}) (m_{\chi_3^0} + m_{\chi_3^0}) \Delta_{h1} \Delta_{h3} \right] + (C\chi_2^0\chi_3^0,h)^2 (C\chi_2^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h2}^2 \Delta_{h2}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 \right. \\ &\left. + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h2} + 3(m_{\chi_3^0} + m_{\chi_2^0}) (m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h2} \Delta_{h2} \right] + \\ &(C\chi_2^0\chi_3^0,h)^2 (C\chi_3^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h2}^2 \Delta_{h3}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h2} \right. \\ &\left. + 3(m_{\chi_3^0} + m_{\chi_3^0}) (m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h2} \Delta_{h3} \right] + (C\chi_3^0\chi_3^0,h)^2 (C\chi_3^0\chi_3^0,h^*)^2 \frac{1}{\Delta_{h3}^2 \Delta_{h3}^2} \times \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_h^2)^2 \right. \\ &\left. + 4m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) (m_{\chi_3^0} + m_{\chi_3^0}) \Delta_{h3} + 3(m_{\chi_3^0} + m_{\chi_3^0}) (m_{\chi_3^0} + m_{\chi_3^0}) \Delta_{h3} \Delta_{h3} \right] \end{aligned}$$

$$\begin{aligned}
& \sim \frac{1}{16\pi} \left[(\tilde{f}^4 \mathcal{V}) \frac{1}{m_{\chi_1^0}^2} + (\tilde{f}^4 \mathcal{V}) \frac{1}{m_{\chi_1^0} m_{\chi_2^0}} + (\tilde{f}^2 \mathcal{V}^{\frac{1}{2}} \mathcal{V}^{-\frac{35}{18}}) \frac{1}{m_{\chi_1^0} m_{\chi_3^0}} + \tilde{f}^4 \mathcal{V} \frac{1}{m_{\chi_2^0}^2} + (\tilde{f}^2 \mathcal{V}^{\frac{1}{2}} \mathcal{V}^{-\frac{35}{18}}) \frac{1}{m_{\chi_2^0} m_{\chi_3^0}} \right. \\
& \quad \left. + (\mathcal{V}^{-\frac{35}{9}}) \frac{1}{m_{\chi_1^0}^2} \right] \\
& \sim \frac{1}{16\pi} \times O(10)^{-36} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5 \text{ and } \tilde{f} \sim 10^{-4}.
\end{aligned} \tag{259}$$

• Higgs (h, H)–neutralino (χ_i^0) interference term:

$$\tilde{a}_{hh}^{(h, H-\chi^0)} = 0, \tag{260}$$

$$\begin{aligned}
\tilde{b}_{hh}^{(h, H-\chi^0)} &= \frac{1}{16\pi} \sum_{i=1}^3 Re \left[\sum_{r=h, H} \left(\frac{C^{hhr} C_S^{\chi\chi r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right)^* C_S^{\chi_i^0 \chi^h} C_S^{\chi_i^0 \chi^h} \right] \\
&\times \frac{[2m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) + 3(m_{\chi_3^0} + m_{\chi_3^0}) \Delta_{hi}]}{\Delta_{hi}^2}.
\end{aligned} \tag{261}$$

$$\begin{aligned}
Re \left[\sum_{r=h, H} \left(\frac{C^{hhr} C_S^{\chi\chi r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right)^* \right] &\sim \left[\left(\frac{C^{hhh} C_S^{\chi\chi h}}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} \right)^* + \left(\frac{C^{hhh} C_S^{\chi\chi h}}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_h m_H} \right)^* \right] \\
&\sim \frac{(\mathcal{V}) \cdot \mathcal{V}^{-\frac{35}{36}} M_p}{m_{\chi_3^0}^2} + \frac{(\mathcal{V}) \cdot \mathcal{V}^{-\frac{35}{36}} M_p}{m_H^2} \sim \frac{\mathcal{V}^{\frac{1}{36}} M_p}{m_{\chi_3^0}^2}.
\end{aligned} \tag{262}$$

Expanding this, we get

$$\begin{aligned}
\tilde{b}_{hh}^{(h, H-\chi^0)} &= \frac{1}{16\pi} \frac{\mathcal{V}^{\frac{1}{36}} M_p}{m_{\chi_3^0}^2} \left[\frac{(C^{\chi_1^0 \chi^h})^2 (2m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) + 3(m_{\chi_3^0} + m_{\chi_1^0}) \Delta_{h1})}{\Delta_{h1}^2} + \right. \\
&\quad \left. \frac{(C^{\chi_2^0 \chi^h})^2 (2m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) + 3(m_{\chi_3^0} + m_{\chi_2^0}) \Delta_{h2})}{\Delta_{h2}^2} + \frac{(C^{\chi_3^0 \chi^h})^2 (2m_{\chi_3^0} (m_{\chi_3^0}^2 - m_h^2) + 3(m_{\chi_3^0} + m_{\chi_3^0}) \Delta_{h3})}{\Delta_{h3}^2} \right] \\
&\sim \frac{1}{16\pi} \frac{\mathcal{V}^{\frac{1}{36}} M_p}{m_{\chi_3^0}^2} \left[\frac{(\tilde{f} \mathcal{V}^{\frac{1}{4}})^2 (3m_{\chi_1^0}^3)}{4m_{\chi_1^0}^4} + \frac{(\tilde{f} \mathcal{V}^{\frac{1}{4}})^2 (3m_{\chi_2^0}^3)}{4m_{\chi_2^0}^4} + \frac{(\mathcal{V}^{-\frac{35}{36}})^2 (5m_{\chi_3^0}^3)}{m_{\chi_3^0}^4} \right] \\
&\sim \frac{1}{16\pi} \frac{\mathcal{V}^{\frac{1}{36}} M_p}{m_{\chi_3^0}^2} \left[\frac{(\mathcal{V}^{-\frac{35}{36}})^2 (5m_{\chi_3^0}^3)}{m_{\chi_3^0}^4} \right] \sim \frac{1}{16\pi} \times O(10)^{-26} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5.
\end{aligned} \tag{263}$$

Utilizing results of equations (256), (259), (263),

$$\begin{aligned}
\tilde{b}_{hh} &= \tilde{b}_{hh}^{(h, H)} + \tilde{b}_{hh}^{(\chi^0)} + \tilde{b}_{hh}^{(h, H-\chi^0)} \sim O(10)^{-26} GeV^{-2} \\
\tilde{a}_{hh} &= \tilde{a}_{hh}^{(h, H)} + \tilde{a}_{hh}^{(\chi^0)} + \tilde{a}_{hh}^{(h, H-\chi^0)} = 0.
\end{aligned} \tag{264}$$

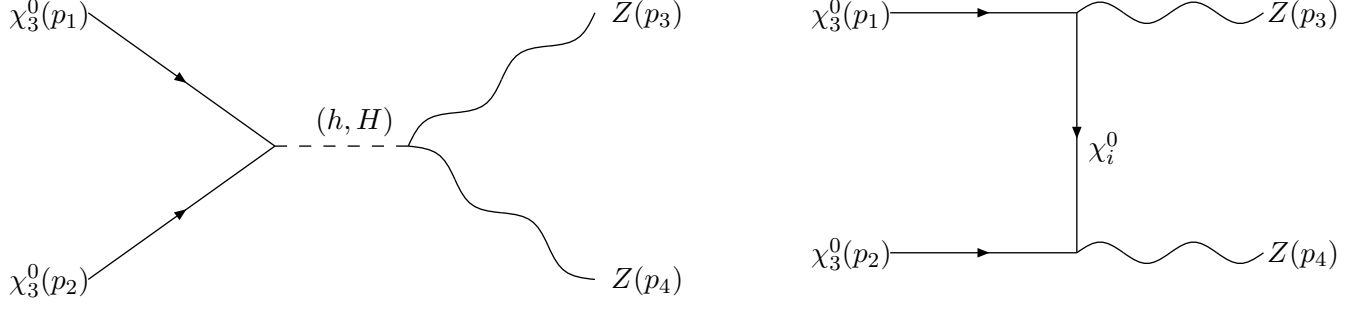


Figure 24: Feynman diagrams for $\chi_3^0 \chi_3^0 \rightarrow ZZ$ via s -channel Higgs exchange and t -channel χ_i^0 exchange.

$$\begin{aligned}
C\chi_3^0\chi_1^0Z &= \tilde{f}C^{\tilde{H}^0\tilde{H}^0Z}, \\
C\chi_3^0\chi_2^0Z &= \tilde{f}C^{\lambda^0\lambda^0Z} + \tilde{f}C^{\tilde{H}^0\tilde{H}^0Z}, \\
C\chi_3^0\chi_3^0Z &= C^{\lambda^0\lambda^0Z} + \tilde{f}^2C^{\tilde{H}^0\tilde{H}^0Z}
\end{aligned} \tag{265}$$

Z Boson-Higgs-Z Boson vertex

$$\mathcal{L} \ni G_{T_B\bar{T}_B} X^{T_B} X^{\bar{T}_B} A^\mu A_\nu \tag{266}$$

where A^μ corresponds to gauge boson and $\mu = 0, 1, 2, 3$ is space time index. The required vertex will be accommodated by $\bar{\partial}_{\bar{z}_k} \partial_{z_i} G_{T_B\bar{T}_B}$. As given in appendix B

$$\bar{\partial}_{\bar{z}_k} \partial_{z_i} G_{T_B\bar{T}_B} \sim \mathcal{O}(1) \bar{\partial}_{\bar{z}_k} \partial_{z_i} G_{T_B\bar{T}_B} \rightarrow \mathcal{V}^{-2} \langle \mathcal{Z}_i \rangle \delta \mathcal{Z}_i \tag{267}$$

and $X = X^B \partial_B = -12i\pi\alpha'\kappa_4^2\mu_7Q_B\partial_{T_B} \sim \mathcal{V}^{-\frac{2}{3}}$. Incorporating values from above, the physical Z Boson-Higgs-Z Boson vertex is proportional to

$$C^{ZZH} \sim \frac{\mathcal{V}^{-2}\tilde{f}^2\mathcal{V}^{-\frac{4}{3}}}{\sqrt{(K_{Z_1Z_1})^2}} \sim \frac{\mathcal{V}^{-2}\tilde{f}^2\mathcal{V}^{-\frac{4}{3}}}{O(10)^{-5}} \sim \tilde{f}^2\mathcal{V}^{-\frac{7}{3}}. \tag{268}$$

Higgsino-Higgsino- Z Boson vertex

$$\mathcal{L} = g_{\mathcal{Z}^I\mathcal{Z}^J} \bar{\chi}^{\mathcal{Z}^I} \not{Z} \text{Im}(X^B K + iD^B) \chi^{\mathcal{Z}^J}, \tag{269}$$

$\chi^{\mathcal{Z}^1}$ corresponds to Higgsino, $X = X^B \partial_B = -12i\pi\alpha'\kappa_4^2\mu_7Q_B\partial_{T_B}$ corresponds to the killing isometry vector and the corresponding D term generated is given by:

$$D^B = \frac{4\pi\alpha'\kappa_4^2\mu_7Q_B v^B}{\mathcal{V}}. \tag{270}$$

Utilizing $g_{\mathcal{Z}_1\bar{\mathcal{Z}}_1} \sim \mathcal{O}(1)g_{z_1\bar{z}_1} \sim \mathcal{V}^{-\frac{2}{3}}$ as given in (124) as well as $v^B \sim \mathcal{V}^{\frac{1}{3}}$ and $Q_B \sim \mathcal{V}^{\frac{1}{3}}(2\pi\alpha')^2\tilde{f}$, yields value of physical Higgsino-Higgsino- Z Boson vertex as:

$$C^{\tilde{H}^0\tilde{H}^0Z} \sim \frac{\left(\mathcal{V}^{-\frac{23}{18}}\right)\tilde{f}}{\left(\sqrt{\hat{K}_{\mathcal{Z}_1\bar{\mathcal{Z}}_1}}\right)^2 \sim O(10)^{-5}} \sim \left(O(10^5)\tilde{f}\mathcal{V}^{-\frac{23}{18}}\right) \sim \tilde{f}\mathcal{V}^{-\frac{16}{15}}. \tag{271}$$

Gaugino-gaugino-Z Bosen vertex

$$\begin{aligned}\mathcal{L} &= \bar{\lambda}^L \not{Z} \text{Im} (X^B K + iD^B) \lambda^L, \\ &\sim \bar{\lambda}^L \gamma \cdot A \left\{ 6\kappa_4^2 \mu_7 2\pi\alpha' Q_B K + \frac{12\kappa_4^2 \mu_7 2\pi\alpha' Q_B v^B}{\mathcal{V}} \right\} \lambda^L\end{aligned}\quad (272)$$

λ^L corresponds to gaugino. Again, Utilizing values of $v^B \sim \mathcal{V}^{\frac{1}{3}}$ and $Q_B \sim \mathcal{V}^{\frac{1}{3}}(2\pi\alpha')^2 \tilde{f}$, yields value of physical Gaugino-gaugino-Z Bosen vertex as:

$$C^{\lambda^0 \lambda^0 Z} \sim \left(\mathcal{V}^{-\frac{11}{18}} \right) \tilde{f} \sim \left(\tilde{f} \mathcal{V}^{-\frac{11}{18}} \right). \quad (273)$$

Using set of results given in equation no (271), (273), (268), the contribution of vertices appearing in equation (274) are as follows:

$$\begin{aligned}C^{\chi_3^0 \chi_1^0 Z} &= \tilde{f}^2 \mathcal{V}^{-\frac{16}{15}}, \\ C^{\chi_3^0 \chi_2^0 Z} &= \tilde{f}^2 \mathcal{V}^{-\frac{11}{18}} + \tilde{f}^2 \mathcal{V}^{-\frac{16}{15}} \sim \tilde{f}^2 \mathcal{V}^{-\frac{11}{18}}, \\ C^{\chi_3^0 \chi_3^0 Z} &= \tilde{f} \mathcal{V}^{-\frac{11}{18}} + \tilde{f}^3 \mathcal{V}^{-\frac{16}{15}} \sim \tilde{f} \mathcal{V}^{-\frac{11}{18}}.\end{aligned}\quad (274)$$

Again, using (and quoting verbatim) the analytical expressions needed to calculate partial wave coefficients for $\chi_3^0 \chi_3^0 \rightarrow ZZ$ annihilation process, we will estimate the values of same in our set up.

• Higgs-boson (h, H) exchange:

$$\tilde{a}_{ZZ}^{(h,H)} = 0, \quad (275)$$

$$\tilde{b}_{ZZ}^{(h,H)} = \frac{3}{64\pi} \left| \sum_{r=h,H} \frac{C^{ZZr} C^{\chi_3^0 \chi_3^0 r}}{s - m_r^2 + i\Gamma_r m_r} \right|^2 \frac{3m_Z^4 - 4m_Z^2 m_{\chi_3^0}^2 + 4m_{\chi_3^0}^4}{m_Z^4}. \quad (276)$$

Expanding the summation

$$\tilde{b}_{ZZ}^{(h,H)} = \frac{3}{64\pi} \left| \frac{C^{ZZh} C^{\chi_3^0 \chi_3^0 h}}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} + \frac{C^{ZZH} C^{\chi_3^0 \chi_3^0 H}}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_H m_H} \right|^2 \frac{3m_Z^4 - 4m_Z^2 m_{\chi_3^0}^2 + 4m_{\chi_3^0}^4}{m_Z^4}. \quad (277)$$

Utilizing the value of mass $m_h = 125\text{GeV}$, $m_H \sim \mathcal{V}^{-\frac{85}{72}} M_p$, $m_{\chi_3^0} \sim \mathcal{V}^{-\frac{4}{3}} M_p$, $m_Z \sim 90\text{GeV}$ and $C^{ZZh} \sim C^{ZZH} \sim \tilde{f}^2 \mathcal{V}^{-4}$, $C^{\chi_3^0 \chi_3^0 h} \sim \mathcal{V}^{-\frac{35}{36}}$ from above, after simplifying, we have:

$$\begin{aligned}\tilde{b}_{ZZ}^{(h,H)} &\sim \frac{3}{64\pi} \left| \frac{\tilde{f}^2 \mathcal{V}^{-\frac{7}{3}} \cdot \mathcal{V}^{-\frac{35}{36}}}{4m_{\chi_3^0}^2} + \frac{\tilde{f}^2 \mathcal{V}^{-\frac{7}{3}} \cdot \mathcal{V}^{-\frac{35}{36}}}{m_H^2} \right|^2 \frac{m_{\chi_3^0}^4}{m_Z^4} m_{pl}^2 \sim \frac{3}{64\pi} \left(\frac{\tilde{f}^2 \mathcal{V}^{-\frac{10}{3}}}{4m_{\chi_3^0}^2} \right)^2 \frac{m_{\chi_3^0}^4}{m_Z^4} m_{pl}^2 \\ &\sim O(10)^{-23} \text{GeV}^{-2} \text{ for } \mathcal{V} \sim 10^5.\end{aligned}\quad (278)$$

Here we assume that $\Gamma_{h,H} < m_{h,H}$ in our set up.

• neutralino (χ_i^0) exchange:

$$\begin{aligned}
\tilde{a}_{ZZ}^{(\chi^0)} &= \frac{1}{4\pi} \sum_{i,j=1}^3 |C\chi_i^0\chi_3^0Z|^2 |C\chi_j^0\chi_3^0Z|^2 \frac{(m_{\chi_3^0}^2 - m_Z^2)}{\Delta_{Zi}\Delta_{Zj}}, \\
&\sim \frac{1}{4\pi} \left[|C\chi_1^0\chi_3^0Z|^2 |C\chi_1^0\chi_3^0Z|^2 \frac{m_{\chi_3^0}^2}{\Delta_{Z1}\Delta_{Z1}} + |C\chi_1^0\chi_3^0Z|^2 |C\chi_2^0\chi_3^0Z|^2 \frac{m_{\chi_3^0}^2}{\Delta_{Z1}\Delta_{Z2}} + |C\chi_1^0\chi_3^0Z|^2 |C\chi_3^0\chi_3^0Z|^2 \frac{m_{\chi_3^0}^2}{\Delta_{Z1}\Delta_{Z3}} \right. \\
&\quad \left. + |C\chi_2^0\chi_3^0Z|^2 |C\chi_3^0\chi_3^0Z|^2 \frac{m_{\chi_3^0}^2}{\Delta_{Z2}\Delta_{Z3}} + |C\chi_3^0\chi_3^0Z|^2 |C\chi_3^0\chi_3^0Z|^2 \frac{m_{\chi_3^0}^2}{\Delta_{Z3}\Delta_{Z3}} \right] \quad (279)
\end{aligned}$$

where $\Delta_{Zi} \equiv m_Z^2 - m_{\chi_3^0}^2 - m_{\chi_i^0}^2$. For the given neutralino mass eigenstates, $\Delta_{Z1} \equiv m_Z^2 - m_{\chi_3^0}^2 - m_{\chi_1^0}^2 \sim -\mathcal{V}^2 m_{\frac{3}{2}}^2$, $\Delta_{Z2} \equiv m_Z^2 - m_{\chi_3^0}^2 - m_{\chi_2^0}^2 \sim -\mathcal{V}^2 m_{\frac{3}{2}}^2$, $\Delta_{Z3} \equiv m_Z^2 - m_{\chi_3^0}^2 - m_{\chi_3^0}^2 \sim -2\mathcal{V}^{\frac{4}{3}} m_{\frac{3}{2}}^2$. Using above values and couplings given in (274), (279) reduces to

$$\begin{aligned}
\tilde{a}_{ZZ}^{(\chi^0)} &\sim \frac{1}{4\pi} m_{\chi_3^0}^2 \left[\frac{\tilde{f}^8 \mathcal{V}^{-\frac{64}{15}}}{\mathcal{V}^4 m_{\frac{3}{2}}^4} + \frac{\tilde{f}^8 \mathcal{V}^{-\frac{10}{3}}}{\mathcal{V}^4 m_{\frac{3}{2}}^4} + \frac{\tilde{f}^6 \mathcal{V}^{-\frac{10}{3}}}{\mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^4} + \frac{\tilde{f}^6 \mathcal{V}^{-\frac{22}{9}}}{\mathcal{V}^{\frac{10}{3}} m_{\frac{3}{2}}^4} + \frac{\tilde{f}^4 \mathcal{V}^{-\frac{22}{9}}}{\mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4} \right] \\
&\sim \frac{1}{4\pi} m_{\chi_3^0}^2 \cdot \frac{\tilde{f}^4 \mathcal{V}^{-\frac{22}{9}}}{\mathcal{V}^{\frac{8}{3}} m_{\frac{3}{2}}^4} \sim O(10)^{-54} GeV^{-2}. \quad (280)
\end{aligned}$$

As from [13], the analytical expression of $\tilde{b}_{ZZ}^{(\chi^0)}$ is defined as following:

$$\begin{aligned}
\tilde{b}_{ZZ}^{(\chi^0)} &= \frac{1}{16\pi} \sum_{i,j=1}^3 \frac{1}{m_Z^4 \Delta_{Zi}^3 \Delta_{Zj}^3} \\
&\times |C\chi_i^0\chi_3^0Z|^2 |C\chi_j^0\chi_3^0Z|^2 \left[D_{ij}^{(1)} \Delta_{Zi}^2 + D_{ij}^{(2)} \Delta_{Zi} \Delta_{Zj} + D_{ij}^{(3)} \Delta_{Zi}^2 \Delta_{Zj} + D_{ij}^{(4)} \Delta_{Zi}^2 \Delta_{Zj}^2 \right], \quad (281)
\end{aligned}$$

where

$$\begin{aligned}
D_{ij}^{(1)} &= 16 m_{\chi_3^0}^2 m_Z^4 (m_Z^2 - m_{\chi_3^0}^2)^2, \\
D_{ij}^{(2)} &= 4 m_{\chi_3^0}^2 (m_Z^2 - m_{\chi_3^0}^2)^2 \{ 3 m_Z^4 + 4 m_{\chi_3^0}^2 m_{\chi_i^0} m_{\chi_j^0} \\
&\quad + m_Z^2 [4 m_{\chi_3^0}^2 + 4 m_{\chi_i^0} m_{\chi_j^0} - 6 m_{\chi_3^0} (m_{\chi_i^0} + m_{\chi_j^0})] \}, \\
D_{ij}^{(3)} &= -4 m_{\chi_3^0} (m_Z^2 - m_{\chi_3^0}^2) \{ 4 m_{\chi_3^0}^3 m_{\chi_i^0} m_{\chi_j^0} + m_Z^4 (7 m_{\chi_3^0} + 3 m_{\chi_i^0} + 6 m_{\chi_j^0}) \\
&\quad + 2 m_Z^2 m_{\chi_3^0} [4 m_{\chi_3^0}^2 - m_{\chi_i^0} m_{\chi_j^0} - m_{\chi_3^0} (m_{\chi_i^0} + 5 m_{\chi_j^0})] \}, \\
D_{ij}^{(4)} &= 8 m_Z^6 + 2 m_Z^2 m_{\chi_3^0}^2 [4 m_{\chi_3^0}^2 - 6 m_{\chi_i^0} m_{\chi_j^0} - 5 m_{\chi_3^0} (m_{\chi_i^0} + m_{\chi_j^0})] \\
&\quad + 4 m_{\chi_3^0}^4 [2 m_{\chi_3^0}^2 + 3 m_{\chi_i^0} m_{\chi_j^0} + 2 m_{\chi_3^0} (m_{\chi_i^0} + m_{\chi_j^0})] \\
&\quad + m_Z^4 [3 m_{\chi_3^0}^2 + 9 m_{\chi_i^0} m_{\chi_j^0} + 5 m_{\chi_3^0} (m_{\chi_i^0} + m_{\chi_j^0})]. \quad (282)
\end{aligned}$$

Solving above,

$$\begin{aligned}
D_{11}^{(1)} &\sim D_{12}^{(1)} \sim D_{13}^{(1)} \sim D_{23}^{(1)} \sim D_{33}^{(1)} \equiv m_Z^4 \mathcal{V}^4 m_{\frac{3}{2}}^4 \\
D_{11}^{(2)} &\sim D_{12}^{(2)} \equiv \mathcal{V}^{\frac{20}{3}} m_{\frac{3}{2}}^{10}, D_{13}^{(2)} \sim D_{23}^{(2)} \equiv \mathcal{V}^{\frac{21}{3}} m_{\frac{3}{2}}^{10}, D_{33}^{(2)} \sim \mathcal{V}^{\frac{20}{3}} m_{\frac{3}{2}}^{10} \\
D_{11}^{(3)} &\sim D_{12}^{(3)} \equiv \mathcal{V}^6 m_{\frac{3}{2}}^8, D_{13}^{(3)} \equiv D_{23}^{(3)} \sim \mathcal{V}^{\frac{17}{3}} m_{\frac{3}{2}}^8, D_{33}^{(3)} \sim \mathcal{V}^{\frac{16}{3}} m_{\frac{3}{2}}^8 \\
D_{11}^{(4)} &\sim D_{12}^{(4)} \equiv \mathcal{V}^{\frac{14}{3}} m_{\frac{3}{2}}^6, D_{13}^{(4)} \equiv D_{23}^{(4)} \sim \mathcal{V}^{\frac{13}{3}} m_{\frac{3}{2}}^6, D_{33}^{(4)} \sim \mathcal{V}^4 m_{\frac{3}{2}}^6.
\end{aligned} \tag{283}$$

Now, incorporating results of equations (283), $\Delta_{Z\ 1,2,3}$ and (274) in (281), the same reduces to:

$$\tilde{b}_{ZZ}^{(\chi^0)} \sim O(10)^{-14} GeV^{-2}. \tag{284}$$

• **Higgs (h, H) –neutralino (χ_i^0) interference term:**

$$\begin{aligned}
\tilde{a}_{ZZ}^{(h,H-\chi^0)} &= 0, \\
\tilde{b}_{ZZ}^{(h,H-\chi^0)} &= \frac{1}{16\pi} \sum_{i=1}^3 Re \left[\left(\sum_{r=h,H} \frac{C^{ZZr} C_S^{\chi_3^0 \chi_3^0 r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right)^* \right] \frac{1}{m_Z^4 \Delta_{Zi}^2} C_S^{\chi_3^0 \chi_3^0 r} \\
&\times \left\{ 2m_{\chi_3^0} (m_{\chi_3^0}^2 - m_Z^2) [-3m_Z^4 - 4m_{\chi_3^0}^3 m_{\chi_i^0} + 2m_Z^2 m_{\chi_3^0} (m_{\chi_3^0} + m_{\chi_i^0})] \right. \\
&\left. + \Delta_{Zi} \left[-4m_{\chi_3^0}^4 (2m_{\chi_3^0} + 3m_{\chi_3^0}) + 2m_Z^2 m_{\chi_3^0}^2 (5m_{\chi_3^0} + 6m_{\chi_i^0}) - m_Z^4 (5m_{\chi_3^0} + 9m_{\chi_i^0}) \right] \right\}. \tag{285}
\end{aligned}$$

Utilizing results for $\Delta_{Z\ 1,2,3}$ and (274), the same reduces to

$$\begin{aligned}
\tilde{b}_{ZZ}^{(h,H-\chi^0)} &= \frac{1}{16\pi} \frac{m_{\chi_3^0}^3}{m_Z^4} \cdot \sum_{i=1}^3 Re \left[\left(\sum_{r=h,H} \frac{C^{ZZr} C^{\chi_3^0 \chi_3^0 r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right)^* \right] C^{\chi_3^0 \chi_3^0 r} M_p \\
&\sim \frac{1}{16\pi} \frac{\mathcal{V}^2 m_{\frac{3}{2}}^3}{m_Z^4} \left(\frac{C^{ZZh} (C^{\chi_3^0 \chi_3^0 h})^2}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} + \frac{C^{ZZH} (C^{\chi_3^0 \chi_3^0 H})^2}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_H m_H} \right) M_p \tag{286}
\end{aligned}$$

Utilizing the value of mass $m_h = 125 GeV$, $m_H \sim \mathcal{V}^{-\frac{85}{72}} M_p$, $m_{\chi_3^0} \sim \mathcal{V}^{-\frac{4}{3}} M_p$, $m_Z \sim 90 GeV$ and $C^{ZZh} \sim C^{ZZH} \sim \tilde{f}^2 \mathcal{V}^{-\frac{7}{3}}$, $C^{\chi_3^0 \chi_3^0 Z} \sim \mathcal{V}^{-\frac{35}{36}}$ from above, after simplifying, we have:

$$\begin{aligned}
\tilde{b}_{ZZ}^{(h,H-\chi^0)} &\sim \frac{1}{16\pi} \frac{\mathcal{V}^2 m_{\frac{3}{2}}^3}{m_Z^4} \left(\frac{\tilde{f}^2 \mathcal{V}^{-\frac{7}{3}} \cdot \mathcal{V}^{-\frac{35}{18}}}{4m_{\chi_3^0}^2} + \frac{\tilde{f}^2 \mathcal{V}^{-\frac{7}{3}} \cdot \mathcal{V}^{-\frac{35}{18}}}{m_H^2} \right) M_p \sim \frac{1}{16\pi} \frac{m_{\frac{3}{2}}^3}{m_Z^4} \left(\frac{\tilde{f}^2 \mathcal{V}^{-\frac{7}{3}}}{4m_{\chi_3^0}^2} \right) M_p \\
&\sim O(10)^{-10} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5. \tag{287}
\end{aligned}$$

Here we assume that $\Gamma_{h,H} < m_{h,H}$ in our set up. Utilizing results of equations (278), (280), (284), (287),

$$\tilde{b}_{ZZ} = \tilde{b}_{ZZ}^{(h,H)} + \tilde{b}_{ZZ}^{(\chi^0)} + \tilde{b}_{ZZ}^{(h,H-\chi^0)} \sim O(10)^{-10} GeV^{-2}; \tag{288}$$

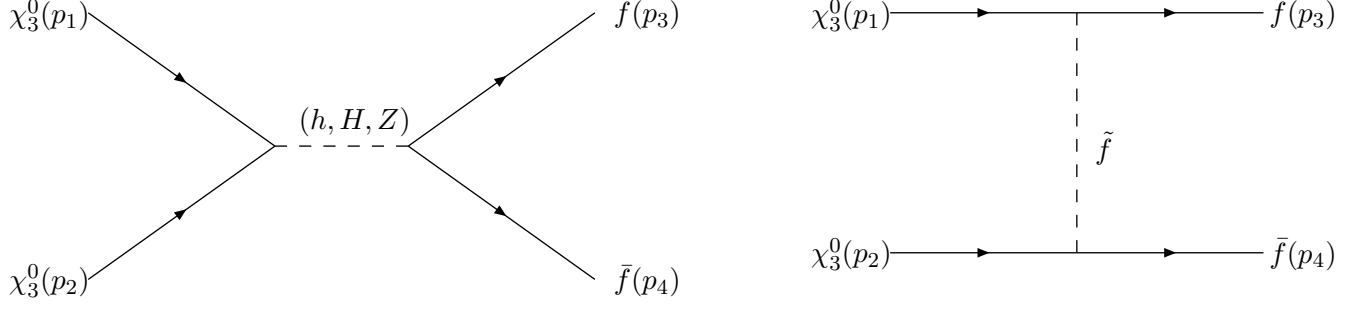


Figure 25: Feynman diagrams for $\chi_3^0 \chi_3^0 \rightarrow f \bar{f}$ via s -channel Higgs/ Z exchange and t -channel \tilde{f} exchange.

$$\tilde{a}_{ZZ} = \tilde{a}_{ZZ}^{(h,H)} + \tilde{a}_{ZZ}^{(\chi^0)} + \tilde{a}_{ZZ}^{(h,H-\chi^0)} \sim O(10)^{-54} GeV^{-2}. \quad (289)$$

Higgs-Fermion-Fermion Interaction

Fig. 25 involves the s -channel Higgs boson (h, H) and Z boson exchange and the t - and u -channel sfermion (\tilde{f}_a) exchange. To get the estimate of thermal cross-section for this process, one needs to evaluate the following partial wave coefficients:

$$\begin{aligned} \tilde{a}_{\bar{f}f} &= \tilde{a}_{\bar{f}f}^{(h,H)} + \tilde{a}_{\bar{f}f}^{(Z)} + \tilde{a}_{\bar{f}f}^{(\tilde{f})} + \tilde{a}_{\bar{f}f}^{(h,H-\tilde{f})} + \tilde{a}_{\bar{f}f}^{(Z-\tilde{f})}, \\ \tilde{b}_{\bar{f}f} &= \tilde{b}_{\bar{f}f}^{(h,H)} + \tilde{b}_{\bar{f}f}^{(Z)} + \tilde{b}_{\bar{f}f}^{(\tilde{f})} + \tilde{b}_{\bar{f}f}^{(h,H-\tilde{f})} + \tilde{b}_{\bar{f}f}^{(Z-\tilde{f})}. \end{aligned} \quad (290)$$

We have calculated below the interaction vertices corresponding to s and t channel of $\chi_3^0 \chi_3^0 \rightarrow f \bar{f}$.

$$\mathcal{L} \ni ig_{\mathcal{A}_I \bar{\mathcal{A}}_I} \bar{\chi}_L^{\bar{\mathcal{A}}_I} \left[\not{\partial} \chi_L^{\mathcal{A}_I} + \Gamma_{\mathcal{Z}_1 \mathcal{A}_I}^{\mathcal{A}_I} \not{\partial} \mathcal{Z}_i \chi_L^{\mathcal{A}_I} + \frac{1}{4} (\partial_{\mathcal{Z}_i} K \not{\partial} \mathcal{Z}_i - \text{c.c.}) \chi_L^{\mathcal{A}_I} \right] \quad (291)$$

$\chi_L^{\mathcal{A}_I}, I = 1, 2$ corresponding to first generation of Left-handed leptons and Left-handed quarks.

Higgs-lepton-lepton interaction

Utilizing $z_1 = \delta z_1 + \mathcal{V}^{\frac{1}{36}} M_p$, for lepton mass $m_{\chi^{\mathcal{A}_1}} \sim O(1) MeV$ and $m_{3/2} = \mathcal{V}^{-2} M_p$, $g_{a_1 \bar{a}_1} \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle z_i \rangle \delta z_i}{M_p^2}, \Gamma_{z_i a_1}^{a_1} \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle z_i \rangle}{M_p}, \partial_{z_i} K \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle z_i \rangle}{M_p}$, one gets:

$$\begin{aligned} g_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \not{\partial} \chi_L^{\mathcal{A}_1} &\sim O(1) g_{a_1 \bar{a}_1} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \not{\partial} \chi_L^{\mathcal{A}_1} \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle \mathcal{Z}_I \rangle}{M_p} \delta \mathcal{Z}_I \bar{\chi}_L^{\bar{\mathcal{A}}_1} \not{p}_{\chi^{\mathcal{A}_1}} \chi_L^{\mathcal{A}_1} \sim \frac{\mathcal{V}^{-\frac{7}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \delta \mathcal{Z}_I \chi_L^{\mathcal{A}_1}; \\ g_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \Gamma_{\mathcal{Z}_i \mathcal{A}_1}^{\mathcal{A}_1} \not{\partial} \mathcal{Z}_i \chi_L^{\mathcal{A}_1} &\sim O(1) g_{a_1 \bar{a}_1} \Gamma_{z_i a_1}^{a_1} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \not{\partial} \mathcal{Z}_i \chi_L^{\mathcal{A}_1} \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle \mathcal{Z}_I \rangle}{M_p} \delta \mathcal{Z}_I \bar{\chi}_L^{\bar{\mathcal{A}}_1} (\not{p}_{\chi^{\mathcal{A}_1}} + \not{p}_{\bar{\chi}^{\mathcal{A}_1}}) \chi_L^{\mathcal{A}_1} \\ &\sim \frac{\mathcal{V}^{-\frac{7}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \delta \mathcal{Z}_I \chi_L^{\mathcal{A}_1}; \\ g_{\mathcal{A}_1 \bar{\mathcal{A}}_1} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \frac{1}{4} (\partial_{\mathcal{Z}_i} K \gamma \cdot \mathcal{Z}_i - \text{c.c.}) \chi_L^{\mathcal{A}_1} &\sim O(1) g_{a_1 \bar{a}_1} \partial_{z_i} K \bar{\chi}_L^{\bar{\mathcal{A}}_1} \not{\partial} \mathcal{Z}_i \chi_L^{\mathcal{A}_1} \rightarrow \frac{\mathcal{V}^{-\frac{2}{9}} \langle \mathcal{Z}_i \rangle}{M_p} \delta \mathcal{Z}_i \bar{\chi}_L^{\bar{\mathcal{A}}_1} (\not{p}_{\chi^{\mathcal{A}_1}} + \not{p}_{\bar{\chi}^{\mathcal{A}_1}}) \chi_L^{\mathcal{A}_1} \\ &\sim \frac{\mathcal{V}^{-\frac{7}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_1} \delta \mathcal{Z}_I \chi_L^{\mathcal{A}_1}. \end{aligned} \quad (292)$$

Incorporating results of (292) in (291), the physical Higgs-lepton-lepton vertex will be given as

$$C^{\mathcal{A}_{1L}\bar{\mathcal{A}}_{1L}h} = \frac{1}{\sqrt{(\hat{K}_{\mathcal{Z}_i\bar{\mathcal{Z}}_i})^2 \cdot (\hat{K}_{\mathcal{A}_1\bar{\mathcal{A}}_1})^2}} \left[\frac{\mathcal{V}^{-\frac{7}{36}}}{M_p} \right] \sim 10^1 \left[\frac{\mathcal{V}^{\frac{7}{36}}}{M_p} \right] \sim \frac{\mathcal{V}^{\frac{1}{180}}}{M_p} \quad (293)$$

Higgs-quark-quark interaction

Utilizing $z_1 = \delta z_1 + \mathcal{V}^{\frac{1}{36}} M_p$, for quark mass $m_{\chi^{\mathcal{A}_2}} \sim \mathcal{O}(5)MeV$ and $m_{3/2} = \mathcal{V}^{-2} M_p$, $g_{a_2\bar{a}_2} \rightarrow \frac{\mathcal{V}^{-\frac{11}{9}} \langle z_i \rangle}{M_p^2}, \Gamma_{z_i a_2}^{a_2} \rightarrow \frac{\mathcal{V}^{-\frac{2}{3}} \langle z_i \rangle}{M_p}, \partial_{z_i} K \rightarrow \frac{\mathcal{V}^{-\frac{2}{3}} \langle z_i \rangle}{M_p}$, one gets:

$$\begin{aligned} g_{\mathcal{A}_2\bar{\mathcal{A}}_2} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \not{\partial} \chi_L^{\mathcal{A}_2} &\sim \mathcal{O}(1) g_{a_2\bar{a}_2} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \not{\partial} \chi_L^{\mathcal{A}_2} \rightarrow \frac{\mathcal{V}^{-\frac{11}{9}} \langle \mathcal{Z}_i \rangle}{M_p} \delta \mathcal{Z}_i \bar{\chi}_L^{\bar{\mathcal{A}}_2} \not{p}_{\chi^{\mathcal{A}_2}} \chi_L^{\mathcal{A}_2} \sim \frac{\mathcal{V}^{-\frac{43}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \delta \mathcal{Z}_i \chi_L^{\mathcal{A}_2}; \\ g_{\mathcal{A}_2\bar{\mathcal{A}}_2} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \Gamma_{\mathcal{Z}_i \mathcal{A}_2}^{\mathcal{A}_2} \not{\partial} \chi_L^{\mathcal{A}_2} &\sim \mathcal{O}(1) g_{a_2\bar{a}_2} \Gamma_{z_i a_2}^{a_2} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \not{\partial} \chi_L^{\mathcal{A}_2} \rightarrow \frac{\mathcal{V}^{-\frac{11}{9}} \langle \mathcal{Z}_i \rangle}{M_p} \delta \mathcal{Z}_i \bar{\chi}_L^{\bar{\mathcal{A}}_2} (\not{p}_{\chi^{\mathcal{A}_2}} + \not{p}_{\bar{\chi}^{\mathcal{A}_2}}) \chi_L^{\mathcal{A}_2} \\ &\sim \frac{\mathcal{V}^{-\frac{43}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \delta \mathcal{Z}_i \chi_L^{\mathcal{A}_2}; \\ g_{\mathcal{A}_2\bar{\mathcal{A}}_2} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \cdot \frac{1}{4} (\partial_{\mathcal{Z}_i} K \gamma \cdot \mathcal{Z}_i - \text{c.c.}) \chi_L^{\mathcal{A}_2} &\sim \mathcal{O}(1) g_{a_2\bar{a}_2} \partial_{z_i} K \bar{\chi}_L^{\bar{\mathcal{A}}_2} \not{\partial} \chi_L^{\mathcal{A}_2} \rightarrow \frac{\mathcal{V}^{-\frac{11}{9}} \langle \mathcal{Z}_i \rangle}{M_p} \delta \mathcal{Z}_i \bar{\chi}_L^{\bar{\mathcal{A}}_2} (\not{p}_{\chi^{\mathcal{A}_2}} + \not{p}_{\bar{\chi}^{\mathcal{A}_2}}) \chi_L^{\mathcal{A}_2} \\ &\sim \frac{\mathcal{V}^{-\frac{43}{36}}}{M_p} \bar{\chi}_L^{\bar{\mathcal{A}}_2} \delta \mathcal{Z}_i \chi_L^{\mathcal{A}_2}. \end{aligned} \quad (294)$$

Incorporating results of (294) in (291), the physical Higgs-quark-quark vertex will be given as

$$\begin{aligned} C^{\mathcal{A}_{2L}\bar{\mathcal{A}}_{2L}h} &= \frac{1}{\sqrt{(\hat{K}_{\mathcal{Z}_i\bar{\mathcal{Z}}_i})^2 \cdot (\hat{K}_{\mathcal{A}_2\bar{\mathcal{A}}_2})^2}} \left[\frac{\mathcal{V}^{-\frac{43}{36}}}{M_p} \right] \\ &\sim 10^{-7} \left[\frac{\mathcal{V}^{-\frac{43}{36}}}{M_p} \right] \sim \frac{\mathcal{V}^{-\frac{13}{5}}}{M_p}. \end{aligned} \quad (295)$$

We would estimate:

$$C^{f\bar{f}h} = \text{Max} \left[C^{\mathcal{A}_{1L}\bar{\mathcal{A}}_{1L}h}, C^{\mathcal{A}_{2L}\bar{\mathcal{A}}_{2L}h} \right] \sim \frac{\mathcal{V}^{\frac{1}{180}}}{M_p}. \quad (296)$$

fermion-fermion- Z Boson vertex

$$\mathcal{L} \ni g_{\mathcal{A}_1\bar{\mathcal{A}}_1} \bar{\chi}^{\mathcal{A}_1} \gamma \cdot A \text{Im} (X^B K + iD^B) \chi^{\mathcal{A}_1} + g_{\mathcal{A}_2\bar{\mathcal{A}}_2} \bar{\chi}^{\mathcal{A}_2} \gamma \cdot A \text{Im} (X^B K + iD^B) \chi^{\mathcal{A}_2}, \quad (297)$$

$\chi^{\mathcal{A}_{1,2}}$ correspond to first generation of leptons and quarks, $X = X^B \partial_B = -12i\pi\alpha' \kappa_4^2 \mu_7 Q_B \partial_{T_B}$ corresponds to the killing isometry vector and D term generated is given by:

$$D^B = \frac{4\pi\alpha' \kappa_4^2 \mu_7 Q_B v^B}{\mathcal{V}}. \quad (298)$$

Utilizing $g_{\mathcal{A}_1\bar{\mathcal{A}}_1} \sim \mathcal{O}(1) g_{a_1\bar{a}_1} \sim \mathcal{V}^{\frac{4}{9}}$, $g_{\mathcal{A}_2\bar{\mathcal{A}}_2} \sim \mathcal{O}(1) g_{a_2\bar{a}_2} \sim \mathcal{V}^{-\frac{5}{9}}$ as given in 124 as well as $v^B \sim \mathcal{V}^{\frac{1}{3}}$ and $Q_B \sim \mathcal{V}^{\frac{1}{3}} (2\pi\alpha')^2 \tilde{f}$, the physical fermion-fermion- Z Boson vertex is estimated as:

$$C^{f\bar{f}Z} \sim \text{Max} \left[\frac{(\mathcal{V}^{-\frac{1}{6}}) \tilde{f}}{\left(\sqrt{\hat{K}_{\mathcal{A}_1\bar{\mathcal{A}}_1}} \right)^2 \sim \mathcal{O}(10^4)}, \frac{(\mathcal{V}^{-\frac{7}{6}}) \tilde{f}}{\left(\sqrt{\hat{K}_{\mathcal{A}_2\bar{\mathcal{A}}_2}} \right)^2 \sim \mathcal{O}(10^{-2})} \right] \sim \tilde{f} \mathcal{V}^{-\frac{23}{36}}. \quad (299)$$

Quoting the analytical expressions as given in [13]:

• **Higgs-boson (h, H) exchange:**

$$\tilde{a}_{\bar{f}f}^{(h,H)} = 0, \quad (300)$$

$$\tilde{b}_{\bar{f}f}^{(h,H)} = \frac{3}{4\pi} \left| \sum_{r=h,H} \frac{C^{ffr} C^{\chi_3^0 \chi_3^0 r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right|^2 (m_{\chi_3^0}^2 - m_f^2). \quad (301)$$

Expanding the summation

$$\tilde{b}_{\bar{f}f}^{(h,H)} = \frac{3}{64\pi} \left| \frac{C^{ffh} C^{\chi_3^0 \chi_3^0 h}}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} + \frac{C^{hhH} C^{\chi_3^0 \chi_3^0 H}}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_H m_H} \right|^2 (m_{\chi_3^0}^2 - m_f^2). \quad (302)$$

Again, utilizing the value of masses as given in above cases and $C^{ffh} \sim \mathcal{V}^{-\frac{18}{5}}$, $C^{\chi_3^0 \chi_3^0 h} \sim C^{\chi_3^0 \chi_3^0 H} \sim \mathcal{V}^{-\frac{35}{36}}$ from above, after simplifying, we have:

$$\begin{aligned} \tilde{b}_{\bar{f}f}^{(h,H)} &\sim \frac{3}{64\pi} \left| \frac{\mathcal{V}^{-\frac{18}{5}} \cdot \mathcal{V}^{-\frac{35}{36}}}{4 \cdot \mathcal{V}^{-\frac{8}{3}} m_{pl}^2} + \frac{\mathcal{V}^{-\frac{18}{5}} \cdot \mathcal{V}^{-\frac{35}{36}}}{\mathcal{V}^{-\frac{85}{36}} m_{pl}^2} \right|^2 (m_{\chi_3^0}^2 - m_f^2) \sim \frac{3}{64\pi} \left(\frac{\mathcal{V}^{-\frac{23}{5}}}{\mathcal{V}^{-\frac{8}{3}} m_{pl}^2} \right)^2 \mathcal{V}^{-\frac{8}{3}} M_p^2 \\ &\sim O(10)^{-48} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5. \end{aligned} \quad (303)$$

• **Z-boson exchange:**

$$\tilde{a}_{\bar{f}f}^{(Z)} = \frac{1}{2\pi} \left| \frac{C^{ffZ} C_A^{\chi\chi Z}}{4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \frac{m_f^2 (m_Z^2 - 4m_{\chi_3^0}^2)^2}{m_Z^4}. \quad (304)$$

Utilizing the value of mass as given above, and $C^{ffZ} \sim (\mathcal{V}^{-\frac{23}{30}})$, $C^{\chi_3^0 \chi_3^0 Z} \sim \mathcal{V}^{-\frac{35}{36}}$ from above, after simplifying, we have

$$\begin{aligned} \tilde{a}_{\bar{f}f}^{(h,H)} &\sim \frac{3}{64\pi} \left| \frac{\mathcal{V}^{-\frac{23}{30}} \cdot \mathcal{V}^{-\frac{35}{36}}}{4 \cdot \mathcal{V}^{-\frac{8}{3}} m_{pl}^2} + \frac{\mathcal{V}^{-\frac{23}{30}} \cdot \mathcal{V}^{-\frac{35}{36}}}{\mathcal{V}^{-\frac{85}{36}} m_{pl}^2} \right|^2 \frac{m_f^2 (m_Z^2 - 4m_{\chi_3^0}^2)^2}{m_Z^4} \sim \frac{3}{64\pi} \left(\frac{\mathcal{V}^{\frac{97}{36}}}{\mathcal{V}^{-\frac{8}{3}} m_{pl}^2} \right)^2 \frac{m_f^2 (4m_{\chi_3^0})^4}{m_Z^4} \\ &\sim O(10)^{-43} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5; \end{aligned} \quad (305)$$

$$\begin{aligned}
\tilde{b}_{ff}^{(Z)} &= \frac{1}{2\pi} \left| \frac{C^{ffZ} C\chi_3^0\chi_3^0Z}{4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \frac{1}{m_Z^2 ((4m_{\chi_3^0}^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2)} \\
&\times \left[2|C^{ffZ}|^2 \left\{ m_Z^2 (m_{\chi_3^0}^2 - m_f^2) (m_Z^2 - 4m_{\chi_3^0}^2)^2 \right. \right. \\
&\quad \left. \left. + \Gamma_Z^2 [m_{\chi_3^0}^2 m_Z^4 + m_f^2 (24m_{\chi_3^0}^4 - 6m_Z^2 m_{\chi_3^0}^2 - m_Z^4)] \right\} \right. \\
&\quad \left. + m_Z^2 |C^{ffZ}|^2 \left\{ (2m_{\chi_3^0}^2 + m_f^2) [(4m_{\chi_3^0}^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2] \right\} \right] \\
&\sim \frac{1}{2\pi} \left| \frac{C^{ffZ} C\chi_3^0\chi_3^0Z}{4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \frac{1}{m_Z^2 (16m_{\chi_3^0}^4)} \times \left[2.16 \mathcal{V}^{-\frac{23}{15}} m_Z^2 m_{\chi_3^0}^6 + \mathcal{V}^{-\frac{23}{15}} m_Z^2 (16m_{\chi_3^0}^6) \right] \\
&\sim \frac{1}{2\pi} \mathcal{V}^{-\frac{23}{15}} m_{\chi_3^0}^2 \cdot \left| \frac{C^{ffZ} C\chi_3^0\chi_3^0Z}{4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \sim \frac{1}{2\pi} \mathcal{V}^{-\frac{23}{15}} \cdot \mathcal{V}^{-\frac{8}{3}} m_{pl}^2 \left(\frac{\mathcal{V}^{-\frac{23}{30}} \cdot \mathcal{V}^{-\frac{35}{36}}}{4 \cdot \mathcal{V}^{-\frac{8}{3}} M_p^2} \right)^2 \\
&\sim \mathcal{O}(10)^{-39} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5.
\end{aligned} \tag{306}$$

• sfermion (\tilde{f}_a) exchange:

$$\tilde{a}_{ff}^{(\tilde{f})} = \frac{1}{32\pi} \sum_{a,b} \frac{(m_f C_+^a + m_{\chi_3^0} D_+^a) (m_f C_+^b + m_{\chi_3^0} D_+^b)}{\Delta_{\tilde{f}_a} \Delta_{\tilde{f}_b}}, \tag{307}$$

a is the index for sfermion mass eigenstates so that $a = 1, \dots, 6$ for squark and charged sleptons and $a = 1, \dots, 3$ corresponds for sneutrino, and:

$$C_{\pm}^a = |\Lambda_{fL}^a|^2 \pm |\Lambda_{fR}^a|^2, \tag{308}$$

$$D_{\pm}^a = \Lambda_{fL}^a (\Lambda_{fR}^a)^* \pm (\Lambda_{fL}^a)^* \Lambda_{fR}^a; \tag{309}$$

Λ_{fL}^a corresponds to the Neutralino-fermion-sfermion interactions mediated by L- handed squarks/sleptons and Λ_{fR}^a corresponds to the Neutralino-fermion-sfermion interactions mediated by R-handed squarks/sleptons.

Using results from **section 3**: $C\chi_3^0 l_L \tilde{l}_L = \tilde{f} \mathcal{V}^{-\frac{1}{2}}$, $C\chi_3^0 u_L \tilde{u}_L \sim \tilde{f} \mathcal{V}^{-\frac{4}{5}}$. With exactly similar procedure, we find: $C\chi_3^0 l_L \tilde{l}_R = \tilde{f} \mathcal{V}^{-\frac{12}{15}}$, $C\chi_3^0 u_L \tilde{u}_R \sim \tilde{f} \mathcal{V}^{-\frac{25}{36}}$. Utilizing above

$$\begin{aligned}
\sum_a C_{\pm}^a &= \text{Max} \left(\left| C\chi_3^0 l_L \tilde{l}_L \right|^2 \pm \left| C\chi_3^0 l_L \tilde{l}_R \right|^2, \left| C\chi_3^0 u_L \tilde{u}_L \right|^2 \pm \left| C\chi_3^0 u_L \tilde{u}_R \right|^2 \right) \sim \tilde{f}^2 \mathcal{V}^{-1} \\
\sum_a D_{\pm}^a &= \text{Max} \left(C\chi_3^0 l_L \tilde{l}_L (C\chi_3^0 l_L \tilde{l}_R)^*, C\chi_3^0 u_L \tilde{u}_L (C\chi_3^0 u_L \tilde{u}_R)^* \right) \pm c.c.. \sim \tilde{f}^2 \mathcal{V}^{-\frac{13}{10}}
\end{aligned} \tag{310}$$

and

$$\Delta_{\tilde{f}_a} \equiv m_f^2 - m_{\chi_3^0}^2 - m_{\tilde{f}_a}^2 \sim -m_{\chi_3^0}^2.$$

Considering only first two generations quarks/sleptons and assuming the universality in scalar masses for first two generations quarks/sleptons, equation (307) reduces to the following simplified form:

$$\begin{aligned}
\tilde{a}_{ff}^{(\tilde{f})} &= \frac{1}{\pi} \left(\frac{m_f \tilde{f}^2 \mathcal{V}^{-1} + m_{\chi_3^0} \tilde{f}^2 \mathcal{V}^{-\frac{13}{10}}}{m_{\chi_3^0}^4} (m_f \tilde{f}^2 \mathcal{V}^{-1} + m_{\chi_3^0} \tilde{f}^2 \mathcal{V}^{-\frac{13}{10}}) \right) \\
&\sim 10^{-52} GeV^{-2}, \text{ for } m_{\chi_3^0} \sim \mathcal{V}^{-\frac{4}{3}} M_p \text{ and } \mathcal{V} \sim 10^5.
\end{aligned} \tag{311}$$

Now, utilizing the numerical estimates of coupling summed over first two generation of squarks/sleptons given above, we will evaluate the value of $\tilde{b}_{ff}^{(\tilde{f})}$. Using (and quoting verbatim) the form of expression from [13]:

$$\begin{aligned}\tilde{b}_{ff}^{(\tilde{f})} = & \frac{1}{64\pi} \sum_{a,b} \frac{1}{\Delta_{fa}^2 \Delta_{fb}^2} \left\{ C_+^a C_+^b \left[8m_f^2 m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) \Delta_{fa}^2 \right. \right. \\ & - 4m_{\chi_3^0}^2 (m_f^4 + m_{\chi_3^0}^2 m_f^2 - 2m_{\chi_3^0}^4) \Delta_{fa} \Delta_{fb} \\ & \left. \left. + 4m_{\chi_3^0}^2 (m_f^2 + 2m_{\chi_3^0}^2) \Delta_{fa}^2 \Delta_{fb}^2 + 4(m_{\chi_3^0}^2 - m_f^2) \Delta_{fa}^2 \Delta_{fb}^2 \right] \right. \\ & + D_+^a D_+^b \left[8m_{\chi_3^0}^4 (m_{\chi_3^0}^2 - m_f^2) \Delta_{fa}^2 + 4m_{\chi_3^0}^2 (m_{\chi_3^0}^4 + m_{\chi_3^0}^2 m_f^2 - 2m_f^4) \Delta_{fa} \Delta_{fb} \right. \\ & \left. \left. + 4m_{\chi_3^0}^2 (5m_{\chi_3^0}^2 - 2m_f^2) \Delta_{fa}^2 \Delta_{fb}^2 - 3(m_f^2 - 3m_{\chi_3^0}^2) \Delta_{fa}^2 \Delta_{fb}^2 \right] \right. \\ & + C_-^a C_-^b \left[8m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2)^2 \Delta_{fa} \Delta_{fb} + 8m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) \Delta_{fa}^2 \Delta_{fb} \right. \\ & \left. \left. + 2(m_f^2 + 2m_{\chi_3^0}^2) \Delta_{fa}^2 \Delta_{fb}^2 \right] \right. \\ & + C_+^a D_+^b + 2m_f m_{\chi_3^0} \left[8m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) \Delta_{fa}^2 + 12m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) \Delta_{fa} \Delta_{fb} \right. \\ & \left. \left. + 3\Delta_{fa}^2 \Delta_{fb}^2 - 2(m_f^2 - 4m_{\chi_3^0}^2) \Delta_{fa} \Delta_{fb}^2 - 6(m_f^2 - 2m_{\chi_3^0}^2) \Delta_{fa}^2 \Delta_{fb} \right] \right\},\end{aligned}$$

for $m_{\chi_3^0} \sim \mathcal{V}^{-\frac{4}{3}} M_p$, $\Delta_{fa} \sim -m_{\chi_3^0}^2$, after solving, one gets:

$$\tilde{b}_{ff}^{(\tilde{f})} = \frac{1}{\pi} \times \frac{\tilde{f}^4 \mathcal{V}^{-2}}{m_{\chi_3^0}^2} \sim O(10)^{-48} GeV^{-2}. \quad (312)$$

• Higgs (h, H)–sfermion (\tilde{f}_a) interference term:

$$\tilde{a}_{ff}^{(h, H-\tilde{f})} = 0, \quad (313)$$

$$\begin{aligned}\tilde{b}_{ff}^{(h, H-\tilde{f})} = & -\frac{1}{8\pi} \sum_a Re \left[\sum_{r=h, H} \frac{C_S^{ffr} C_S^{\chi\chi r}}{4m_{\chi_3^0}^2 - m_r^2 + i\Gamma_r m_r} \right] \frac{(m_{\chi_3^0}^2 - m_f^2)}{\Delta_{fa}^2} \\ & \times \left[C_+^a + 2m_f m_{\chi_3^0} + D_+^a (2m_{\chi_3^0}^2 + 3\Delta_{fa}) \right];\end{aligned} \quad (314)$$

expanding summation

$$\begin{aligned}& \sim \frac{1}{8\pi} \left| \frac{C^{ffh} C^{\chi_3^0 \chi_3^0 h}}{4m_{\chi_3^0}^2 - m_h^2 + i\Gamma_h m_h} + \frac{C^{hhH} C^{\chi_3^0 \chi_3^0 H}}{4m_{\chi_3^0}^2 - m_H^2 + i\Gamma_H m_H} \right|^2 \frac{(m_{\chi_3^0}^2 - m_f^2)}{\Delta_{fa}^2} \\ & \times \left[C_+^a + 2m_f m_{\chi_3^0} + D_+^a (2m_{\chi_3^0}^2 + 3\Delta_{fa}) \right] \\ & \sim \frac{1}{\pi} \left| \frac{\mathcal{V}^{-\frac{23}{36}} \mathcal{V}^{-\frac{35}{36}} m_{pl}}{4\mathcal{V}^{-\frac{8}{3}} m_{pl}^2} + \frac{\mathcal{V}^{-\frac{23}{36}} \mathcal{V}^{-\frac{35}{36}} m_{pl}}{\mathcal{V}^{-\frac{85}{36}} m_{pl}^2} \right|^2 \mathcal{V}^{-\frac{13}{10}} \tilde{f}^2 \sim \frac{1}{\pi} \tilde{f}^2 \mathcal{V}^{-\frac{13}{10}} \left(\frac{\mathcal{V}^{\frac{19}{18}}}{m_{pl}} \right)^2 \\ & \sim O(10)^{-42} GeV^{-2} \text{ for } \mathcal{V} \sim 10^5.\end{aligned} \quad (315)$$

• Z-sfermion (\tilde{f}_a) interference term:

$$\tilde{a}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} = -\frac{1}{4\pi} \sum_a \text{Re} \left[\frac{C^{ffZ} C^{\chi_3^0 \chi_3^0 Z}}{4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z} \right] \frac{m_f (m_Z^2 - 4m_{\chi_3^0}^2)}{m_Z^2} \frac{(m_f C_+^a + m_{\chi_3^0} D_+^a)}{\Delta_{\tilde{f}_a}}; \quad (316)$$

utilizing the value of mass $m_Z = 90\text{GeV}$, $m_{\chi_3^0} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} \sim \mathcal{V}^{-\frac{4}{3}} M_p$; $C^{ffZ} \sim (\mathcal{V}^{-\frac{23}{36}})$, $C^{\chi_3^0 \chi_3^0 Z} \sim \tilde{f} \mathcal{V}^{-\frac{11}{18}}$ and equation no 310, after simplifying, we have

$$\begin{aligned} \tilde{a}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} &= -\frac{1}{4\pi} \left[\frac{\mathcal{V}^{-\frac{23}{36}} \tilde{f} \mathcal{V}^{-\frac{11}{18}}}{4m_{\chi_3^0}^2} \right] \frac{m_f (m_Z^2 - 4m_{\chi_3^0}^2)}{m_Z^2} \frac{(m_f C_+^a + m_{\chi_3^0} D_+^a)}{\Delta_{\tilde{f}_a}}, \\ &\sim O(10)^{-42} \text{GeV}^{-2} \end{aligned} \quad (317)$$

$$\begin{aligned} \tilde{b}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} &= -\frac{1}{8\pi} \sum_a \text{Re} \left[\left(\frac{C^{\chi_3^0 \chi_3^0 Z}}{(4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z)^2} \right) \frac{1}{m_Z^2 \Delta_{\tilde{f}_a}^3} \right. \\ &\times \left[C^{ffZ} C_-^a \left\{ 2m_Z^2 P_Z \Delta_{\tilde{f}_a} [2m_{\chi_3^0}^2 (m_{\chi_3^0}^2 + \Delta_{\tilde{f}_a}) + m_f^2 (-2m_{\chi_3^0}^2 + \Delta_{\tilde{f}_a})] \right\} \right. \\ &+ C^{ffZ} \left\{ C_+^a \left[2m_f^2 m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) (m_Z^2 - 4m_{\chi_3^0}^2) P_Z m_{\chi_3^0}^2 [m_f^2 m_Z^2 + 2m_{\chi_3^0}^2 (m_Z^2 - 6m_f^2)] P_Z \Delta_{\tilde{f}_a} \right. \right. \\ &+ 2m_Z \{-m_Z (m_{\chi_3^0}^2 - m_f^2) (m_Z^2 - 4m_{\chi_3^0}^2) + i\Gamma_Z [m_Z^2 m_{\chi_3^0}^2 - m_f^2 (m_Z^2 + 3m_{\chi_3^0}^2)] \} \Delta_{\tilde{f}_a}^2 \Big] \\ &+ m_f m_{\chi_3^0} D_+^a \left[4m_{\chi_3^0}^2 (m_{\chi_3^0}^2 - m_f^2) (m_Z^2 - 4m_{\chi_3^0}^2) P_Z + 2[6m_Z^2 m_{\chi_3^0}^2 - 16m_{\chi_3^0}^4 - m_f^2 (3m_Z^2 - 4m_{\chi_3^0}^2)] P_Z \Delta_{\tilde{f}_a} \right. \\ &\left. \left. \left. - 3[(m_Z^2 - 4m_{\chi_3^0}^2)^2 - i m_Z \Gamma_Z (m_Z^2 - 8m_{\chi_3^0}^2)] \Delta_{\tilde{f}_a}^2 \right] \right\} \right] \Bigg]; \end{aligned} \quad (318)$$

where $P_Z \equiv 4m_{\chi_3^0}^2 - m_Z^2 + i\Gamma_Z m_Z$. Again using the numerical values of masses and relevant couplings and assuming $\Gamma_Z m_Z \ll m_{\chi_3^0}^2$, the above expression reduces to

$$\tilde{b}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} = \mathcal{O}(10)^{-50} \text{GeV}^{-2}. \quad (319)$$

Utilizing the results from equation no (303), (306), (312), (315), (319) and (305), (317):

$$\tilde{b}_{\tilde{f}\tilde{f}} = \tilde{b}_{\tilde{f}\tilde{f}}^{(h,H)} + \tilde{b}_{\tilde{f}\tilde{f}}^{(Z)} + \tilde{b}_{\tilde{f}\tilde{f}}^{(\tilde{f})} + \tilde{b}_{\tilde{f}\tilde{f}}^{(h,H-\tilde{f})} + \tilde{b}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} \sim O(10)^{-42} \text{GeV}^{-2}; \quad (320)$$

$$\tilde{a}_{\tilde{f}\tilde{f}} = \tilde{a}_{\tilde{f}\tilde{f}}^{(h,H)} + \tilde{a}_{\tilde{f}\tilde{f}}^{(Z)} + \tilde{a}_{\tilde{f}\tilde{f}}^{(\tilde{f})} + \tilde{a}_{\tilde{f}\tilde{f}}^{(h,H-\tilde{f})} + \tilde{a}_{\tilde{f}\tilde{f}}^{(Z-\tilde{f})} \sim O(10)^{-42} \text{GeV}^{-2}. \quad (321)$$

Relative velocity $v_{f_1 f_2}$ is defined as

$$v_{f_1 f_2} \equiv \left[1 - \frac{(m_{f_1} + m_{f_2})^2}{4m_{\chi_3^0}^2} \right]^{1/2} \left[1 - \frac{(m_{f_1} - m_{f_2})^2}{4m_{\chi_3^0}^2} \right]^{1/2}. \quad (322)$$

For $f_1, f_2 = hh, v_{hh} \equiv 1$, $f_1, f_2 = ZZ, v_{ZZ} \equiv 1$, $f_1, f_2 = f\bar{f}, v_{f\bar{f}} \equiv 1$. Having estimated the partial wave coefficients for each possible annihilation processes, summing up their contribution as according to (243):

$$\begin{aligned} a &= \tilde{a}_{hh} + \tilde{a}_{ZZ} + \tilde{a}_{f\bar{f}} \equiv O(10)^{-29} GeV^{-2} \\ b &= \tilde{b}_{hh} + \tilde{b}_{ZZ} + \tilde{b}_{f\bar{f}} \equiv O(10)^{-10} GeV^{-2} \end{aligned} \quad (323)$$

$$J(x_f) \equiv \int_0^{x_f} dx \langle \sigma v_{M\phi} \rangle(x) = \int_0^{x_f} dx (a + bx_f) = ax_f + b \frac{x_f^2}{2} \quad (324)$$

where $x = T/m_\chi$. The value of x_f can be calculated by solving iteratively the equation

$$x_f^{-1} = \ln \left(\frac{m_\chi}{2\pi^3} \sqrt{\frac{45}{2g_* G_N}} \langle \sigma v_{M\phi} \rangle(x_f) x_f^{1/2} \right), \quad (325)$$

where g_* represents the effective number of degrees of freedom at freeze-out ($\sqrt{g_*} \simeq 9$). Solving above equation, $x_f \equiv T_f/m_{\chi_3^0}$ comes out to be around $1/33$

The analytical expression of relic abundance is given as [35]

$$\Omega_\chi h^2 = \frac{1}{\mu^2 \sqrt{g_*} J(x_F)} \quad (326)$$

where $\mu = 1.2 \times 10^5 GeV$ For $J(x_f) \sim a(x_f) + b \frac{x_f^2}{2} \sim 10^{-10} \frac{x_f^2}{2} GeV^{-2}$ and $x_f = \frac{1}{33}$, $\sqrt{g_*} = 9$,

$$\Omega_{\chi_3^0} h^2 \sim \frac{2 \cdot (33)^2}{1.44 \times 10^{10} \cdot 9 \cdot 10^{-10}} \equiv 168. \quad (327)$$

For $m_{\frac{3}{2}} \sim 10^8 GeV$ and $m_{\chi_3^0} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} \sim 10^{11} GeV$, relic abundance of gravitino is given as :

$$\Omega_{\tilde{G}} h^2 = \Omega_{\chi_3^0} h^2 \times \frac{m_{\frac{3}{2}}}{m_{\chi_3^0}} = \Omega_{\chi_3^0} \times \mathcal{V}^{-\frac{2}{3}} \sim 0.16, \quad (328)$$

clearly a very desirable value!

5.2 Slepton relic density calculations

For the case of slepton NLSPs, the dominant annihilation channel possible in our set up are: $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ZZ$, $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow Zh$, $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh$, $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma\gamma$, $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma h$, $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ll$.

The analytical expressions for $\tilde{w}(s)$ are given in [14]. Once again, the approach is to first calculate required vertices in the context of $\mathcal{N} = 1$ gauged supergravity and then utilize the same to calculate partial wave coefficients.

(a) $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ZZ$

Slepton-slepton- Higgs vertex

Expanding effective supergravity potential $V = \exp^K G^{TsTs} |D_{Ts} W|^2$ in the fluctuations around $\mathcal{Z}_i \rightarrow \mathcal{Z}_i + \mathcal{V}^{\frac{1}{36}} M_p$, $\mathcal{A}_1 \rightarrow \mathcal{A}_1 + \mathcal{V}^{-\frac{2}{9}} M_p$, contribution of term quadratic in \mathcal{A}_1 as well as \mathcal{Z}_i is of the order $\mathcal{V}^{\frac{-89}{36}} < \mathcal{Z}_i >$, which after giving VEV to one of the \mathcal{Z}_i , will be given as:

$$C^{\tilde{\ell}_a \tilde{\ell}_b h} \sim \frac{1}{\sqrt{(\hat{K}_{\mathcal{Z}_i \bar{\mathcal{Z}}_i})^2 (\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1})^2}} \left[\mathcal{V}^{\frac{-89}{36}} < \mathcal{Z}^i > \bar{\mathcal{Z}}^i \mathcal{A}^1 \mathcal{A}^{*1} \right] \sim O(\mathcal{V}^{\frac{-34}{15}}). \quad (329)$$

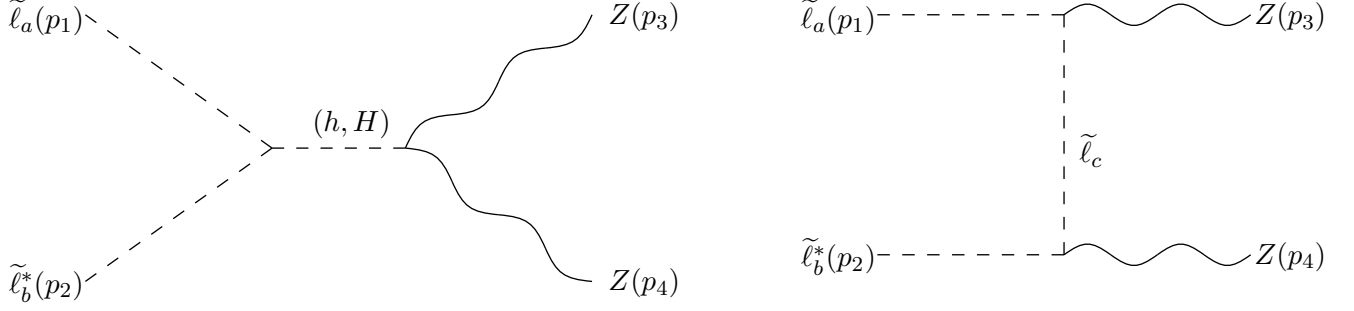


Figure 26: Feynman diagrams for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ZZ$ via s -channel Higgs exchange and t -channel $\tilde{\ell}_c$ exchange.

Similarly

$$C^{\tilde{\ell}_a \tilde{\ell}_b hh} \sim \frac{1}{\sqrt{(\hat{K}_{\tilde{Z}_i \tilde{Z}_i})^2 (\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1})^2}} \left[\mathcal{V}^{-\frac{89}{36}} \tilde{Z}^i \tilde{Z}^j \mathcal{A}^1 \mathcal{A}^{*1} \right] \sim O(\mathcal{V}^{-\frac{41}{18}}). \quad (330)$$

Slepton-slepton-Z Boson- Z Boson vertex

In the context of supergravity action, contribution of required vertex will be given by: $\bar{\partial}_{\bar{\mathcal{A}}_1} \partial_{\mathcal{A}_1} G_{T_B \bar{T}_B} X^{T_B} X^{\bar{T}_B} A^\mu A_\nu$. As given in appendix **B**

$$\bar{\partial}_{\bar{\mathcal{A}}_1} \partial_{\mathcal{A}_1} G_{T_B \bar{T}_B} \sim O(1) \bar{\partial}_{\bar{a}_1} \partial_{a_1} G_{T_B \bar{T}_B} \sim \mathcal{V}^{-\frac{8}{9}} \mathcal{A}_1^* \mathcal{A}_1, \quad (331)$$

$$X = X^B \partial_B = -12i\pi\alpha' \kappa_4^2 \mu_7 Q_B \partial_{T_B} \sim \mathcal{V}^{-\frac{2}{3}}.$$

Incorporating values from above, the physical Slepton-slepton-Z Boson- Z Boson vertex is proportional to

$$C^{\tilde{\ell}_a \tilde{\ell}_b^* ZZ} \sim \frac{\mathcal{V}^{-\frac{8}{9}} \tilde{f}^2 \mathcal{V}^{-\frac{4}{3}}}{\sqrt{(K_{\mathcal{A}_1 \bar{\mathcal{A}}_1})^2}} \sim \frac{\tilde{f}^2 \mathcal{V}^{-\frac{20}{9}}}{O(10)^4} \sim \tilde{f}^2 \mathcal{V}^{-3}. \quad (332)$$

Slepton- slepton- Z Boson vertex

The gauge kinetic term for slepton-slepton- Z Boson vertex, relevant to the second Feynman graph in Fig.26 will be given by $\frac{e^K}{\kappa_4^2} G^{T_B \bar{T}_B} \tilde{\nabla}_\mu T_B \tilde{\nabla}^\mu \bar{T}_B$. This implies that the following term generates the required squark-squark-gauge boson vertex:

$$\begin{aligned} & \frac{6i\kappa_4^2 \mu_7 2\pi\alpha' Q_B G^{T_B \bar{T}_B}}{\kappa_4^2 \mathcal{V}^2} \kappa_4^2 A^\mu \partial_\mu (\kappa_4^2 \mu_7 (2\pi\alpha')^2 C_{1\bar{1}} \mathcal{A}_1 \bar{\mathcal{A}}_{\bar{1}}) \xrightarrow[G_{T_B \bar{T}_B \sim \mathcal{V}^{\frac{4}{3}}]{\kappa_4^2 \mu_7 (2\pi\alpha')^2 C_{1\bar{1}} \sim \mathcal{V}^{\frac{10}{9}}} \frac{\mathcal{V}^{\frac{7}{9}}}{\left(\sqrt{\hat{K}_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}\right)^2} \\ & \sim O(10^{-4}) \tilde{f} \mathcal{V}^{\frac{7}{9}} \sim \tilde{f}, \text{ for } \mathcal{V} \sim 10^5 \end{aligned} \quad (333)$$

Quoting directly the analytical expressions given in [14], we obtain a numerical estimate of \tilde{w} :

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ZZ} = \tilde{w}_{ZZ}^{(h,H,P)} + \tilde{w}_{ZZ}^{(\tilde{\ell})} + \tilde{w}_{ZZ}^{(h,H,P-\tilde{\ell})}. \quad (334)$$

- Higgs (h, H) exchange (+ Point interaction):

$$\begin{aligned} \tilde{w}_{ZZ}^{(h,H,P)} &= \left| \sum_{r=h,H} \frac{C^{ZZr} C^{\tilde{\ell}_b^* \tilde{\ell}_a r}}{s - m_h^2 + i\Gamma_h m_h} - C^{\tilde{\ell}_b^* \tilde{\ell}_a ZZ} \right|^2 \frac{s^2 - 4m_Z^2 s + 12m_Z^4}{8m_Z^4} \\ &\sim \left| \frac{\tilde{f}^2 \mathcal{V}^{-\frac{23}{5}} m_{pl}^2}{s - m_r^2 + i\Gamma_r m_r} + \frac{\tilde{f}^2 \mathcal{V}^{-\frac{23}{5}} m_{pl}^2}{s - m_H^2 + i\Gamma_H m_H} - \tilde{f}^2 \mathcal{V}^{-3} \right|^2 \frac{s^2 - 4m_Z^2 s + 12m_Z^4}{8m_Z^4}; \end{aligned} \quad (335)$$

- slepton ($\tilde{\ell}_c$) exchange:

$$\begin{aligned}
\tilde{w}_{ZZ}^{(\tilde{\ell})} &= \frac{1}{m_Z^4} \sum_{c,d=1}^2 C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} C^{\tilde{\ell}_d^* \tilde{\ell}_a Z^*} \\
&\times \left[\mathcal{T}_4 - 2(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 + 2m_Z^2) \mathcal{T}_3 + [m_{\tilde{\ell}_a}^4 + m_{\tilde{\ell}_b}^4 + 4m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 + 2m_Z^2(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 + 3m_Z^2)] \mathcal{T}_2 \right. \\
&- 2[(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2)(m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 - m_Z^4) + m_Z^2(m_{\tilde{\ell}_a}^4 + m_{\tilde{\ell}_b}^4 - 4m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 + 2m_Z^4)] \mathcal{T}_1 \\
&+ (m_{\tilde{\ell}_a}^2 - m_Z^2)^2 (m_{\tilde{\ell}_b}^2 - m_Z^2)^2 \mathcal{T}_0 - \mathcal{Y}_4 + [s(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 2m_Z^2) - 2(m_{\tilde{\ell}_a}^2 - m_Z^2)(m_{\tilde{\ell}_b}^2 - m_Z^2)] \mathcal{Y}_2 \\
&- [s^2(m_{\tilde{\ell}_a}^2 - m_Z^2)(m_{\tilde{\ell}_b}^2 - m_Z^2) + s\{-m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 (m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2) + 3m_Z^2(m_{\tilde{\ell}_a}^4 + m_{\tilde{\ell}_b}^4) - 3m_Z^4(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2) + 2m_Z^6\} \\
&\left. + (m_{\tilde{\ell}_a}^2 - m_Z^2)^2 (m_{\tilde{\ell}_b}^2 - m_Z^2)^2] \mathcal{Y}_0 \right]. \tag{336}
\end{aligned}$$

Considering only first-generation squarks,

$$\sum_{c,d=1}^2 C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} C^{\tilde{\ell}_d^* \tilde{\ell}_a Z^*} \sim \tilde{f}^4 \tag{337}$$

(336) yields:

$$\begin{aligned}
\tilde{w}_{ZZ}^{(\tilde{\ell})} &= \frac{\tilde{f}^4}{m_Z^4} \left[\mathcal{T}_4 - 4m_{\tilde{\ell}_a}^2 \mathcal{T}_3 + 6m_{\tilde{\ell}_a}^4 \mathcal{T}_2 - 4m_{\tilde{\ell}_a}^6 \mathcal{T}_1 + m_{\tilde{\ell}_a}^8 \mathcal{T}_0 - \mathcal{Y}_4 + [2sm_{\tilde{\ell}_a}^2 - 2m_{\tilde{\ell}_a}^4] \mathcal{Y}_2 - \right. \\
&\left. [s^2 m_{\tilde{\ell}_a}^4 - 2sm_{\tilde{\ell}_a}^6 + m_{\tilde{\ell}_a}^8] \mathcal{Y}_0 \right]; \tag{338}
\end{aligned}$$

$\mathcal{T}_4, \mathcal{T}_3, \mathcal{Y}_0 \dots$ etc are auxiliary functions defined in the appendix of [14]. Given that $m_{\tilde{\ell}_a}^2 \sim \mathcal{V}^{-3} M_p, m_Z \sim 90 \text{ GeV}$, on solving and simplifying the same, we get

$$\begin{aligned}
\mathcal{T}_0 &= \frac{1}{-\mathcal{V}^{\frac{42}{5}} - 4\mathcal{V}^5 + (\mathcal{V}^{\frac{21}{5}} + s) \mathcal{V}^{\frac{21}{5}} + \mathcal{V}^{\frac{8}{5}}} \\
\mathcal{T}_1 &\equiv \frac{\sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} \mathcal{V}^{\frac{13}{5}} + (-4\mathcal{V}^{\frac{17}{5}} + s\mathcal{V}^{\frac{13}{5}} + 1) \log \left(\frac{s - 2\mathcal{V}^{\frac{4}{5}} - \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - 2\mathcal{V}^{\frac{4}{5}} + \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right)}{\sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} (-4\mathcal{V}^{\frac{17}{5}} + s\mathcal{V}^{\frac{13}{5}} + 1) \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}; \\
\mathcal{T}_2 &\equiv - \frac{4 \left(\sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} (\mathcal{V}^{\frac{34}{5}} + s\mathcal{V}^{\frac{13}{5}}) \mathcal{V}^{\frac{8}{5}} + (-8\mathcal{V}^{\frac{46}{5}} + 2s\mathcal{V}^{\frac{42}{5}}) \log \left(\frac{s - 2\mathcal{V}^{\frac{4}{5}} - \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - 2\mathcal{V}^{\frac{4}{5}} + \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right) \right)}{\sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} (16\mathcal{V}^5 - 4s\mathcal{V}^{\frac{21}{5}} - 4\mathcal{V}^{\frac{8}{5}})}; \\
\mathcal{T}_3 &\equiv \frac{2 \left(6 (4\mathcal{V}^{\frac{17}{5}} - s\mathcal{V}^{\frac{13}{5}}) \log \left(\frac{s - 2\mathcal{V}^{\frac{4}{5}} - \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - 2\mathcal{V}^{\frac{4}{5}} + \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right) \mathcal{V}^{10} + \sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} (-2\mathcal{V}^{11} - 6s\mathcal{V}^{\frac{34}{5}} + s^2\mathcal{V}^{\frac{13}{5}}) \mathcal{V}^{\frac{8}{5}} \right)}{\sqrt{s - 4\mathcal{V}^{\frac{4}{5}}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} (16\mathcal{V}^5 - 4s\mathcal{V}^{\frac{21}{5}} - 4\mathcal{V}^{\frac{8}{5}})};
\end{aligned}$$

$$\mathcal{T}_4 \equiv \frac{4 \log \left(\frac{s-2\mathcal{V}^{\frac{4}{5}} - \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}}{s-2\mathcal{V}^{\frac{4}{5}} + \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} \right) \mathcal{V}^{\frac{63}{5}}}{\sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} + \frac{1}{3} \left(\frac{3\mathcal{V}^{\frac{76}{5}}}{-4\mathcal{V}^{\frac{17}{5}} + s\mathcal{V}^{13/5} + 1} + 18\mathcal{V}^{42/5} - 7s\mathcal{V}^{17/5}\mathcal{V}^{4/5} + s^2 \right);$$

$$\mathcal{Y}_0 \equiv \frac{2 \log \left(\frac{s-2\mathcal{V}^{\frac{4}{5}} - \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}}{s-2\mathcal{V}^{\frac{4}{5}} + \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} \right)}{\sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \left(s-2\mathcal{V}^{\frac{4}{5}} \right) \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}};$$

$$\mathcal{Y}_2 \equiv \frac{2 \left(s - \mathcal{V}^{\frac{4}{5}} \left(\mathcal{V}^{\frac{17}{5}} + 2 \right) \right) \log \left(\frac{s-2\mathcal{V}^{\frac{4}{5}} - \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}}{s-2\mathcal{V}^{\frac{4}{5}} + \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} \right) \mathcal{V}^{\frac{21}{5}}}{\sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \left(s-2\mathcal{V}^{\frac{4}{5}} \right) \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} + 1;$$

$$\mathcal{Y}_4 \equiv \frac{1}{6} \left(-s^2 + 2\mathcal{V}^{\frac{4}{5}} \left(5\mathcal{V}^{\frac{17}{5}} + 2 \right) s - 2 \left(6\mathcal{V}^{\frac{42}{5}} + 8\mathcal{V}^5 + 3\mathcal{V}^{\frac{8}{5}} \right) \right).$$

- Higgs (h, H) (+ Point) – slepton ($\tilde{\ell}_c$) interference:

$$\begin{aligned} \tilde{w}_{ZZ}^{(h,H,P-\tilde{\ell})} &= \frac{1}{2m_Z^4} \sum_{c=1}^2 \text{Re} \left[\left(\sum_{r=h,H} \frac{C^{ZZr} C^{\tilde{\ell}_b^* \tilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\tilde{\ell}_b^* \tilde{\ell}_a ZZ} \right)^* C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} \right] \\ &\times \left[s^2 + s(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 2m_{\tilde{\ell}_c}^2 - 4m_Z^2) + 2m_Z^2(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 2m_{\tilde{\ell}_c}^2 + 2m_Z^2) \right. \\ &- 2[s(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_c}^2 - m_Z^2)(m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}_c}^2 - m_Z^2) \\ &+ 2m_Z^2\{m_{\tilde{\ell}_a}^4 + m_{\tilde{\ell}_b}^4 - m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 + m_{\tilde{\ell}_c}^2(m_{\tilde{\ell}_c}^2 - m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 - 2m_Z^2) \\ &\left. - m_Z^2(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - m_Z^2)\}]\mathcal{F} \right]. \end{aligned} \quad (339)$$

Using the universality in squark masses as obtained in Appendix B, the simplification leads to:

$$\begin{aligned} \tilde{w}_{ZZ}^{(h,H,P-\tilde{\ell})} &\equiv \frac{1}{2m_Z^4} \text{Re} \left[\left(\sum_{r=h,H} \frac{\tilde{f}^2 \mathcal{V}^{-\frac{23}{5}} m_{pl}^2}{s - m_r^2 + i\Gamma_r m_r} - \tilde{f}^2 \mathcal{V}^{-3} \right)^* \tilde{f}^2 \right] \\ &\times \left[(s^2 - 4sm_Z^2 + 4m_Z^4) - 2(sm_Z^4 - 8m_Z^4 m_{\tilde{\ell}_a}^2) \mathcal{F} \right] \end{aligned} \quad (340)$$

where

$$\mathcal{F} \equiv \frac{\log \left(\frac{s-2\mathcal{V}^{\frac{4}{5}} - \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}}{s-2\mathcal{V}^{\frac{4}{5}} + \sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}} \right)}{\sqrt{s-4\mathcal{V}^{\frac{4}{5}}} \sqrt{s-4\mathcal{V}^{\frac{21}{5}}}}.$$

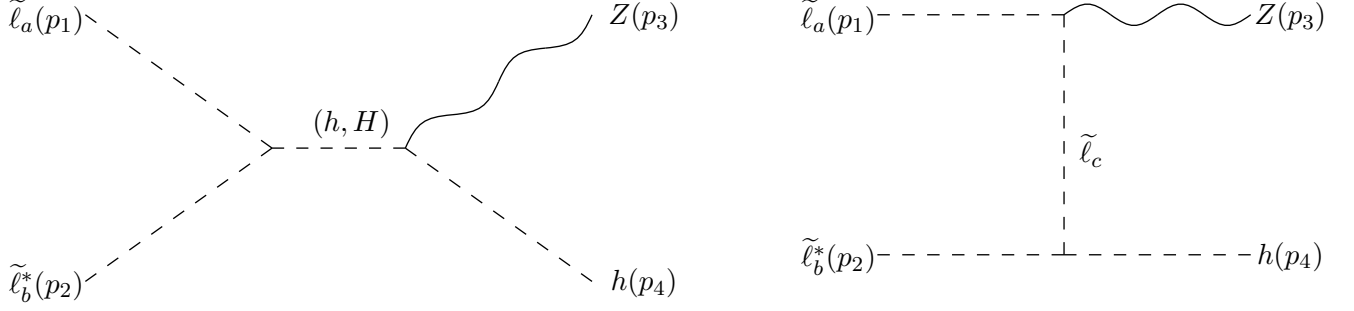


Figure 27: Feynman diagrams for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow Zh$ via s -channel Higgs exchange and t -channel $\tilde{\ell}_c$ exchange.

Summing up the contribution of \tilde{w} for all s , t and u -channels as according to equation (334):

$$\tilde{w}_{ZZ}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{\frac{122}{45}}, \tilde{w}'_{ZZ}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{\frac{122}{45}}. \quad (341)$$

The “reduced” coefficients \tilde{a}_{ZZ} and \tilde{b}_{ZZ} will be given by ⁵

$$\tilde{a}_{ZZ} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{\frac{122}{45}} \sim 10^{-9} GeV^{-2}; \tilde{b}_{ZZ} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{\frac{122}{45}} \sim 10^{-9} GeV^{-2}. \quad (342)$$

(b) $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow Zh$

The process $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow Zh$ involves the s -channel Z -boson exchange, and the t - and u -channel slepton ($\tilde{\ell}_a$, $a = 1, 2$) exchange:

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow Zh} = \tilde{w}_{Zh}^{(Z)} + \tilde{w}_{Zh}^{(\tilde{\ell})} + \tilde{w}_{Zh}^{(Z-\tilde{\ell})}. \quad (343)$$

• Z exchange:

$$\begin{aligned} \tilde{w}_{Zh}^{(Z)} = & \frac{1}{12m_Z^6} \left| \frac{C^{ZZh} C^{\tilde{\ell}_b^* \tilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \\ & \times \left[s^2 \left\{ 3(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 + m_Z^4 \right\} - 2s \left\{ 3(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 (m_h^2 + 2m_Z^2) + m_Z^4 (m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 + m_h^2 - 5m_Z^2) \right\} \right. \\ & + m_Z^4 \left\{ (m_h^2 - m_Z^2)^2 + 4(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2)(m_h^2 - 5m_Z^2) \right\} + (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 (3m_h^4 + 6m_Z^2 m_h^2 + 19m_Z^4) \\ & \left. - \frac{2}{s} m_Z^2 \left\{ (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 (3m_h^4 - 2m_Z^2 m_h^2 + m_Z^4) + m_Z^2 (m_h^2 - m_Z^2)^2 (m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2) \right\} + \frac{4}{s^2} m_Z^4 (m_h^2 - m_Z^2)^2 (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 \right]. \end{aligned} \quad (344)$$

For $C^{ZZh} \sim \tilde{f}^2 \mathcal{V}^{-\frac{7}{3}}$, $C^{\tilde{\ell}_b^* \tilde{\ell}_a Z} \sim \tilde{f}$ and assuming universality in slepton masses; after simplification, above expression reduces to:

$$\tilde{w}_{Zh}^{(Z)} = \frac{1}{12m_Z^6} \left| \frac{\tilde{f}^3 \mathcal{V}^{-\frac{7}{3}}}{s - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \times \left[s^2 m_Z^4 - 4s m_{\tilde{\ell}_a}^2 m_Z^4 + 8m_Z^4 m_{\tilde{\ell}_a}^2 m_h^2 - \frac{2}{s} m_Z^4 m_{\tilde{\ell}_a}^2 m_h^4 \right]. \quad (345)$$

⁵Here, we are considering: $\tilde{f} \sim 10^{-5}$

• slepton ($\tilde{\ell}_c$) exchange:

$$\begin{aligned}
\tilde{w}_{Zh}^{(\tilde{\ell})} &= \frac{1}{m_Z^2} \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} C^{\tilde{\ell}_d^* \tilde{\ell}_a Z^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d h^*} \left[\mathcal{T}_2^t - 2(m_{\tilde{\ell}_a}^2 + m_Z^2) \mathcal{T}_1^t + (m_{\tilde{\ell}_a}^2 - m_Z^2)^2 \mathcal{T}_0^t \right] \\
&+ \frac{1}{m_Z^2} \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} \left[\mathcal{T}_2^u - 2(m_{\tilde{\ell}_b}^2 + m_Z^2) \mathcal{T}_1^u + (m_{\tilde{\ell}_b}^2 - m_Z^2)^2 \mathcal{T}_0^u \right] \\
&+ \frac{1}{m_Z^2} \text{Re} \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} \\
&\times \left[-2\mathcal{Y}_2 + \frac{1}{s}(s - m_h^2 + m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) \mathcal{Y}_1 + \left\{ s(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 2m_Z^2) - (m_{\tilde{\ell}_a}^4 + m_{\tilde{\ell}_b}^4) - 2m_Z^2 m_h^2 \right. \right. \\
&\left. \left. + (3m_Z^2 - m_h^2)(m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2) + \frac{1}{s}(m_h^2 - m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 - \frac{1}{2s^2}(m_h^2 - m_Z^2)^2(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2)^2 \right\} \mathcal{Y}_0 \right]. \quad (346)
\end{aligned}$$

Assuming universality in slepton masses of first two generations

$$\sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} \sim \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} \sim \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d Z^*} \equiv \tilde{f}^2 \mathcal{V}^{-\frac{68}{15}} M_p^2.$$

Therefore, (346) reduces to:

$$\tilde{w}_{Zh}^{(\tilde{\ell})} \sim \frac{\tilde{f}^2 \mathcal{V}^{-\frac{68}{15}} m_{pl}^2}{m_Z^2} \times \left[(\mathcal{T}_2^t + \mathcal{T}_2^u) - m_{\tilde{\ell}_a}^2 (\mathcal{T}_1^t + \mathcal{T}_1^u) + m_{\tilde{\ell}_a}^4 (\mathcal{T}_0^t + \mathcal{T}_0^u) - 2\mathcal{Y}_2 + (2s m s l i^2 - 2m_{\tilde{\ell}_a}^4) \mathcal{Y}_0 \right]. \quad (347)$$

• Z - slepton ($\tilde{\ell}_c$) interference:

$$\begin{aligned}
\tilde{w}_{Zh}^{(Z-\tilde{\ell})} &= \frac{1}{m_Z^4} \text{Re} \sum_{c=1}^2 \left(\frac{C^{ZZh} C^{\tilde{\ell}_b^* \tilde{\ell}_a Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right)^* \left[C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} \left\{ - (s - m_h^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) \right. \right. \\
&+ 2m_Z^2(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_c}^2 + m_Z^2) - \frac{1}{s} m_Z^2(m_h^2 - m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) + \left[s(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 + m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_c}^2 - m_Z^2) \right. \\
&+ m_Z^2 \{ -2m_{\tilde{\ell}_c}^4 + m_{\tilde{\ell}_c}^2(3m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 + 3m_Z^2) - 3m_{\tilde{\ell}_a}^4 \\
&+ 3m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 - 2m_{\tilde{\ell}_b}^4 + m_Z^2(4m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2) - m_Z^4 \} - m_h^2(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 + m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_c}^2 + m_Z^2) \left. \right] \mathcal{F}^t \left. \right\} \\
&+ C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c Z} \left\{ (s - m_h^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) + 2m_Z^2(m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}_c}^2 + m_Z^2) + \frac{1}{s} m_Z^2(m_h^2 - m_Z^2)(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) \right. \\
&+ \left[-s(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 - m_Z^2)(m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}_c}^2 - m_Z^2) + m_Z^2 \{ -2m_{\tilde{\ell}_c}^4 + m_{\tilde{\ell}_c}^2(3m_{\tilde{\ell}_b}^2 + m_{\tilde{\ell}_a}^2 + 3m_Z^2) - 3m_{\tilde{\ell}_b}^4 + 3m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 \right. \\
&\left. \left. - 2m_{\tilde{\ell}_a}^4 + m_Z^2(4m_{\tilde{\ell}_b}^2 + m_{\tilde{\ell}_a}^2) - m_Z^4 \} + m_h^2(m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 - m_Z^2)(m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}_c}^2 + m_Z^2) \right] \mathcal{F}^u \left. \right\} \left. \right]. \quad (348)
\end{aligned}$$

For $C^{ZZh} \sim \tilde{f}^2 \mathcal{V}^{-\frac{7}{3}}$, $C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} \sim \tilde{f}$, $C^{\tilde{\ell}_b^* \tilde{\ell}_c h} \sim \mathcal{V}^{-\frac{34}{15}}$, after simplification, (348) reduces to

$$\tilde{w}_{Zh}^{(Z-\tilde{\ell})} \equiv \frac{1}{m_Z^4} \text{Re} \sum_{c=1}^2 \left(\frac{\tilde{f}^3 \mathcal{V}^{-\frac{7}{3}} M_p}{s - m_Z^2 + i\Gamma_Z m_Z} \right)^* C^{\tilde{\ell}_c^* \tilde{\ell}_a Z} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} \left[4m_Z^4 + (-2sm_Z^4 + m_{\tilde{\ell}_a}^2 m_Z^4) (\mathcal{F}^t + \mathcal{F}^u) \right]$$

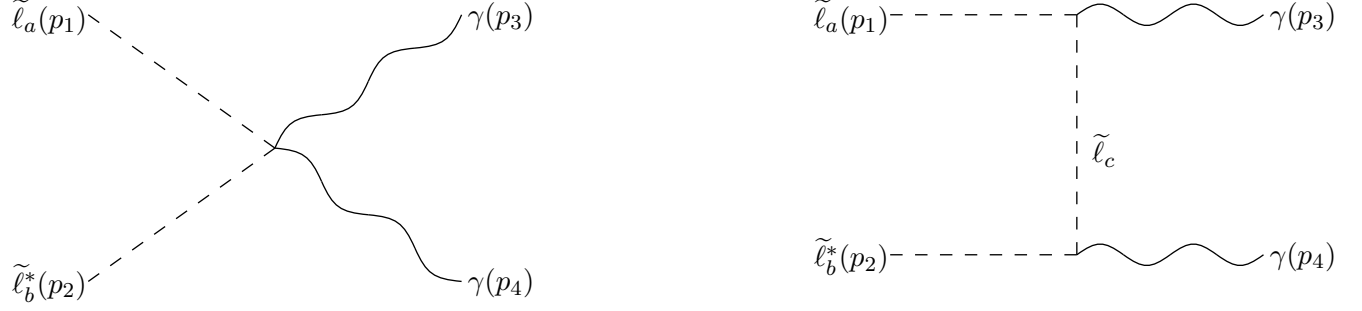


Figure 28: Feynman diagrams for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma\gamma$ via point interaction and t-channel $\tilde{\ell}_c^*$ exchange.

where

$$\mathcal{F}^t \sim \mathcal{F}^u \equiv \frac{3 \log \left(\frac{3s-8V^{\frac{4}{5}} - \sqrt{\frac{3s+2(-4+\sqrt{15})V^{\frac{4}{5}}}{s}} \sqrt{3s-2(4+\sqrt{15})V^{\frac{4}{5}}} \sqrt{s-4V^{\frac{21}{5}}} }{3s-8V^{\frac{4}{5}} + \sqrt{\frac{3s+2(-4+\sqrt{15})V^{\frac{4}{5}}}{s}} \sqrt{3s-2(4+\sqrt{15})V^{\frac{4}{5}}} \sqrt{s-4V^{\frac{21}{5}}} } \right)}{\sqrt{\frac{3s+2(-4+\sqrt{15})V^{\frac{4}{5}}}{s}} \sqrt{3s-2(4+\sqrt{15})V^{\frac{4}{5}}} \sqrt{s-4V^{\frac{21}{5}}}}.$$

Summing up the contribution of \tilde{w} for all s, t and u-channels as according to equation (348)

$$\tilde{w}_{Zh}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{\frac{4}{15}}, \tilde{w}'_{Zh}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{\frac{4}{15}}. \quad (349)$$

The “reduced” coefficients \tilde{a}_{Zh} and \tilde{b}_{Zh} will be given by

$$\tilde{a}_{Zh} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{\frac{4}{15}} \sim 10^{-22} GeV^{-2}; \tilde{b}_{Zh} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{\frac{4}{15}} \sim 10^{-22} GeV^{-2}. \quad (350)$$

(c) $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma\gamma$

The four-point contact interaction needs to be included along with the t - and u -channel slepton ($\tilde{\ell}_a$, $a = 1, 2$) exchange:

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma\gamma} = \tilde{w}_{\gamma\gamma}^{(P)} + \tilde{w}_{\gamma\gamma}^{(\tilde{\ell})} + \tilde{w}_{\gamma\gamma}^{(P-\tilde{\ell})}.$$

- Contact interaction:

$$\tilde{w}_{\gamma\gamma}^{(P)} \sim (C^{\tilde{\ell}_a \tilde{\ell}_b^* \gamma\gamma})^2 \sim e^4 (\tilde{f}^2 \mathcal{V}^{-3})^2 \delta_{ab} \quad (351)$$

- slepton ($\tilde{\ell}_a$) exchange:

$$\begin{aligned} \tilde{w}_{\gamma\gamma}^{(\tilde{\ell})} &= (C^{\tilde{\ell}_a \tilde{\ell}_b^* \gamma})^4 \delta_{ab} \left[4(\mathcal{T}_2 + 2m_{\tilde{\ell}_a}^2 \mathcal{T}_1 + m_{\tilde{\ell}_a}^4 \mathcal{T}_0) - (s - 4m_{\tilde{\ell}_a}^2)^2 \mathcal{Y}_0 \right] \\ &\sim \tilde{f}^4 \left[4(\mathcal{T}_2 + 2m_{\tilde{\ell}_a}^2 \mathcal{T}_1 + m_{\tilde{\ell}_a}^4 \mathcal{T}_0) - (s - 4m_{\tilde{\ell}_a}^2)^2 \mathcal{Y}_0 \right] \end{aligned} \quad (352)$$

where

$$\mathcal{T}_0 \equiv \frac{1}{V^{21/5} (V^{21/5} + s) - V^{42/5}}; \mathcal{T}_1 \equiv \frac{\log \left(\frac{\sqrt{s} - \sqrt{s-4V^{21/5}}}{\sqrt{s} + \sqrt{s-4V^{21/5}}} \right)}{\sqrt{s} \sqrt{s-4V^{21/5}}} + \frac{1}{s}$$

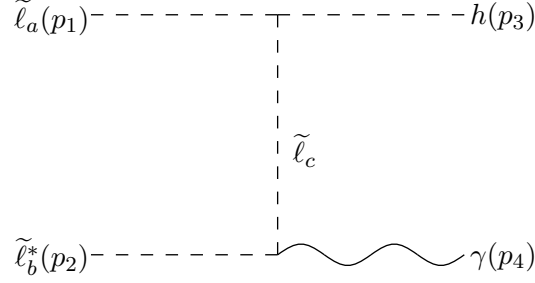


Figure 29: Feynman diagrams for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow h \gamma$ via t-channel $\tilde{\ell}_c$ exchange.

$$\mathcal{T}_2 \equiv \frac{2 \log \left(\frac{\sqrt{s} - \sqrt{s - 4V^{21/5}}}{\sqrt{s} + \sqrt{s - 4V^{21/5}}} \right) V^{21/5}}{\sqrt{s} \sqrt{s - 4V^{21/5}}} + \frac{V^{21/5}}{s} + 1; \mathcal{Y}_0 \equiv \frac{2 \log \left(\frac{\sqrt{s} - \sqrt{s - 4V^{21/5}}}{\sqrt{s} + \sqrt{s - 4V^{21/5}}} \right)}{s^{3/2} \sqrt{s - 4V^{21/5}}}$$

• Contact – slepton ($\tilde{\ell}_c$) interference:

$$\tilde{w}_{\gamma\gamma}^{(P-\tilde{\ell})} = 2C_{\tilde{\ell}_a \tilde{\ell}_b^* \gamma\gamma} (C_{\tilde{\ell}_a \tilde{\ell}_b^* \gamma})^2 e^4 \delta_{ab} \left[-4 + (s - 8m_{\tilde{\ell}_a}^2) \mathcal{F} \right] \sim (\tilde{f}^4 \mathcal{V}^{-5}) e^4 \delta_{ab} \left[-4 + (s - 8m_{\tilde{\ell}_a}^2) \mathcal{F} \right] \quad (353)$$

and

$$\mathcal{F} \equiv \frac{\log \left(\frac{\frac{1}{2}\sqrt{s} \sqrt{s - 4V^{21/5}} - \frac{s}{2}}{-\frac{s}{2} - \frac{1}{2}\sqrt{s - 4V^{21/5}} \sqrt{s}} \right)}{\sqrt{s} \sqrt{s - 4V^{21/5}}} \quad (354)$$

Summing up the contribution of \tilde{w} for all channels: (351)

$$\tilde{w}_{\gamma\gamma}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}, \tilde{w}'_{\gamma\gamma}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}, \quad (355)$$

the “reduced” coefficients $\tilde{a}_{\gamma\gamma}$ and $\tilde{b}_{\gamma\gamma}$ will be given by

$$\tilde{a}_{\gamma\gamma} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V} \sim 10^{-18} GeV^{-2}; \tilde{b}_{\gamma\gamma} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V} \sim 10^{-18} GeV^{-2}. \quad (356)$$

(c) $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma h$

• slepton ($\tilde{\ell}_c$) exchange:

With

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow \gamma h} = \tilde{w}_{\gamma h}^{(\tilde{\ell})} : \quad (357)$$

$$\begin{aligned} \tilde{w}_{\gamma h}^{(\tilde{\ell})} &= -2 \left| C_{\tilde{\ell}_b^* \tilde{\ell}_a \gamma} \right|^2 \left| C_{\tilde{\ell}_b^* \tilde{\ell}_a h} \right|^2 \left[(\mathcal{T}_1^t + m_{\tilde{\ell}_a}^2 \mathcal{T}_0^t) + (\mathcal{T}_1^u + m_{\tilde{\ell}_b}^2 \mathcal{T}_0^u) \right. \\ &\quad \left. + (s + m_h^2 - 2m_{\tilde{\ell}_a}^2 - 2m_{\tilde{\ell}_b}^2) \mathcal{Y}_0 \right] \\ &\sim (\tilde{f} \mathcal{V}^{-34/15})^2 \left[(\mathcal{T}_1^t + m_{\tilde{\ell}_a}^2 \mathcal{T}_0^t) + (\mathcal{T}_1^u + m_{\tilde{\ell}_b}^2 \mathcal{T}_0^u) + (s + m_h^2 - 4m_{\tilde{\ell}_a}^2) \mathcal{Y}_0 \right] \end{aligned}$$

where

$$\begin{aligned}
\mathcal{T}_0^t &\equiv \mathcal{V} \frac{s}{s\mathcal{V}^{\frac{42}{5}} + s \left(\mathcal{V}^{21/5} - \mathcal{V}^{\frac{4}{5}} + s \right) \mathcal{V}^{\frac{21}{5}} + \left(-2s\mathcal{V}^{\frac{21}{5}} + \mathcal{V}^{\frac{8}{5}} - s\mathcal{V}^{\frac{4}{5}} \right) \mathcal{V}^{21/5}}, \\
\mathcal{T}_0^u &\equiv \mathcal{V} \frac{s}{s\mathcal{V}^{\frac{42}{5}} + s \left(\mathcal{V}^{\frac{21}{5}} - \mathcal{V}^{\frac{4}{5}} + s \right) \mathcal{V}^{\frac{21}{5}} + \left(-2s\mathcal{V}^{\frac{21}{5}} + \mathcal{V}^{\frac{8}{5}} - s\mathcal{V}^{\frac{4}{5}} \right) \mathcal{V}^{\frac{21}{5}}}, \\
\mathcal{T}_1^t &\equiv \mathcal{V} \frac{\log \left(\frac{s - \mathcal{V}^{\frac{4}{5}} - \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - \mathcal{V}^{\frac{4}{5}} + \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right) \left(s - \mathcal{V}^{\frac{4}{5}} \right)^2 + s \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} \sqrt{s - \mathcal{V}^{\frac{4}{5}}}}{\left(s - \mathcal{V}^{\frac{4}{5}} \right)^{5/2} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}, \\
\mathcal{T}_1^u &\equiv \mathcal{V} \frac{\log \left(\frac{s - \mathcal{V}^{\frac{4}{5}} - \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - \mathcal{V}^{\frac{4}{5}} + \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right) \left(s - \mathcal{V}^{\frac{4}{5}} \right)^2 + s \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} \sqrt{s - \mathcal{V}^{\frac{4}{5}}}}{\left(s - \mathcal{V}^{\frac{4}{5}} \right)^{5/2} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}, \\
\mathcal{Y}_0 &\equiv \mathcal{V} \frac{2 \log \left(\frac{s - \mathcal{V}^{\frac{4}{5}} - \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{s - \mathcal{V}^{\frac{4}{5}} + \sqrt{s - \mathcal{V}^{\frac{4}{5}}} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right)}{\left(s - \mathcal{V}^{\frac{4}{5}} \right)^{3/2} \sqrt{1 - \frac{\mathcal{V}^{\frac{4}{5}}}{s}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}.
\end{aligned}$$

For $s = 4m_{\tilde{\ell}_a}^2$

$$\tilde{w}_{\gamma h}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{98}{15}}, \tilde{w}'_{ZZ}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{47}{15}}. \quad (358)$$

The “reduced” coefficients $\tilde{a}_{\gamma h}$ and $\tilde{b}_{\gamma h}$ will be given by

$$\tilde{a}_{\gamma h} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{98}{15}} \sim 10^{-55} GeV^{-2}; \tilde{b}_{\gamma h} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{47}{15}} \sim 10^{-38} GeV^{-2}. \quad (359)$$

(d) $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh$

Now,

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh} = \tilde{w}_{hh}^{(h,H,P)} + \tilde{w}_{hh}^{(\tilde{\ell})} + \tilde{w}_{hh}^{(h,H,P-\tilde{\ell})} \quad (360)$$

where

- Higgs (h, H) exchange (+ Point interaction):

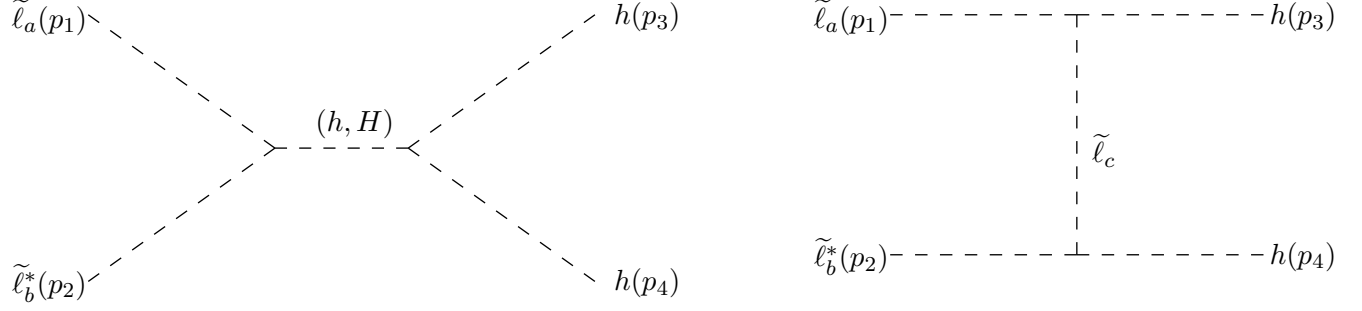


Figure 30: Feynman diagrams for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh$ via s -channel Higgs exchange and t -channel $\tilde{\ell}_c^*$ exchange.

$$\tilde{w}_{hh}^{(h,H,P)} = \left| \sum_{r=h,H} \frac{C^{hHr} C^{\tilde{\ell}_b^* \tilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\tilde{\ell}_b^* \tilde{\ell}_a hh} \right|^2. \quad (361)$$

For $m_h = 125 \text{ GeV}$, $m_H \sim \mathcal{V}^{\frac{59}{72}} m_{\frac{3}{2}}$, $m_{\chi_3^0} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}} \sim \mathcal{V}^{-\frac{4}{3}} M_p$. and $C^{hhh} \sim (\mathcal{V}^{-1})$, $C^{\tilde{\ell}_a \tilde{\ell}_b h} \sim C^{\tilde{\ell}_a \tilde{\ell}_b H} \sim \mathcal{V}^{-\frac{34}{15}}$, $C^{\tilde{\ell}_a \tilde{\ell}_b hh} \sim \mathcal{V}^{-\frac{41}{18}}$ from above, after simplifying, we have

$$\tilde{w}_{hh}^{(h,H,P)} = \left| \frac{\mathcal{V}^{-\frac{49}{15}} m_{pl}^2}{s - m_h^2 + i\Gamma_h m_h} + \frac{\mathcal{V}^{-\frac{49}{15}}}{s - m_H^2 + i\Gamma_H m_H} - \mathcal{V}^{-\frac{41}{18}} \right|^2. \quad (362)$$

- slepton ($\tilde{\ell}_c$) exchange:

$$\begin{aligned} \tilde{w}_{hh}^{(\tilde{\ell})} &= \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c H} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d H^*} \mathcal{T}_0^t + \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a H} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} C^{\tilde{\ell}_d^* \tilde{\ell}_a H^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d h^*} \mathcal{T}_0^u \\ &\quad - 2\text{Re} \sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c H} C^{\tilde{\ell}_d^* \tilde{\ell}_a H^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d h^*} \mathcal{Y}_0. \end{aligned} \quad (363)$$

Strictly speaking we are considering first generation of squarks which get identified with same modulus \mathcal{A}_1 . Therefore,

$$\sum_{c,d=1}^2 C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c H} C^{\tilde{\ell}_d^* \tilde{\ell}_a h^*} C^{\tilde{\ell}_b^* \tilde{\ell}_d H^*} \sim (\mathcal{V}^{-\frac{34}{15}})^4 \sim \mathcal{V}^{-9} M_p^4.$$

Hence

$$\tilde{w}_{hh}^{(\tilde{\ell})} = \mathcal{V}^{-9} (\mathcal{T}_0^t + \mathcal{T}_0^u - \mathcal{Y}_0) M_p^4. \quad (364)$$

The contribution of $\mathcal{T}_0^t, \mathcal{T}_0^u, \mathcal{Y}_0$ for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh$ is of the same order as $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow ZZ$.

- Higgs (h, H) (+ Point) – slepton ($\tilde{\ell}_c$) interference:

$$\tilde{w}_{hh}^{(h,H,P-\tilde{\ell})} = 2\text{Re} \sum_{c=1}^2 \left(\sum_{r=h,H} \frac{C^{hHr} C^{\tilde{\ell}_b^* \tilde{\ell}_a r}}{s - m_r^2 + i\Gamma_r m_r} - C^{\tilde{\ell}_b^* \tilde{\ell}_a hh} \right)^* \left[C^{\tilde{\ell}_c^* \tilde{\ell}_a h} C^{\tilde{\ell}_b^* \tilde{\ell}_c H} \mathcal{F}^t + C^{\tilde{\ell}_c^* \tilde{\ell}_a H} C^{\tilde{\ell}_b^* \tilde{\ell}_c h} \mathcal{F}^u \right], \quad (365)$$

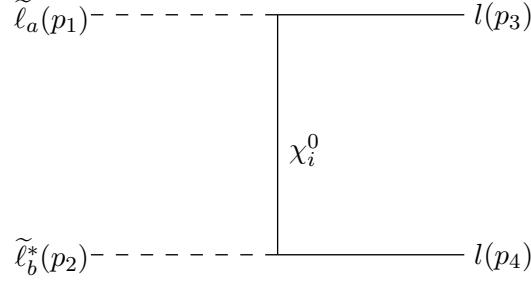


Figure 31: Feynman diagram for $\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow hh$ via t-channel $\tilde{\ell}_c^*$ exchange.

which expanding and incorporating in the values for the couplings,

$$\begin{aligned} \tilde{w}_{hh}^{(h,H,P-\tilde{\ell})} &= 2\mathcal{V}^{-\frac{68}{15}} \left(\frac{\mathcal{V}^{-\frac{49}{15}} M_p^2}{s - m_r^2 + i\Gamma_r m_r} + \frac{\mathcal{V}^{-\frac{49}{15}}}{s - m_r^2 + i\Gamma_r m_r} - \mathcal{V}^{-\frac{41}{18}} \right)^* [\mathcal{F}^t + \mathcal{F}^u] \\ \text{where } \mathcal{F}^t = \mathcal{F}^u &\equiv \frac{\log \left(\frac{-4\mathcal{V}^2 - 2\mathcal{V}^{2/5} + s - \sqrt{s - 4\mathcal{V}^{2/5}} \sqrt{s - 12\mathcal{V}^2}}{-4\mathcal{V}^2 - 2\mathcal{V}^{2/5} + s + \sqrt{s - 4\mathcal{V}^{2/5}} \sqrt{s - 12\mathcal{V}^2}} \right)}{\sqrt{s - 4\mathcal{V}^{2/5}} \sqrt{s - 12\mathcal{V}^2}}. \end{aligned} \quad (366)$$

Summing up the contribution of \tilde{w} for all channels:

$$\tilde{w}_{hh}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{8}{15}}, \tilde{w}'_{hh}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{8}{15}}, \quad (367)$$

and the “reduced” coefficients \tilde{a}_{hh} and \tilde{b}_{hh} will be given by

$$\tilde{a}_{hh} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{8}{15}} \sim 10^{-25} \text{GeV}^{-2}; \tilde{b}_{hh} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{8}{15}} \sim 10^{-25} \text{GeV}^{-2} \quad (368)$$

(e) $\tilde{\ell}_a \tilde{\ell}_b \rightarrow \ell\ell$

This process proceeds only t - and u -channel neutralino (χ_i^0 , $i = 1, 2, 3$) exchange ($\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b \rightarrow \ell\ell}^{(\chi_i^0)} = \tilde{w}_{\ell\ell}^{(\chi_i^0)}$).

• neutralino (χ_i^0) exchange:

$$\begin{aligned} \tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b \rightarrow \ell\ell}^{(\chi_i^0)} &= \sum_{i,j=1}^3 \left[D_{LLLL}^{abij} m_{\chi_i^0} m_{\chi_j^0} (s - 2m_{\tilde{\ell}}^2) \mathcal{T}_0 - 2m_{\tilde{\ell}}^2 [D_{LLRR}^{abij} m_{\chi_i^0} m_{\chi_j^0} \mathcal{T}_0 + D_{LRRL}^{abij} \mathcal{T}_1] + D_{LRLR}^{abij} [-\mathcal{T}_2 - (s - m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2) \mathcal{T}_1 \right. \\ &\quad \left. - (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}}^2) \mathcal{T}_0] - m_{\ell} m_{\chi_i^0} [D_{LLLL}^{abij} \{\mathcal{T}_1 - (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}}^2) \mathcal{T}_0\} + D_{LLRL}^{abij} \{\mathcal{T}_1 - (m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}}^2) \mathcal{T}_0\}] \right. \\ &\quad \left. - m_{\ell} m_{\chi_j^0} [D_{LRLR}^{abij} \{\mathcal{T}_1 - (m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}}^2) \mathcal{T}_0\} + D_{RLLL}^{abij} \{\mathcal{T}_1 - (m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}}^2) \mathcal{T}_0\}] \right] \\ &+ \frac{1}{2} \sum_{i,j=1}^3 \left[-2D_{LLLL}^{abij} m_{\chi_i^0} m_{\chi_j^0} (s - 2m_{\tilde{\ell}}^2) \mathcal{Y}_0 + 4D_{LLRR}^{abij} m_{\chi_i^0} m_{\chi_j^0} m_{\tilde{\ell}}^2 \mathcal{Y}_0 - 2D_{LRLR}^{abij} m_{\tilde{\ell}}^2 (m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 2m_{\tilde{\ell}}^2) \mathcal{Y}_0 \right. \\ &\quad \left. + 2D_{LRRL}^{abij} [-\mathcal{Y}_2 - (m_{\tilde{\ell}_a}^2 m_{\tilde{\ell}_b}^2 - m_{\tilde{\ell}}^4) \mathcal{Y}_0] - m_{\ell} m_{\chi_i^0} [D_{LLLR}^{abij} \{\mathcal{Y}_1 + (s + m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 - 4m_{\tilde{\ell}}^2) \mathcal{Y}_0\} \right. \\ &\quad \left. + D_{LLRL}^{abij} \{\mathcal{Y}_1 + (s - m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 4m_{\tilde{\ell}}^2) \mathcal{Y}_0\}] + m_{\ell} m_{\chi_j^0} [D_{LRLR}^{abij} \{\mathcal{Y}_1 - (s + m_{\tilde{\ell}_a}^2 - m_{\tilde{\ell}_b}^2 - 4m_{\tilde{\ell}}^2) \mathcal{Y}_0\} \right. \\ &\quad \left. + D_{RLLL}^{abij} \{\mathcal{Y}_1 - (s - m_{\tilde{\ell}_a}^2 + m_{\tilde{\ell}_b}^2 - 4m_{\tilde{\ell}}^2) \mathcal{Y}_0\}] \right], \end{aligned} \quad (369)$$

where

$$\begin{aligned}
D_{LLLL}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_L^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_L^{\chi_j^0 \tilde{\ell}_b^* \ell} C_L^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \rightarrow R), \\
D_{LLRR}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_L^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_R^{\chi_j^0 \tilde{\ell}_b^* \ell} C_R^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{LRRR}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_R^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_R^{\chi_j^0 \tilde{\ell}_b^* \ell} C_L^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{LRLR}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_R^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_L^{\chi_j^0 \tilde{\ell}_b^* \ell} C_R^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{LLLL}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_L^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_L^{\chi_j^0 \tilde{\ell}_b^* \ell} C_R^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{LLRL}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_L^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_R^{\chi_j^0 \tilde{\ell}_b^* \ell} C_L^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{LRLR}^{abij} &= C_L^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_R^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_L^{\chi_j^0 \tilde{\ell}_b^* \ell} C_L^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R), \\
D_{RLLL}^{abij} &= C_R^{\chi_i^0 \tilde{\ell}_a^* \ell^*} C_L^{\chi_i^0 \tilde{\ell}_b^* \ell^*} C_L^{\chi_j^0 \tilde{\ell}_b^* \ell} C_L^{\chi_j^0 \tilde{\ell}_a^* \ell} + (L \leftrightarrow R).
\end{aligned} \tag{370}$$

Given that $m_{\chi_3^0} \sim \mathcal{V}^{-\frac{4}{3}} M_p$, $m_{\chi_1^0} = m_{\chi_2^0} \sim \mathcal{V}^{-1} M_p$, $C_R^{\chi_i^0 \tilde{\ell}_a^* \ell^*} \sim \tilde{f} \mathcal{V}^{-\frac{1}{2}}$, $C_R^{\chi_i^0 \tilde{\ell}_a^* \ell^*} \sim \tilde{f} \mathcal{V}^{-\frac{12}{15}}$, by expanding summation, the above expression reduces to

$$\tilde{w}_{\tilde{\ell}_a \tilde{\ell}_b \rightarrow \ell \ell}^{(\chi^0)} \sim \frac{\sqrt{s} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}} \left(8s\mathcal{V}^2 - 160\mathcal{V}^{\frac{7}{5}} \right) - 2 \left(-160\mathcal{V}^{\frac{7}{5}} \mathcal{V}^{\frac{21}{5}} + 9s^2 + s8\mathcal{V}^{\frac{31}{5}} \right) \log \left(\frac{\sqrt{s} - \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}{\sqrt{s} + \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}} \right)}{2s^{3/2} \mathcal{V}^{\frac{29}{5}} \sqrt{s - 4\mathcal{V}^{\frac{21}{5}}}}.$$

For $s = 4m_{\tilde{\ell}_a}^2$

$$\tilde{w}_{\ell \ell}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{19}{5}}, \tilde{w}'_{\ell \ell}|_{s=(4m_{\tilde{\ell}_a}^2)} \sim \mathcal{V}^{-\frac{19}{5}}. \tag{371}$$

The “reduced” coefficients $\tilde{a}_{\ell \ell}$ and $\tilde{b}_{\ell \ell}$ will be given by

$$\tilde{a}_{\ell \ell} \sim \frac{1}{32\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{19}{5}} \sim 10^{-41} \text{GeV}^{-2}; \tilde{b}_{\ell \ell} \sim \frac{3}{64\pi m_{\tilde{\ell}_a}^2} \mathcal{V}^{-\frac{19}{5}} \sim 10^{-41} \text{GeV}^{-2}. \tag{372}$$

Summing up the contribution of partial wave coefficients for each possible annihilation processes:

$$\begin{aligned}
a_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow f_1 f_2} &= \tilde{a}_{ZZ} + \tilde{a}_{hZ} + \tilde{a}_{h\gamma} + \tilde{a}_{\gamma\gamma} + \tilde{a}_{hh} + \tilde{a}_{ll} \equiv O(10)^{-9} \text{GeV}^{-2} \\
b_{\tilde{\ell}_a \tilde{\ell}_b^* \rightarrow f_1 f_2} &= \tilde{b}_{ZZ} + \tilde{b}_{hZ} + \tilde{b}_{h\gamma} + \tilde{b}_{\gamma\gamma} + \tilde{b}_{hh} + \tilde{b}_{ll} \equiv O(10)^{-9} \text{GeV}^{-2},
\end{aligned} \tag{373}$$

and

$$J(x_f) \equiv \int_0^{x_f} dx \langle \sigma v_{M\emptyset} \rangle(x) = \int_0^{x_f} dx (a + bx_f) = ax_f + b \frac{x_f^2}{2},$$

where $x = T/m_{\tilde{\ell}_a}$.

Using the analytical expression of relic abundance as given in (326)

$$\Omega_{\tilde{\ell}_a} = \frac{1}{\mu^2 \sqrt{g_*} J(x_F)} \tag{374}$$

where $\mu = 1.2 \times 10^5 \text{ GeV}$. For $J(x_f) \sim a(x_f) + b \frac{x_f^2}{2} \sim 10^{-9} x_f \text{ GeV}^{-2}$ and $x_f = \frac{1}{33}$, $\sqrt{g_*} = 9$

$$\Omega_{\tilde{\ell}_a} \sim \frac{33}{1.44 \times 10^{10} \cdot 9 \cdot 10^{-9}} \equiv 0.25, \quad (375)$$

for $m_{\frac{3}{2}} \sim 10^8 \text{ GeV}$ and $m_{\tilde{\ell}_a} \sim \mathcal{V}^{\frac{1}{2}} m_{\frac{3}{2}}$, relic abundance of gravitino will be given as :

$$\Omega_{\tilde{G}} = \Omega_{\tilde{\ell}_a} \times \frac{m_{\frac{3}{2}}}{m_{\tilde{\ell}_a}} = 0.25 \times \mathcal{V}^{-\frac{1}{2}} \sim 0.001 \text{ for } \mathcal{V} \sim 10^5. \quad (376)$$

From (328) and (376), it appears that relic abundance of gravitino turns out to be suppressed in case of slepton (NLSP) (co-)annihilations as compared to relic abundance of gravitino in case of neutralino (NLSP) annihilations, for almost similar value of thermal cross-section. The reason, as explained in [8], is based on the fact that in the context of partial wave expansion approach, neutralino annihilation, because of the Majorana nature of the neutralino, occurs in P-wave while slepton annihilation occurs in S-wave. Therefore relic abundance of sleptons gets suppressed as compared to that of neutralino by a factor of $\frac{2}{x_f}$.

6 Results and Discussion

In most of the supersymmetric models, gravitino appears as most natural candidate of dark matter, the stability of which is governed by R-parity conservation that was initially proposed to explain the non-observation of proton decays in collider physics. However in split SUSY models, the constraints appearing from R-parity conservation are relaxed because of the fact that high scalar masses help to avoid the fast proton decay. Therefore it is interesting to see if gravitino appears as a stable dark matter candidate by taking into consideration trilinear R-parity violating couplings.

Large volume Swiss-Cheese type IIB compactifications [39] have been of wide interest, both from cosmological as well as phenomenological points of view. In this paper, we have investigated, in detail, the possibility of gravitino as a viable dark matter candidate in the framework of “L(arge) V(olume) μ -split SUSY” scenario. The presented scenario includes four Wilson line moduli on the world volume of space-time filling $D7$ -branes wrapped around the “big divisor” and two position moduli of a mobile space-time filling $D3$ -brane restricted to (nearly) a special Lagrangian sub-manifold. We have shown that the $\mathcal{N} = 1$ gauged supergravity fermionic mass term corresponding to fermionic super-partners of two (\mathcal{A}_1 and \mathcal{A}_3) of the aforementioned four Wilson line moduli matches very well with the order of Dirac mass of the electron, and the $\mathcal{N} = 1$ gauged supergravity fermionic mass term corresponding to fermionic super-partners of the remaining two (\mathcal{A}_2 and \mathcal{A}_4) of the aforementioned four Wilson line moduli, matches very well with the order of Dirac masses of the first generation SM-like quarks, such that the fermionic superpartners of \mathcal{A}_1 and \mathcal{A}_3 get identified, respectively with e_L and e_R , and the fermionic superpartners of \mathcal{A}_2 and \mathcal{A}_4 get identified, respectively with the first generation quarks: u/d_L and u/d_R . In doing so we also show that the RG-flow equations for the “effective” Yukawa couplings relevant to the fermionic mass calculations, change by $\mathcal{O}(1)$ in flowing from the string to the EW scale; it has been shown in [10] that with some fine tuning, it is possible to RG flow the value of $\langle z_i \rangle$ (identified with Higgs VEV) $\sim V^{\frac{1}{36}} M_p$ at string scale to $\langle z_i \rangle \sim 246 \text{ MeV}$ at the EW scale. In fact, using the RG-flow arguments of [10], one can also show that the Weinberg-type dimension-five Majorana-mass generating operator: $\mathcal{O}(\langle z_i \rangle^2)$ coefficient in $\frac{e^{\frac{K}{2}} \partial^2 W}{\sqrt{K^2_{\bar{z}_i \bar{z}_i} K^2_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}} \left(\bar{\chi}_L^{\mathcal{A}_1} \mathcal{Z}_i \right)^2$ or in fact $\frac{e^{\frac{K}{2}} \mathcal{D}_{\bar{\mathcal{A}}_1} D_{\mathcal{A}_1} \bar{W}}}{\sqrt{K^2_{\bar{z}_i \bar{z}_i} K^2_{\mathcal{A}_1 \bar{\mathcal{A}}_1}}} \left(\bar{\chi}_L^{\mathcal{A}_1} \mathcal{Z}_i \right)^2$ produces the correct first-generation neutrino mass scale of slightly less than 1 eV for $\langle z_i \rangle \sim \mathcal{O}(1) \mathcal{V}^{\frac{1}{36}}$.

We also show that it is possible to generate the massive gauge boson mass scales corresponding to the W/Z-bosons, under a similar RG flow. Building up on this set up, we have also evaluated masses of all supersymmetric as well as soft SUSY breaking parameters obtainable from bulk F -terms in the context of gravity mediation. Using RG-flow solutions of scalar masses, similar to the ones for a single Wilson line modulus set up of [36], one can again show that values of same do not change from string scale down to EW scale. On evaluation of the soft supersymmetry breaking parameters, we find a universality in the squarks/sleptons masses and assuming non-universality in the $D3$ -brane position moduli (to be identified with the neutral components of the Higgs doublets) masses, on diagonalizing the Higgs mass matrix we obtain, at the EW scale, one light Higgs of the order 125GeV and one heavy Higgs while the Higgsino mass parameter comes out to be heavy. The lightest neutralino is predominantly gaugino(Wino/Bino)-type formed by linear a combination of gaugino and Higgsino. The mass scales of various SM and superpartners are given in table 1. After calculating the masses of various SM and their superpartners, it appears that gravitino is the Lightest Supersymmetric particle (LSP) for Calabi-Yau volume $\mathcal{V} \sim 10^5$ which motivates the query: can we have gravitino DM in gravity mediation scenarios ?

Quark mass	$M_q \sim \mathcal{O}(5)\text{MeV}$
Lepton mass	$M_l \sim \mathcal{O}(1)\text{MeV}$
Gravitino mass	$m_{\frac{3}{2}} \sim \mathcal{V}^{-\frac{n^s}{2}-1}$
Gaugino mass	$M_{\tilde{g}} \sim V^{\frac{2}{3}} m_{\frac{3}{2}}$
Neutralino mass	$M_{\chi_3^0} \sim V^{\frac{2}{3}} m_{\frac{3}{2}}$
$D3$ -brane position moduli (Higgs) mass	$m_{Z_i} \sim \mathcal{V}^{\frac{59}{72}} m_{\frac{3}{2}}$
Wilson line moduli mass	$m_{\tilde{A}_I} \sim \mathcal{V}^{\frac{1}{2}} m_{\frac{3}{2}}$ $I = 1, 2, 3, 4$
A-terms	$A_{pqr} \sim n^s \mathcal{V}^{\frac{37}{36}} m_{\frac{3}{2}}$ $\{p, q, r\} \in \{\tilde{A}_I, Z_i\}$
Physical μ -terms (Higgsino mass)	$\hat{\mu}_{Z_i Z_j} \sim \mathcal{V}^{\frac{37}{36}} m_{\frac{3}{2}}$
Physical $\hat{\mu}B$ -terms	$(\hat{\mu}B)_{Z_1 Z_2} \sim \mathcal{V}^{\frac{37}{18}} m_{\frac{3}{2}}^2$

Table 1: Mass scales of first generation of SM as well supersymmetric and soft SUSY parameters

In big bang cosmology, gravitino population depends on two kinds of mechanisms: thermal as well as non-thermal production. We assume that reheating temperature is low enough to produce the effective relic abundance of gravitino in agreement with experimental observations, therefore almost all of the gravitinos are produced by electromagnetic as well as hadronic decays of unstable NLSP. The scale of masses of various superpartners suggest that because of an $\mathcal{O}(1)$ difference between the masses of the sleptons/squarks and the lightest neutralino and given that in our calculations we have not bothered about such $\mathcal{O}(1)$ factors, both can exist as valid NLSP candidate if life-times of the same are such that they do not spoil the bounds given by Big Bang Nucleosynthesis(BBN). Based on this fact, by considering the contributions of all required couplings in $\mathcal{N} = 1$ gauged supergravity action and assuming that similar to the aforementioned effective Yukawa couplings, there is only an $\mathcal{O}(1)$ multiplicative change in the three-point interaction vertices in RG-flowing from the string to the EW

scales, in section **3.1** we have discussed in detail the leading two body decays of gaugino/neutralino $\tilde{B} \rightarrow \psi_\mu Z, \mu\gamma$ and three body $\lambda_{1,2}^0 \rightarrow \psi_\mu u\bar{u}, \psi_\mu W^+W^-$ decays and determine that life-time of these decays comes out to be too short to affect the prediction of BBN. In **3.1.2**, we have (re)calculated the tree-level as well as one-loop gluino decays into neutralino as well as Gravitino/Goldstino in four-Wilson-line-moduli set up, large life time(s) of which satisfy one of the important phenomenological features of “ μ - split SUSY” but can not be considered as an appropriate NLSP’s because late decays of the same can surely elude the constraints coming from BBN. Furthermore, since we are considering R-parity violating couplings into account, in addition to NLSP decaying into LSP, there might be chances that neutralino(NLSP) directly decays into SM particles via R-parity violating couplings and hence affect the relic abundance of gravitino produced by neutralino. To ensure that this does not happen, in **3.2**, we have studied in detail the three-body decay width of three-body decay of neutralino into SM particles ($\chi_3^0 \rightarrow u_L \bar{d}_R l_L$), which comes out to be very much less than the two- and three-body decay widths of ordinary neutralino decays into LSP. Therefore, we show that R-parity violating couplings do not affect relic abundance of gravitino. On a similar ground, we have calculated three body decay width of sleptons $\tilde{l} \rightarrow \tilde{G}l, \tilde{l} \rightarrow l'\tilde{G}V$, which similar to neutralino decays are short enough to effect bounds on BBN. In order to satisfy the requirement of DM to have life time of the order of/greater than the age of the universe, in section **4**, we have estimated the mean lifetime by calculating two-body and three-body decay widths of Gravitino decaying into SM particles, which in case of two-body decays, comes out to be 10^{17} seconds and in case of three-body decays, comes out to be 10^{21} seconds, i.e., greater than the age of the universe. The numerical estimates of various N(LSP)’s are provided in table 6. In short, the explicit calculation of life times of both, LSP and NLSP, confirms that gravitino can be considered as an appropriate dark matter candidate. However this is not the end of the story. The viable dark matter candidate should also have the right order of relic abundance in the range provided by WMAP data and other direct as well as indirect experiments. With the assumption that the next-lightest supersymmetric particle (NLSP) freezes out with its thermal relic density before decaying to the gravitino and then eventually decays to gravitino, relic density of gravitino will be completely given in terms of relic density of NLSP. Therefore, in section **5**, utilizing the partial wave expansion approach, we explicitly calculate thermal annihilation cross-section of sleptons and neutralino which ultimately give gravitino relic abundance from sleptons to be 0.001 while the same from neutralino comes out to be 0.16, almost in agreement with the value of $\Omega_C h^2$ suggested by the WMAP 7-year CMB anisotropy observation [2].

An interesting outcome of our current project is that for trilinear R-parity violating couplings λ'_{ijk} (to be interpreted as an effective Yukawa couplings in $\mathcal{N} = 1$ gauged supergravity evaluated in **4**) $\sim \mathcal{V}^{-\frac{5}{3}}$ and λ''_{ijk} (to be interpreted as an effective Yukawa couplings in $\mathcal{N} = 1$ gauged supergravity evaluated in **4**) $\sim \mathcal{V}^{-\frac{43}{30}}$, according to the analytic expression given in [37], the rough estimate of proton decay width comes out to be

$$\Gamma_{p \rightarrow e^+ \pi^0} \sim m_{proton}^5 \frac{|\lambda'_{ijk} \lambda''_{ijk}|^2}{m_{\tilde{q}_I}^4} \sim \frac{\mathcal{V}^{-6.1}}{m_{\tilde{q}_I}^4} \sim 10^{-73} GeV, \quad (377)$$

thus giving life time of about 10^{42} years, which explicitly governs the stability of proton in the presence of small R-parity violating couplings discussed in μ -split SUSY scenario.

To summarize, we conclude that the gravitino qualifies as a potential dark matter candidate in Large volume “ μ split SUSY” scenario.

Decay channels	Average life time
Gaugino decays: $\tilde{B} \rightarrow \psi_\mu Z/\gamma$	$10^{-30} s$
$\tilde{B} \xrightarrow{Z} \psi_\mu u \bar{u}$	$10^{-13} s$
$\tilde{W} \xrightarrow{\tilde{q}} \psi_\mu u \bar{u}$	$10^{-15} s$
$\tilde{W}^0 \rightarrow \psi_\mu W^+ W^-$	$10^{-66} s$
Gluino decays: $\tilde{g} \rightarrow \chi_n^0 q_I \bar{q}_J$	$10^{-5} s$
$\tilde{g} \rightarrow \tilde{\chi}_3^0 g$	$10^2 s$
$\tilde{g} \rightarrow \psi_\mu q_I \bar{q}_J$	$10^4 s$
$\tilde{g} \rightarrow \psi_\mu g$	$10^{11} s$
RPV Neutralino decay: $\chi_3^0 \rightarrow u \bar{d} e^-$	$10^1 s$
Slepton decays: $\tilde{l} \rightarrow l' G V$	$10^{-28} s$
$\tilde{l}/\tilde{q} \rightarrow l/q \psi_\mu$	$10^{-25.5} s$
Gravitino decays: $\psi_\mu \rightarrow h \nu_e$	$10^{17} s$
$\psi_\mu \rightarrow \nu \gamma, \nu Z$	$10^{21} s$
$\psi_\mu \rightarrow L_i L_j E_k^c$	$10^{21} s$
$\psi_\mu \rightarrow L_i Q_j D_k^c$	$10^{20} s$
$\psi_\mu \rightarrow U_i^c D_j^c D_k^c$	$10^{18} s$

Table 2: Life time estimates of various N(LSP) decay channels

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A Geometric Kähler Potential

The crux of the Donaldson's algorithm is that the sequence

$$\frac{1}{k\pi} \partial_i \bar{\partial}_{\bar{j}} \left(\ln \sum_{\alpha, \beta} h^{\alpha \bar{\beta}} s_\alpha \bar{s}_{\bar{\beta}} \right)$$

on $P(\{z_i\})$, in the $k \rightarrow \infty$ -limit - which in practice implies $k \sim 10$ - converges to a unique Calabi-Yau metric for the given Kähler class and complex structure; $h_{\alpha \bar{\beta}}$ is a balanced metric on the line bundle $\mathcal{O}_{P(\{z_i\})}(k)$ (with sections s_α) for any $k \geq 1$, i.e.,

$$T(h)_{\alpha \bar{\beta}} \equiv \frac{N_k}{\sum_{j=1} w_j} \sum_i \frac{s_\alpha(p_i) \overline{s_\beta(p_i)} w_i}{h^{\gamma \bar{\delta}} s_\gamma(p_i) \overline{s_\delta(p_i)}} = h_{\alpha \bar{\beta}},$$

where the weight at point p_i , $w_i \sim \frac{i^*(J_{GLSM}^3)}{\Omega \wedge \bar{\Omega}}$ with the embedding map $i : P(\{z_i\}) \hookrightarrow \mathbf{WCP}^4$ and the number of sections is denoted by N_k . The defining hypersurface of the Swiss-Cheese Calabi-Yau in the $x_2 = 1$ -coordinate patch in $\mathbf{WCP}^4[1, 1, 1, 6, 9]$ is given by:

$$1 + z_1^{18} + z_2^{18} + z_3^3 + z_4^2 - \psi z_1 z_2 z_3 z_4 - 3\phi z_1^6 z_2^6 = 0. \quad (\text{A1})$$

In the large volume limit, the above can be satisfied if, e.g., $1 + z_1^{18} + z_2^{18} \sim -z_3^3$, $z_4^2 \sim \psi z_1 z_2 z_3 z_4 + 3\phi z_1^6 z_2^6$. For $z_{1,2} \sim \mathcal{V}^{\frac{1}{36}}$, one sees the same are satisfied for $z_{3,4} \sim \mathcal{V}^{\frac{1}{6}}$ provided $\psi \mathcal{V}^{\frac{1}{18}} \sim 3\phi$. Therefore:

$$\begin{aligned} h_{1\bar{z}_i} &\sim \frac{\mathcal{V}^{\frac{1}{36}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{1\bar{z}_4} \sim \frac{\mathcal{V}^{\frac{1}{6}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{1\bar{z}_i^2} &\sim \frac{\mathcal{V}^{\frac{1}{18}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{1\bar{z}_4^2} \sim \frac{\mathcal{V}^{\frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{1\bar{z}_i \bar{z}_4} &\sim \frac{\mathcal{V}^{\frac{1}{6} + \frac{1}{36}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_i \bar{z}_j} \sim \frac{\mathcal{V}^{\frac{1}{18}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_i \bar{z}_4} &\sim \frac{\mathcal{V}^{\frac{1}{36} + \frac{1}{6}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_i \bar{z}_j \bar{z}_k} \sim \frac{\mathcal{V}^{\frac{1}{12}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_i \bar{z}_4^2} &\sim \frac{\mathcal{V}^{\frac{1}{36} + \frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_i \bar{z}_j \bar{z}_4} \sim \frac{\mathcal{V}^{\frac{1}{18} + \frac{1}{6}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_4 \bar{z}_4} &\sim \frac{\mathcal{V}^{\frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_4 \bar{z}_i^2} \sim \frac{\mathcal{V}^{\frac{1}{18} + \frac{1}{6}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_4 \bar{z}_4^2} &\sim \frac{\sqrt{\mathcal{V}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_4 \bar{z}_i \bar{z}_4} \sim \frac{\mathcal{V}^{\frac{1}{36} + \frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_i z_j \bar{z}_k \bar{z}_l} &\sim \frac{\mathcal{V}^{\frac{1}{9}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_i z_j \bar{z}_4^2} \sim \frac{\mathcal{V}^{\frac{1}{18} + \frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}} \\ h_{z_i z_j \bar{z}_k \bar{z}_4} &\sim \frac{\mathcal{V}^{\frac{1}{12} + \frac{1}{6}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_4^2 \bar{z}_4^2} \sim \frac{1}{h^{z_4^2 \bar{z}_4^2}} \\ h_{z_4^2 \bar{z}_i \bar{z}_4} &\sim \frac{\mathcal{V}^{\frac{1}{36} + \frac{1}{2}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \quad h_{z_i z_4 \bar{z}_i \bar{z}_4} \sim \frac{\mathcal{V}^{\frac{1}{18} + \frac{1}{3}}}{h^{z_4^2 \bar{z}_4^2 \mathcal{V}^{\frac{2}{3}}}}, \end{aligned} \quad (\text{A2})$$

which on being inverted gives:

$$h^{\alpha\bar{\beta}} \sim \begin{pmatrix} h^{11} & h^{1\bar{z}_i} & h^{1\bar{z}_4} & h^{1\bar{z}_i^2} & h^{1\bar{z}_4^2} & h^{1\bar{z}_i\bar{z}_4} \\ h^{z_i 1} & h^{z_i \bar{z}_j} & h^{z_i \bar{z}_4} & h^{z_i \bar{z}_j \bar{z}_k} & h^{z_i \bar{z}_4^2} & h^{z_i \bar{z}_j \bar{z}_4} \\ h^{z_4 1} & h^{z_4 \bar{z}_i} & h^{z_4 \bar{z}_4} & h^{z_4 \bar{z}_i \bar{z}_j} & h^{z_4 \bar{z}_4^2} & h^{z_4 \bar{z}_i \bar{z}_4} \\ h^{z_i z_j 1} & h^{z_i z_j \bar{z}_k} & h^{z_i z_j \bar{z}_4} & h^{z_i z_j \bar{z}_k \bar{z}_l} & h^{z_i z_j \bar{z}_4^2} & h^{z_i z_j \bar{z}_k \bar{z}_4} \\ h^{z_4^2 1} & h^{z_4^2 \bar{z}_i} & h^{z_4^2 \bar{z}_4} & h^{z_4^2 \bar{z}_i \bar{z}_j} & h^{z_4^2 \bar{z}_4^2} & h^{z_4^2 \bar{z}_i \bar{z}_4} \\ h^{z_i z_4 1} & h^{z_i z_4 \bar{z}_k} & h^{z_i z_4 \bar{z}_4} & h^{z_i z_4 \bar{z}_j \bar{z}_k} & h^{z_i z_4 \bar{z}_4^2} & h^{z_i z_4 \bar{z}_j \bar{z}_4} \end{pmatrix} \begin{pmatrix} h^{z_4^2 \bar{z}_4^2} V^{2/3} & h^{z_4^2 \bar{z}_4^2} V^{23/36} & h^{z_4^2 \bar{z}_4^2} \sqrt{V} & h^{z_4^2 \bar{z}_4^2} V^{11/18} & h^{z_4^2 \bar{z}_4^2} \sqrt[3]{V} & h^{z_4^2 \bar{z}_4^2} V^{17/36} \\ h^{z_4^2 \bar{z}_4^2} V^{23/36} & h^{z_4^2 \bar{z}_4^2} V^{11/18} & h^{z_4^2 \bar{z}_4^2} V^{17/36} & h^{z_4^2 \bar{z}_4^2} V^{7/12} & h^{z_4^2 \bar{z}_4^2} V^{11/36} & h^{z_4^2 \bar{z}_4^2} V^{4/9} \\ h^{z_4^2 \bar{z}_4^2} \sqrt{V} & h^{z_4^2 \bar{z}_4^2} V^{17/36} & h^{z_4^2 \bar{z}_4^2} \sqrt[3]{V} & h^{z_4^2 \bar{z}_4^2} V^{4/9} & h^{z_4^2 \bar{z}_4^2} \sqrt[6]{V} & h^{z_4^2 \bar{z}_4^2} V^{11/36} \\ h^{z_4^2 \bar{z}_4^2} V^{11/18} & h^{z_4^2 \bar{z}_4^2} V^{7/12} & h^{z_4^2 \bar{z}_4^2} V^{4/9} & h^{z_4^2 \bar{z}_4^2} V^{5/9} & h^{z_4^2 \bar{z}_4^2} V^{5/18} & h^{z_4^2 \bar{z}_4^2} V^{5/12} \\ h^{z_4^2 \bar{z}_4^2} \sqrt[3]{V} & h^{z_4^2 \bar{z}_4^2} V^{11/36} & h^{z_4^2 \bar{z}_4^2} \sqrt[6]{V} & h^{z_4^2 \bar{z}_4^2} V^{5/18} & h^{z_4^2 \bar{z}_4^2} & h^{z_4^2 \bar{z}_4^2} V^{5/36} \\ h^{z_4^2 \bar{z}_4^2} V^{17/36} & h^{z_4^2 \bar{z}_4^2} V^{4/9} & h^{z_4^2 \bar{z}_4^2} V^{11/36} & h^{z_4^2 \bar{z}_4^2} V^{5/12} & h^{z_4^2 \bar{z}_4^2} V^{5/36} & h^{z_4^2 \bar{z}_4^2} V^{5/18} \end{pmatrix}. \quad (\text{A3})$$

Using (A3), one hence obtains the g Kähler potential ansatz of (25).

B $K_{T_B \bar{T}_B}$, the Massive Gauge Boson mass and Soft SUSY Parameters Miscellania

The relevant term in the $\mathcal{N} = 1$ SUGRA action of [12], assuming that for multiple $D7$ -brane stacks, the non-abelian killing isometry of the moduli space gets identified with the SM gauge group, the mass squared of the W/Z -bosons are given by: $g_{T_B \bar{T}_B} (X^{T_B})^2$. Now,

$$G_{T_B \bar{T}_B} \sim \frac{1}{\sqrt{T_B + \bar{T}_B - C_{I\bar{J}} a_I \bar{a}_{\bar{J}} - \mu_3 l^2 \mathcal{V}^{\frac{1}{18}} \Sigma}} - \frac{3 \left(T_B + \bar{T}_B - C_{I\bar{J}} a_I \bar{a}_{\bar{J}} - \mu_3 l^2 \mathcal{V}^{\frac{1}{18}} \right)}{\Sigma^2}, \quad (\text{B1})$$

where $\Sigma \equiv \left(T_B + \bar{T}_B - C_{I\bar{J}} a_I \bar{a}_{\bar{J}} - \mu_3 l^2 \mathcal{V}^{\frac{1}{18}} \right)^{\frac{3}{2}} - \left(T_B + \bar{T}_B - \mu_3 l^2 \mathcal{V}^{\frac{1}{18}} \right)^{\frac{3}{2}} + \sum_{\beta} n_{\beta}^0(\dots)$. So, to get a mass of 90 GeV or so at the EW scale, one needs to show that (B1) can RG-flow down to the required very small value of the square of the same; lets call this small value as $G_{T_B \bar{T}_B}^{EW} \sim 10^{-20}$. This would imply the following relation between $\mathcal{T}_B \equiv T_B + \bar{T}_B - C_{I\bar{J}} a_I \bar{a}_{\bar{J}} - \mu_3 l^2 \mathcal{V}^{\frac{1}{18}}$ and Σ :

$$\frac{1}{\sqrt{\mathcal{T}_B}} - \frac{3\mathcal{T}_B}{\Sigma^2} = G_{T_B \bar{T}_B}^{EW}$$

$$\text{or : } \Sigma^2 - \frac{\Sigma}{\delta \sqrt{\mathcal{T}_B}} + \frac{3\mathcal{T}_B}{\Sigma} = 0, \quad (\text{B2})$$

whose valid solution is given by:

$$\Sigma = \frac{1}{2} \left(\frac{1}{G_{T_B \bar{T}_B}^{EW} \sqrt{\mathcal{T}_B}} - \sqrt{1 - \frac{12\mathcal{T}_B}{G_{T_B \bar{T}_B}^{EW}}} \right) \approx 3\mathcal{T}_B^{\frac{3}{2}} + 9\mathcal{T}_B^{\frac{7}{2}} G_{T_B \bar{T}_B}^{EW} + \mathcal{O}(G_{T_B \bar{T}_B}^{EW^2}). \quad (\text{B3})$$

The above is equivalent to:

$$\mathcal{T}_B(EW) \sim 0.6\mathcal{V}^{\frac{2}{3}}, \quad (\text{B4})$$

which can be satisfied, e.g., if

$$\langle a_I \rangle(EW) \sim \frac{1}{\mathcal{O}(1)} \langle a_I \rangle(M_S). \quad (\text{B5})$$

Now,

$$\partial_{a^I} G_{T_B \bar{T}_B} \sim -\frac{C_{I\bar{J}} \bar{a}^{\bar{J}}}{2\mathcal{T}_B^{\frac{3}{2}} \Sigma} - \frac{9C_{I\bar{J}} \bar{a}^{\bar{J}}}{2\Sigma^2} + 9\frac{\mathcal{T}_B^{\frac{3}{2}} C_{I\bar{J}} \bar{a}^{\bar{J}}}{\Sigma^3}. \quad (\text{B6})$$

Similarly

$$\partial_{z^i} G_{T_B \bar{T}_B} \sim -\frac{\mu_3 l^2 \bar{z}^{\bar{j}}}{2\mathcal{T}_B^{\frac{3}{2}} \Sigma} - \frac{9\mu_3 l^2 \bar{z}^{\bar{j}}}{2\Sigma^2} + 9\frac{\mathcal{T}_B^{\frac{3}{2}} \mu_3 l^2 \bar{z}^{\bar{j}}}{\Sigma^3}. \quad (\text{B7})$$

Therefore,

$$\begin{aligned} \bar{\partial}_{\bar{a}_{\bar{K}}} \partial_{a_I} G_{T_B \bar{T}_B} &\sim \frac{C_{I\bar{K}}}{2\mathcal{T}_B^{\frac{3}{2}} \Sigma} + \frac{3}{4} \frac{C_{I\bar{J}} \bar{a}^{\bar{J}} C_{L\bar{K}} a^L}{\mathcal{T}_B^{\frac{5}{2}}} + \frac{9}{4} \frac{C_{I\bar{J}} \bar{a}^{\bar{J}} C_{L\bar{K}} a^L}{\Sigma^2 \mathcal{T}_B} - \frac{9C_{I\bar{K}}}{2\Sigma^2} + \frac{27}{2} \frac{C_{I\bar{J}} \bar{a}^{\bar{J}} C_{L\bar{K}} a^L \sqrt{\mathcal{T}_B}}{\Sigma^3} + \frac{9C_{I\bar{K}} \mathcal{T}_B^{\frac{3}{2}}}{\Sigma^3} \\ &\quad - \frac{81}{2} \frac{\mathcal{T}_B^2 C_{I\bar{J}} \bar{a}^{\bar{J}} C_{L\bar{K}} a^L}{\Sigma^4} + \frac{27}{2} \frac{\sqrt{\mathcal{T}_B} C_{I\bar{J}} \bar{a}^{\bar{J}} C_{L\bar{K}} a^L}{\Sigma^3}. \end{aligned} \quad (\text{B8})$$

Now, for $I = K = a_1$, $C_{L\bar{a}_1} \langle a^L \rangle(M_S) \sim \mathcal{V}^{\frac{8}{9}}$; which because of (B5), implies that at the EW scale,

$$\bar{\partial}_{\bar{a}_{\bar{K}}} \partial_{a_I} G_{T_B \bar{T}_B} \Big|_{\Sigma(EW) \sim 3\mathcal{T}_B^{\frac{3}{2}}(EW), \mathcal{T}_B(EW) \sim 0.6\mathcal{V}^{\frac{2}{3}}} \sim \mathcal{V}^{-\frac{8}{9}}. \quad (\text{B9})$$

Similarly

$$\begin{aligned} \bar{\partial}_{\bar{z}_{\bar{k}}} \partial_{z_i} G_{T_B \bar{T}_B} &\sim \frac{\mu_3 l^2}{2\mathcal{T}_B^{\frac{3}{2}} \Sigma} + \frac{3}{4} \frac{(\mu_3 l^2)^2 \bar{z}^{\bar{j}} z^l}{\mathcal{T}_B^{\frac{5}{2}}} + \frac{9}{4} \frac{(\mu_3 l^2)^2 \bar{z}^{\bar{j}} z^l}{\Sigma^2 \sqrt{\mathcal{T}_B}} - \frac{\mu_3 l^2}{\Sigma^2} + \frac{27}{2} \frac{(\mu_3 l^2)^2 \bar{z}^{\bar{j}} z^L}{\Sigma^2 \mathcal{T}_B} - \frac{\mathcal{T}_B^{\frac{3}{2}} \mu_3 l^2}{\Sigma^3} \\ &\quad + 9\frac{\mathcal{T}_B^{\frac{3}{2}} \mu_3 l^2}{\Sigma^3} - \frac{81}{2} \frac{\mathcal{T}_B^2 (\mu_3 l^2)^2 \bar{z}^{\bar{j}} z^l}{\Sigma^4} + \frac{27}{2} \frac{\sqrt{\mathcal{T}_B} (\mu_3 l^2)^2 \bar{z}^{\bar{j}} z^l}{\Sigma^3} \end{aligned} \quad (\text{B10})$$

Now, for $i = k = z_1$, at the EW scale will be

$$\bar{\partial}_{\bar{z}_{\bar{k}}} \partial_{z_i} G_{T_B \bar{T}_B} \Big|_{\mathcal{T}_B(EW) \sim 0.6\mathcal{V}^{\frac{2}{3}}} \sim \frac{\mu_3 l^2}{2\mathcal{T}_B^{\frac{3}{2}} \Sigma} \Big|_{\mathcal{T}_B(EW) \sim 0.6\mathcal{V}^{\frac{2}{3}}} \sim \mathcal{V}^{-2}. \quad (\text{B11})$$

We will briefly describe evaluation of various soft supersymmetry breaking parameters in our current setup involving four Wilson line moduli. To begin with, in order to evaluate the gaugino masses, one needs to evaluate the bulk F -terms which in turn entails evaluating the bulk metric. Writing the Kähler sector of the Kähler potential as: $K \sim -2\ln \left[(\sigma_B + \bar{\sigma}_B - \gamma K_{\text{geom}})^{\frac{3}{2}} - (\sigma_S + \bar{\sigma}_S - \gamma K_{\text{rmgeom}})^{\frac{3}{2}} + \sum_{\beta \in H_2^-(CY_3)} n_{\beta}^0 \sum_{(n,m)} \cos(ink \cdot (G - \bar{G})g_s + mk \cdot (G + \bar{G})) \right]$, and working near

$\sin(ink \cdot (G - \bar{G})g_s + mk \cdot (G + \bar{G})) = 0$ - corresponding to a local minimum - generates the following

components of the bulk metric: $G_{m\bar{n}} \sim \begin{pmatrix} \mathcal{V}^{-\frac{37}{36}} & \mathcal{V}^{-\frac{59}{36}} & 0 & 0 \\ \mathcal{V}^{-\frac{59}{36}} & \mathcal{V}^{-\frac{4}{3}} & 0 & 0 \\ 0 & 0 & \mathcal{O}(1) & \mathcal{O}(1) \\ 0 & 0 & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$, which therefore produces

the following inverse: $G^{m\bar{n}} \sim \begin{pmatrix} \mathcal{V}^{\frac{37}{36}} & \mathcal{V}^{\frac{13}{18}} & 0 & 0 \\ \mathcal{V}^{\frac{13}{18}} & \mathcal{V}^{\frac{4}{3}} & 0 & 0 \\ 0 & 0 & \mathcal{O}(1) & \mathcal{O}(1) \\ 0 & 0 & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$. Given that bulk F -terms are defined as:

$F^m = e^{\frac{K}{2}} G^{m\bar{n}} D_{\bar{n}} \bar{W}$, one obtains:

$$F^{\sigma_S} \sim \mathcal{V}^{-\frac{n^s}{2} + \frac{1}{36}} M_p^2, \quad F^{\sigma_B} \sim \mathcal{V}^{-\frac{n^s}{2} - \frac{5}{18}} M_p^2, \quad F^{G^a} \sim \mathcal{V}^{-\frac{n^s}{2} - 1} M_p^2, \quad (\text{B12})$$

implying that the gaugino mass is:

$$m_{\tilde{g}} = \frac{F^m \partial_m T_B}{\text{Re} T_B} \lesssim \mathcal{V}^{\frac{2}{3}} m_{3/2}, \quad (\text{B13})$$

where the gravitino mass, $m_{3/2} \sim \mathcal{V}^{-\frac{n^s}{2} - 1} M_p$.

To calculate the mixed double derivatives of the Kähler potential (25) with respect to (31), we use (32) and:

$$\begin{aligned} \partial_{z_i} K &\sim \frac{\sqrt{\mathcal{T}_B} \mu_3 l^2 z_i}{\Sigma} - \frac{\sqrt{\mathcal{T}_S} \mu_3 l^2 z_i}{\Sigma}, \\ \partial_{z_i} \bar{\partial}_{\bar{z}_j} K &\sim \frac{\mu_3^2 l^4 z_i z_j}{\sqrt{\mathcal{T}_B} \Sigma} - \frac{\sqrt{\mathcal{T}_B} \sqrt{\mathcal{T}_S} \mu_3^2 z_i z_j}{\Sigma^2} + \frac{\sqrt{\mathcal{T}_B} \mu_3 \delta_{ij}}{\Sigma}, \\ \partial_{z_i} \bar{\partial}_{\bar{a}_I} K &\sim \frac{\mu_3 l^2 C^{J\bar{I}} a_J}{\Sigma \sqrt{\mathcal{T}_B}} - \frac{\sqrt{\mathcal{T}_B} \sqrt{\mathcal{T}_S} \mu_3 l^2 z_i C^{J\bar{I}} a_J}{\Sigma^2} + \frac{\sqrt{\mathcal{T}_S} \sqrt{\mathcal{T}_B} C^{J\bar{I}} a_J}{\Sigma^2}; \\ \partial_{a_I} K &\sim \frac{\sqrt{\mathcal{T}_B} C^{I\bar{J}} a_{\bar{J}}}{\Sigma}, \\ \partial_{a_I} \bar{\partial}_{\bar{a}_J} K &\sim \frac{C^{I\bar{K}} \bar{a}_{\bar{K}} C^{L\bar{J}} a_L}{\Sigma \sqrt{\mathcal{T}_B}} - \frac{\mathcal{T}_B C^{I\bar{K}} \bar{a}_{\bar{K}} C^{L\bar{J}} a_L}{\Sigma^2} - \frac{\sqrt{\mathcal{T}_B} C^{I\bar{J}}}{\Sigma}, \end{aligned} \quad (\text{B14})$$

we see that:

$$\begin{aligned} \partial_{z_i} \bar{\partial}_{\bar{z}_j} K &= \frac{\partial z_k}{\partial \bar{z}_i} \frac{\bar{\partial} \bar{z}_l}{\bar{\partial} \bar{z}_j} \partial_{z_k} \bar{\partial}_{\bar{z}_l} K + \frac{\partial a_I}{\partial \bar{z}_i} \frac{\bar{\partial} \bar{a}_J}{\bar{\partial} \bar{z}_j} \partial_{a_I} \bar{\partial}_{\bar{a}_J} K + \frac{\partial a_I}{\partial \bar{z}_i} \frac{\bar{\partial} \bar{z}_l}{\bar{\partial} \bar{z}_j} \partial_{a_I} \bar{\partial}_{\bar{z}_l} K + \frac{\partial z_l}{\partial \bar{z}_i} \frac{\bar{\partial} \bar{a}_J}{\bar{\partial} \bar{z}_j} \partial_{z_l} \bar{\partial}_{\bar{a}_J} K \\ &\sim \frac{\sqrt{\mathcal{T}_B} \mu_3 l^2}{\Sigma} \sim \mathcal{V}^{-\frac{2}{3}}, \end{aligned} \quad (\text{B15})$$

implying:

$$\begin{aligned}
\partial_{\sigma_B} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \frac{\mathcal{T}_B \mu_3 l^2}{\Sigma^2} \sim \mathcal{V}^{-\frac{4}{3}}, \\
\partial_{\sigma_B} \bar{\partial}_{\sigma_B} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-2} \mu_3 l^2, \\
\partial_{\sigma_B} \bar{\partial}_{\bar{\sigma}_S} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-\frac{43}{36}} \mu_3 l^2, \\
\partial_{\sigma_S} \bar{\partial}_{\bar{\sigma}_S} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-\frac{11}{12}} \mu_3 l^2, \\
\partial_{G^a} \bar{\partial}_{\sigma_{B,S}} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\Bigg|_{\sin(ink \cdot (G - \bar{G}) g_s + mk \cdot (G + \bar{G})) = 0} = 0, \\
\partial_{G^a} \bar{\partial}_{\bar{G}^b} \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\Bigg|_{\sin(ink \cdot (G - \bar{G}) g_s + mk \cdot (G + \bar{G})) = 0} \sim \mathcal{V}^{-\frac{2}{3}}.
\end{aligned} \tag{B16}$$

From (B12) and (B16), one obtains:

$$\begin{aligned}
|F^{\sigma_S}|^2 \partial_{\sigma_S} \bar{\partial}_{\bar{\sigma}_S} \ln \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-\frac{85}{36}}, \\
|F^{\sigma_B}|^2 \partial_{\sigma_B} \bar{\partial}_{\bar{\sigma}_B} \ln \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-\frac{35}{9}}, \\
F^{\sigma_S} \bar{F}^{\bar{\sigma}_B} \partial_{\sigma_S} \bar{\partial}_{\bar{\sigma}_B} \ln \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\sim \mathcal{V}^{-\frac{59}{18}}, \\
F^{G^a} \bar{F}^{\bar{G}^b} \partial_{G^a} \bar{\partial}_{\bar{G}^b} \ln \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\Bigg|_{\sin(ink \cdot (G - \bar{G}) g_s + mk \cdot (G + \bar{G})) = 0} \sim \mathcal{V}^{-4},
\end{aligned} \tag{B17}$$

implying that

$$m_{z_i} \sim \mathcal{V}^{\frac{59}{72}} m_{3/2}. \tag{B18}$$

Similarly, utilizing:

$$\begin{aligned}
\partial_{\mathcal{A}_1} \bar{\partial}_{\bar{\mathcal{A}}_1} K &= \frac{\partial a_I}{\partial \mathcal{A}_1} \frac{\bar{\partial} \bar{a}_J}{\bar{\partial} \bar{\mathcal{A}}_1} \partial_{a_I} \bar{\partial}_{\bar{a}_J} K + \frac{\partial a_I}{\partial \mathcal{A}_1} \frac{\bar{\partial} \bar{z}_j}{\bar{\partial} \bar{\mathcal{A}}_1} \partial_{a_I} \bar{\partial}_{\bar{z}_j} K + \frac{\partial z_i}{\partial \mathcal{A}_1} \frac{\bar{\partial} \bar{a}_J}{\bar{\partial} \bar{\mathcal{A}}_1} \partial_{z_i} \bar{\partial}_{\bar{a}_J} K + \frac{\partial z_i}{\partial \mathcal{A}_1} \frac{\bar{\partial} \bar{z}_j}{\bar{\partial} \bar{\mathcal{A}}_1} \partial_{z_i} \bar{\partial}_{\bar{z}_j} K \\
&\sim \frac{\mathcal{V}^{\frac{10}{9}} \sqrt{\mathcal{T}_B}}{\Sigma},
\end{aligned} \tag{B19}$$

along with:

$$\begin{aligned}
|F^{\sigma_S}|^2 \partial_{\sigma_S} \bar{\partial}_{\bar{\sigma}_S} \ln \left(\partial_{\mathcal{A}_1} \bar{\partial}_{\bar{\mathcal{A}}_1} K \right) &\sim \mathcal{V}^{-\frac{107}{36}}, \\
|F^{\sigma_B}|^2 \partial_{\sigma_B} \bar{\partial}_{\bar{\sigma}_B} \ln \left(\partial_{\mathcal{A}_1} \bar{\partial}_{\bar{\mathcal{A}}_2} K \right) &\sim \mathcal{V}^{-\frac{35}{9}}, \\
F^{\sigma_S} \bar{F}^{\bar{\sigma}_B} \partial_{\sigma_S} \bar{\partial}_{\bar{\sigma}_B} \ln \left(\partial_{\mathcal{A}_1} \bar{\partial}_{\bar{\mathcal{A}}_2} K \right) &\sim \mathcal{V}^{-\frac{35}{9}}, \\
F^{G^a} \bar{F}^{\bar{G}^b} \partial_{G^a} \bar{\partial}_{\bar{G}^b} \ln \left(\partial_{z_i} \bar{\partial}_{\bar{z}_j} K \right) &\Bigg|_{\sin(ink \cdot (G - \bar{G}) g_s + mk \cdot (G + \bar{G})) = 0} \sim \mathcal{V}^{-4},
\end{aligned} \tag{B20}$$

yields:

$$m_{\mathcal{A}_1} \sim \sqrt{\mathcal{V}} m_{3/2}. \tag{B21}$$

Due to the logarithmic derivatives in the definition of the open moduli masses, in fact, one can show that there is a universality in the open moduli masses and that $m_{\mathcal{A}_I} \sim \sqrt{\mathcal{V}} m_{3/2}$.

We now show that the universality in the trilinear A -couplings that was seen in the case of the $D3$ -brane position moduli and a single Wilson line modulus in [9], is preserved even for the current four-Wilson-line moduli setup.

Using,

$$A_{\mathcal{I}\mathcal{J}\mathcal{K}} = F^m \left(\partial_m K + \partial_m \ln Y_{\mathcal{I}\mathcal{J}\mathcal{K}} + \partial_m \ln (K_{\mathcal{I}\bar{\mathcal{I}}} K_{\mathcal{J}\bar{\mathcal{J}}} K_{\mathcal{K}\bar{\mathcal{K}}}) \right), \quad (\text{B22})$$

and noting:

$$\begin{aligned} F^m \partial_m K &\sim \mathcal{V}^{-\frac{35}{18}}, \\ F^m \partial_m \ln Y_{\mathcal{Z}_i \mathcal{Z}_j \mathcal{Z}_k} &\Bigg|_{\sin(\text{ink} \cdot (G - \bar{G}) g_s + m k \cdot (G + \bar{G})) = 0} \sim \mathcal{V}^{-\frac{35}{36}}, \\ F^m \partial_m \ln (K_{\mathcal{Z}_i \bar{\mathcal{Z}}_i} K_{\mathcal{Z}_j \bar{\mathcal{Z}}_j} K_{\mathcal{Z}_k \bar{\mathcal{Z}}_k}) &\Bigg|_{\sin(\text{ink} \cdot (G - \bar{G}) g_s + m k \cdot (G + \bar{G})) = 0} \sim \mathcal{V}^{-\frac{4}{3}}, \\ F^m \partial_m \ln K_{\mathcal{A}_1 \bar{\mathcal{A}}_1} &= F^m \partial_m \ln K_{\mathcal{A}_2 \bar{\mathcal{A}}_2} = F^m \partial_m \ln K_{\mathcal{A}_3 \bar{\mathcal{A}}_3} = \mathcal{V}^{-2}, \end{aligned} \quad (\text{B23})$$

one sees that one obtains a universal trilinear A -coupling:

$$A_{\mathcal{I}\mathcal{J}\mathcal{K}} \sim \mathcal{V}^{\frac{37}{36}} m_{3/2} \sim \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}. \quad (\text{B24})$$

One can show that

$$\hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2} = \frac{e^{\frac{K}{2}} \mu_{\mathcal{Z}_1 \mathcal{Z}_2}}{\sqrt{K_{\mathcal{Z}_1 \bar{\mathcal{Z}}_1} K_{\mathcal{Z}_2 \bar{\mathcal{Z}}_2}}} \sim \mathcal{V}^{\frac{19}{18}} m_{3/2}. \quad (\text{B25})$$

Further,

$$\begin{aligned} (\hat{\mu} B)_{\mathcal{Z}_1 \mathcal{Z}_2} &= \frac{e^{-i \arg(W) + \frac{K}{2}}}{\sqrt{K_{\mathcal{Z}_1 \bar{\mathcal{Z}}_1} K_{\mathcal{Z}_2 \bar{\mathcal{Z}}_2}}} F^m \left(\partial_m K \mu_{\mathcal{Z}_1 \mathcal{Z}_2} + \partial_m \mu_{\mathcal{Z}_1 \mathcal{Z}_2} - \mu_{\mathcal{Z}_1 \mathcal{Z}_2} \partial_m \ln (K_{\mathcal{Z}_1 \bar{\mathcal{Z}}_1} K_{\mathcal{Z}_2 \bar{\mathcal{Z}}_2}) \right) \\ &\sim \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2} \left(F^m \partial_m K + F^{\sigma_S} - F^m \partial_m \ln (K_{\mathcal{Z}_1 \bar{\mathcal{Z}}_1} K_{\mathcal{Z}_2 \bar{\mathcal{Z}}_2}) \right) \\ &\sim \mathcal{V}^{\frac{19}{18} + \frac{37}{36}} m_{3/2}^2 \sim \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}^2, \end{aligned} \quad (\text{B26})$$

an observation which was very useful in obtaining a light Higgs of mass 125 GeV for a two Wilson line moduli set up of [11] - a conclusion which is still true using the same as shown below.

The Higgs masses after soft supersymmetry breaking is given by $(m_{\mathcal{Z}_i}^2 + \hat{\mu}_{\mathcal{Z}_i}^2)^{1/2}$ (where m_z 's correspond to mobile $D3$ - Brane position moduli masses (to be identified with soft Higgs scalar mass parameter)) and the Higgsino mass is given by $\hat{\mu}_{\mathcal{Z}_i}$. Had the supersymmetry been unbroken, Higgs(sino) masses would have had been degenerate with coefficient $\hat{\mu}_{\mathcal{Z}_i}$. Nevertheless we have defined SUSY breaking but we are still justified to use RG flow equation' solutions because $\hat{\mu}_{\mathcal{Z}_i} \gg m_{\mathcal{Z}_i}$. However, due to lack of universality in moduli masses but universality in trilinear A_{ijk} couplings, we need to use solution of RG flow equation for moduli masses as given in [38].

$$m_{\mathcal{Z}_1}^2(t) = m_o^2(1 + \delta_1) + m_{1/2}^2 g(t) + \frac{3}{5} S_0 p, \quad (\text{B27})$$

where

$$S_0 = \text{Tr}(Ym^2) = m_{\mathcal{Z}_2}^2 - m_{\mathcal{Z}_1}^2 + \sum_{i=1}^{n_g} (m_{\tilde{q}_L}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{l}_L}^2 + m_{\tilde{e}_R}^2) \quad (\text{B28})$$

in which all the masses are at the string scale and n_g is the number of generations. p is defined by $p = \frac{5}{66}[1 - (\frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(M_s)})]$ where $\tilde{\alpha}_1 \equiv g_1^2/(4\pi)^2$ and g_1 is the $U(1)_Y$ gauge coupling constant. Further,

$$m_{\mathcal{Z}_2}^2(t) = m_0^2 \Delta_{\mathcal{Z}_2} + m_{1/2}^2 e(t) + A_o m_{1/2} f(t) + m_o^2 h(t) - k(t) A_o^2 - \frac{3}{5} S_0 p \quad (\text{B29})$$

where $\Delta_{\mathcal{Z}_2}$ is given by

$$\Delta_{\mathcal{Z}_2} = \frac{(D_0 - 1)}{2} (\delta_2 + \delta_3 + \delta_4) + \delta_2; D_0 = 1 - 6\mathcal{Y}_t \frac{F(t)}{E(t)} \quad (\text{B30})$$

Here $\mathcal{Y}_t \equiv \hat{Y}_t^2(M_s)/(4\pi)^2$ where $\hat{Y}_t(M_s)$ is the physical top Yukawa coupling at the string scale which following [?] will be set to 0.08, and

$$E(t) = (1 + \beta_3 t)^{\frac{16}{3b_3}} (1 + \beta_2 t)^{\frac{3}{b_2}} (1 + \beta_1 t)^{\frac{13}{9b_1}} \quad (\text{B31})$$

In equation (B31) $\beta_i \equiv \alpha_i(M_s) b_i / 4\pi$ ($\alpha_1 = (5/3)\alpha_Y$), b_i are the one loop beta function coefficients defined by $(b_1, b_2, b_3) = (33/5, 1, -3)$, and $F(t) = \int_0^t E(t) dt$.

In the dilute flux approximation, $g_1^2(M_s) = g_2^2(M_s) = g_3^2(M_s)$. To ensure $E(t) \in \mathbf{R}$, the $SU(3)$ -valued $1 + \beta_3 t > 0$, which for $t = 57$ implies that $g_3^2(M_s) < \frac{(4\pi)^2}{3 \times 57} \sim \mathcal{O}(1)$. Hence, $g_3^2(M_s) = 0.4$ is what we will be using (See [11]). From (B27), the previous results in this appendix, one sees that:

$$m_{\mathcal{Z}_1}^2(M_{EW}) \sim m_{\mathcal{Z}_1}^2(M_s) + (0.39)\mathcal{V}^{\frac{4}{3}} m_{3/2}^2 + \frac{1}{22} \times \frac{19\pi}{100} \times S_0, \quad (\text{B32})$$

and

$$m_{\mathcal{Z}_2}^2(M_{EW}) \sim (0.32)\mathcal{V}^{\frac{4}{3}} m_{3/2}^2 + (-0.03)n^s \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2} \mathcal{V}^{\frac{2}{3}} m_{3/2} + (0.96)m_0^2 - (0.01)(n^s)^2 \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2} - \frac{19\pi}{2200} \times S_0, \quad (\text{B33})$$

where we used $A_{\mathcal{Z}_i \mathcal{Z}_i \mathcal{Z}_i} \sim n^s \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}$ (B24). The solution for RG flow equation for $\hat{\mu}^2$ to one loop order is given by [38]:

$$\hat{\mu}_{\mathcal{Z}_i \mathcal{Z}_i}^2 = - \left[m_0^2 C_1 + A_0^2 C_2 + m_{\frac{1}{2}}^2 C_3 + m_{\frac{1}{2}} A_0 C_4 - \frac{1}{2} M_Z^2 + \frac{19\pi}{2200} \left(\frac{\tan^2 \beta + 1}{\tan^2 \beta - 1} \right) S_0 \right], \quad (\text{B34})$$

wherein

$$C_1 = \frac{1}{\tan^2 \beta - 1} \left(1 - \frac{3D_0 - 1}{2} \tan^2 \beta \right) + \frac{1}{\tan^2 \beta - 1} \left(\delta_1 - \delta_2 \tan^2 \beta - \frac{D_0 - 1}{2} (\delta_2 + \delta_3 + \delta_4) \tan^2 \beta \right);$$

$$C_2 = -\frac{\tan^2 \beta}{\tan^2 \beta - 1} k(t); C_3 = -\frac{1}{\tan^2 \beta - 1} (g(t) - \tan^2 \beta e(t)); C_4 = -\frac{\tan^2 \beta}{\tan^2 \beta - 1} f(t), \quad (\text{B35})$$

and where the functions $e(t), f(t), g(t), k(t)$ are as defined in the appendix of [11]. In the large $\tan \beta$ (but less than 50)-limit and assuming $\delta_1 = \delta_2 = 0$, one sees that:

$$\hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}^2 \sim - \left[-m_0^2 - (0.01)(n^s)^2 \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}^2 + (0.32)\mathcal{V}^{\frac{4}{3}} m_{3/2}^2 - 1/2 M_{EW}^2 + (0.03)\mathcal{V}^{\frac{2}{3}} n^s \hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2} m_{3/2} + \frac{19\pi}{2200} S_0 \right]. \quad (\text{B36})$$

From (B33) and (B36) one therefore sees that the mass-squared of one of the two Higgs doublets, $m_{H_2}^2$, at the EW scale is given by:

$$m_{H_2}^2 = m_{Z_2}^2 + \hat{\mu}_{Z_1 Z_2}^2 = 2m_0^2 - (0.06)\mathcal{V}^{\frac{2}{3}}n^s\hat{\mu}_{Z_1 Z_2}m_{3/2} + \frac{1}{2}M_{EW}^2 - \frac{19\pi}{1100}S_0. \quad (B37)$$

From [9], we notice:

$$\mathcal{V}^{\frac{2}{3}}\hat{\mu}_{Z_1 Z_2}m_{3/2} \sim m_0^2, \quad (B38)$$

using which in (B37), one sees that for an $\mathcal{O}(1)$ n^s ,

$$m_{H_2}^2(M_{EW}) \sim 1.94m_0^2 + \frac{1}{2}M_{EW}^2 - \frac{19\pi}{1100}S_0 \quad (B39)$$

We have assumed at $m_{Z_2}^2(M_s) \sim m_0^2$ (implying $\delta_2 = 0$ but $\delta_{1,3,4} \neq 0$). Further,

$$\begin{aligned} m_{H_1}^2(M_{EW}) &= (m_{Z_1}^2 + \hat{\mu}_{Z_1 Z_2}^2)(M_{EW}) \sim m_{Z_1}^2(M_s)(1 + \delta_1) + \frac{1}{2}M_{EW}^2 + m_0^2 - (0.03)\mathcal{V}^{\frac{2}{3}}n^s\hat{\mu}_{Z_1 Z_2}m_{3/2} \\ &+ (0.01)(n^s)^2\hat{\mu}_{Z_1 Z_2}^2 \sim (1.97 + \delta_1)m_0^2 + \frac{1}{2}M_{EW}^2 + (0.01)(n^s)^2\hat{\mu}_{Z_1 Z_2}^2 \end{aligned} \quad (B40)$$

By assuming $(\hat{\mu}B)_{Z_1 Z_2} \sim \hat{\mu}_{Z_1 Z_2}^2$ - see (B26) - to be valid at the string and EW scales, the Higgs mass matrix at the EW -scale can thus be expressed as:

$$\begin{pmatrix} m_{H_1}^2 & \hat{\mu}B \\ \hat{\mu}B & m_{H_2}^2 \end{pmatrix} \sim \begin{pmatrix} m_{H_1}^2 & \xi\hat{\mu}^2 \\ \xi\hat{\mu}^2 & m_{H_2}^2 \end{pmatrix}, \quad (B41)$$

ξ being an appropriately chosen $\mathcal{O}(1)$ constant - see (B45). The eigenvalues are given by:

$$\frac{1}{2} \left(m_{H_1}^2 + m_{H_2}^2 \pm \sqrt{(m_{H_1}^2 - m_{H_2}^2)^2 + 4\xi^2\hat{\mu}^4} \right). \quad (B42)$$

As (for $\mathcal{O}(1)$ n^s)

$$\begin{aligned} m_{H_1}^2 + m_{H_2}^2 &\sim (3.91 + \delta_1)m_0^2 - 0.06S_0 + \dots \\ m_{H_1}^2 - m_{H_2}^2 &\sim (0.03 + \delta_1)m_0^2 + 0.06S_0 + \dots, \\ \hat{\mu}_{Z_1 Z_2}^2 &\sim 0.97m_0^2 - 0.03S_0 + \dots \end{aligned} \quad (B43)$$

one sees that the eigenvalues are:

$$(3.91 + \delta_1)m_0^2 - 0.06S_0 + \dots \pm \sqrt{((0.03 + \delta_1)m_0^2 + 0.06S_0 + \dots)^2 + \xi^2(1.94m_0^2 - 0.06S_0)^2}. \quad (B44)$$

Hence, assuming non universality w.r.t. to both $D3$ -brane position moduli masses ($m_{Z_{1,2}}$) and squark/slepton masses, if S_0 and ξ are fine tuned as follows:

$$(0.03 + \delta_1)m_0^2 \sim -0.06S_0 \text{ and } \xi \sim 2 + \frac{1}{8} \frac{m_{EW}^2}{m_0^2}, \quad (B45)$$

one sees that one obtains one light Higgs doublet (corresponding to the negative sign of the square root) and one heavy Higgs doublet (corresponding to the positive sign of the square root). Note, however, the squared Higgsino mass parameter $\hat{\mu}_{Z_1 Z_2}$ then turns out to be heavy with a value, at the EW scale of around $0.01\mathcal{V}m_{3/2}$ i.e to the order of squark/slepton mass squared scale which is possible in case of μ split SUSY scenario discussed above. This shows the possibility of realizing μ split SUSY scenario in the context of LVS phenomenology named as large volume “ μ -split SUSY” scenario.

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